Why do Forecasters Disagree? Lessons from the Term Structure of Cross-Sectional Dispersion*

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4 May 2010

Abstract

Using data on cross-sectional dispersion in forecasters' long- and short-run predictions of macroeconomic variables, we identify key sources of disagreement. Dispersion among forecasters is highest at long horizons where private information is of limited value and lower at short forecast horizons. Moreover, differences in views persist through time. Such differences in opinion cannot be explained by differences in information sets; our results indicate they stem from heterogeneity in priors or models. We also find evidence that differences in opinion move countercyclically, with heterogeneity being strongest during recessions where forecasters appear to place greater weight on their prior beliefs.

Keywords: Dispersion in beliefs, heterogeneous information, term structure of forecasts.

J.E.L. Codes: E37, C53, C32.

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*We thank the Editor and an anonymous referee for many constructive comments. We also thank Roy Batchelor, Steve Cecchetti, Jonas Dovern, Mike McCracken, Hashem Pesaran, Shaun Vahey, Michela Verardo, Mark Watson and seminar participants at the Board of Governors of the Federal Reserve, Cambridge, City University London, Duke, European Central Bank, London School of Economics, NBER Summer Institute, ESAM08 Meetings in Wellington, Oxford, Universite Libre Bruxelles (ECARES), Tilburg and Stanford (SITE workshop) for helpful comments and suggestions. Patton: Department of Economics, Duke University, 213 Social Sciences Building, Box 90097, Durham NC 27708-0097. Email: andrew.patton@duke.edu. Timmermann: Rady School, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0553. Email: atimmerm@ucsd.edu. Timmermann acknowledges support from CREATES, funded by the Danish National Research Foundation.

1 Introduction

Differences in agents' beliefs play an important role in macroeconomic analysis. In models where agents observe noisy private and public information, heterogeneity in beliefs has been offered as an explanation for why monetary policy shocks can have real and persistent effects on output growth due to limited capacity for processing information (Woodford (2003), Mackowiak and Wiederholt (2009)), infrequent updating of beliefs (Mankiw and Reis (2002)) or slow aggregate learning arising from dispersed information (Lorenzoni (2009)). Differences in beliefs also play a key role in determining the effect of public information signals in the literature on the social value of information where agents have a coordination motive due to the strategic complementarity of their actions (Morris and Shin (2002), Amador and Weill (2009)).[1]

While heterogeneity in agents' beliefs can be an important determinant of the "average opinion" about macroeconomic conditions, the reasons why agents disagree are not well understood. This is important since differences in agents' priors versus differences in their private information signals need not display the same degree of persistence and thus may influence macroeconomic dynamics very differently. Moreover, a better understanding of what determines heterogeneity in agents' beliefs and how this heterogeneity evolves over time can facilitate sharper tests of macroeconomic models for which subjective beliefs are a driver of economic activity. This point is highlighted by the sensitivity of some of the conclusions drawn from models with heterogeneous information to the type of signals observed by agents (e.g., Hellwig and Venkateswaran (2009)).

Hence it is important to establish empirically why agents disagree and how this disagreement evolves over time and across different states of the economy. In this paper we explore survey data on differences in agents' subjective views at several forecast horizons and develop a novel approach for comparing these to model-based predictions of forecast dispersion. This allows us to address to what extent agents disagree, whether this disagreement has diminished over time, whether the primary source of disagreement is differences in models or differences in information, and to what extent disagreements depend on the state of the economy.

We make use of a unique data set on forecasts of GDP growth and inflation for a given year recorded at different forecast horizons. Fixing the time period and varying the forecast horizon

[1] Heterogenous beliefs and information asymmetries also play an important role in models of financial markets (Brunnermeier (2001)) and foreign exchange rate models (Bacchetta and van Wincoop (2006)).

allows us to identify the source of disagreement among forecasters. This holds because heterogeneity in private signals versus heterogeneity in model priors have very different effects on the cross-sectional dispersion of beliefs at long, medium and short forecast horizons. If instead we used the conventional approach of fixing the forecast horizon and varying the time period, variations in disagreement might simply reflect changes in the volatility of the underlying variable (e.g., the "Great Moderation", McConnell and Perez-Quiros (2000)) and so the two effects would be difficult to disentangle.

Our analysis accomplishes five objectives. First, we document empirically how the dispersion among agents' beliefs varies over time as well as across different forecast horizons, whether there is any relation between "average" beliefs and dispersion in beliefs, and how persistent differences in individual agents' beliefs tend to be.

Second, we address the question from the title, namely the key sources of disagreement among forecasters. At the most basic level of analysis, agents may disagree either because of differences in their information signals or because of differences in their priors or models. Intuitively, in a stationary world differences among agents' information signals should matter most at short forecast horizons and less so at long horizons since variables will revert to their mean. Conversely, differences in prior beliefs about long-run inflation or output growth, or differences in their models of these quantities, should matter relatively more at long horizons where signals are weaker. If cross-sectional dispersion was only available for a single horizon it would not be possible to infer the relative magnitude of priors versus information signals underlying the cross-sectional dispersion. By studying the term-structure of dispersion in beliefs−i.e., differences in forecasts at long, medium and short horizons−we can therefore identify the key sources of disagreement. Empirically, we find that heterogeneity in information signals is not a major factor in explaining the cross-sectional dispersion in forecasts of GDP growth and inflation: heterogeneity in priors or models is more important.

Third, we develop an approach for comparing the observed dispersion in subjective beliefs to that implied by a simple reduced-form model (whose moments are matched as closely as possible to the survey data) for how uncertainty about macroeconomic variables evolves. Our analysis uncovers evidence of "excess dispersion" in inflation forecasts at short horizons: at horizons of less than six months the observed disagreement between agents' predictions of inflation is high relative to the degree of uncertainty about inflation implied by our model. In contrast, the benchmark model does a good job at matching the empirically observed dispersion in views about GDP growth.

Fourth, we generalize our model to incorporate the effect of economic state variables on time-variation in the (conditional) cross-sectional dispersion measured at different horizons. Theoretical models such as van Nieuwerburgh and Veldkamp (2006) suggest that macroeconomic uncertainty and dispersion in beliefs should be greater during recessions, where fewer information signals are received, than during expansions. Consistent with this, we find empirically that differences in opinion move counter-cyclically, with disagreements being larger in recessions than in expansions. Our analysis suggests that greater differences in opinion are not due to increased heterogeneity in information signals but may be related to a shift toward agents putting more weight on model-based forecasts during recessions.

Fifth, our analysis offers a variety of methodological contributions. We develop a model that incorporates heterogeneity in agents' prior beliefs and information sets while accounting for measurement errors and the overlapping nature of the forecasts for various horizons. We employ a simulation-based method of moments (SMM) framework for estimating the parameters of our model in a way that accounts for how agents update their beliefs as new information arrives. We view the shape of the cross-sectional dispersion in forecasts at different horizons as the object to be fitted and use SMM estimation to account for the complex covariance patterns arising in forecasts recorded at different (overlapping) horizons.

The plan of the paper is as follows. Section 2 takes a first look at the data. Section 3 presents our framework for modelling the evolution in the cross-sectional dispersion among forecasters across multiple forecast horizons in a way that allows for heterogeneity in agents' information and their prior beliefs. Section 4 develops our econometric approach. Empirical findings on the cross-sectional forecast dispersion are presented in Section 5 and Section 6 presents results for a model of time-varying dispersion. Section 7 concludes. Additional details on the estimation of the model are presented in a technical appendix.

2 A first look at the data

Before setting up a formal model, it is useful to take a first look at the data we will be analyzing. Our data is taken from the Consensus Economics Inc. forecasts which comprise quantitative predictions of private sector forecasters. Each month survey participants are asked for their forecasts of a range of macroeconomic and financial variables for the major economies. The number of survey respondents for the variables we study varies between 15 and 33 during our sample, with an average of 26 respondents. Our analysis focuses on US real GDP growth and CPI inflation for the current and subsequent calendar year. This gives 24 monthly next-year and current-year forecasts over the period 1991-2008 or a total of $24 \times 18 = 432$ monthly observations. We refer to $t = 1991, ..., 2008$ as the target date for the predicted variable and to $h = 1, ..., 24$ months as the forecast horizon.

To document how the spread in individual forecasters' views around the mean depends on the forecast horizon, and to see how it evolves through time, Figures 1 and 2 plot for each year in our sample the individual forecasts against the consensus (average) forecast at horizons of $h = 1, 6, 12$ and 24 months. Movements in mean forecasts from year to year tend to be very smooth at the longest forecast horizon but are more volatile at shorter forecast horizons. Conversely, the cross-sectional spread in forecasts is highest at the 24-month horizon and is sharply reduced as the horizon shrinks, with the dispersion being particularly low at the one-month horizon. Since agents' information signals can be expected to be of less value at the long horizons where disagreement seems to be greatest, these plots provide an early indication that differences in opinion are not primarily driven by differences in information.

In the heterogeneous information approach to macroeconomics differences in information are a key to the formation of "average opinion" about macroeconomic conditions. It is therefore of interest to see whether there is a relation between the mean forecast and the dispersion in beliefs. To this end we compute, for each horizon, the correlation between the consensus forecast and the dispersion in forecasts. Table 1 shows the results. For GDP growth we find a strong negative correlation−with 23 of 24 correlation estimates being negative and 14 significant at the 10% level−indicating higher dispersion in beliefs during years with low economic growth, i.e. countercyclical movements in disagreements about GDP growth. Conversely, for inflation we find a positive relation between the dispersion in beliefs and the consensus view−with 22 of 24 correlation estimates being positive and 6 being significant at the 10% level−suggesting that dispersion grows with the average expected inflation rate.

To gain further insight into the sources of differences in opinion, we study the extent to which individual forecasters are regularly above or below the mean forecast. Differences in prior beliefs might suggest persistent patterns in individual forecasters' 'optimism' or 'pessimism' relative to the average forecaster, whereas differences in private information are perhaps more suggestive of short-lived differences. As a first illustration, Figure 3 plots for all horizons the time-series average of four individual forecasters' positions in the cross-sectional distribution of forecasts (with 0.1 meaning that a forecaster is at the 10^{th} percentile of this distribution; 0.5 means the forecaster is at the median; 0.9 means the forecaster is at the 90^{th} percentile, etc.) If differences in beliefs across forecasters were short-lived, the percentiles of the individual forecasters should be tightly clustered around the median. This is not what we find, particularly at the longest horizons where some of the forecasters are consistently optimistic or pessimistic. However, as the forecast horizon gets shorter, views tend to become more densely clustered around the median, particularly for the inflation series.

To address persistence in (relative) views more systematically, we rank all individual forecasters according to whether, in a given year, t, their forecast is in the bottom, middle or top tercile. We repeat this exercise for all years in the sample and then compute transition probabilities to see whether forecasters who were in, say, the top tercile (i.e., the most "optimistic" forecasters in the case of GDP growth) in year t continue to be in the top tercile in year $t+1$. Results from this exercise, conducted separately at short, (1-12 months) and long (13-24 months) forecast horizons are reported in Table 2. In the absence of persistence in the relative views of individual forecasters, the entries in this table should all be approximately one-third (0.33). In contrast, if differences in forecasters' views persist, terms on the diagonal should be significantly higher than 0.33 and off-diagonal terms smaller than 0.33. We find strong evidence that disagreements among forecasters tend to persist: for GDP growth, at the short horizon, there is a 63% chance (nearly twice what is expected under no persistence) that the most optimistic forecasters continue to be relatively optimistic in the following period, while the most pessimistic forecasters repeat with a 45% probability. At the long forecast horizons we find even greater persistence in the relative ranking of forecasters by their degree of optimism/pessimism with repeat probabilities always above 50%. Similar conclusions hold for inflation. In all cases the estimated probabilities of remaining in the same tercile are significantly greater than 33%.

Forecasters enter and exit from our sample, and variations in the length of time a forecaster has been reporting to the survey may be a source of cross-sectional dispersion beyond the two channels that we model in the next section (differences in signals and differences in models or priors). However, this does not appear to be a main concern here: The probability of a forecaster remaining in the sample conditional on having reported in the previous month is 95% (i.e., there is only a 5% probability of leaving the sample), while the probability of remaining out of the sample if previously excluded is 0.90, so there is a 10% chance of re-entering the following month. Moreover, the cross-sectional dispersion is very similar whether calculated with or without new entrants in the sample.

To get an early indication of whether learning effects are important in the sample, we use the number of reported forecasts as a crude indicator of 'experience'. This is admittedly an imperfect measure of experience since a forecaster may have produced predictions long before being included in the Consensus Economics survey. At each point in time we sort our forecasters into two groups according to whether the number of their reported forecasts is higher or lower than the median number of reports filed up to that point. We then compute separate measures of cross-sectional dispersion among the 'most experienced' and 'least experienced' forecasters. In unreported results (available upon request) we find that the cross-sectional dispersion in the two groups is almost identical, with only mild evidence of slightly higher dispersion among the 'most experienced' group of inflation forecasters.

We derive three conclusions from this brief look at the data. First, differences in opinions among forecasters tend to be much greater at long forecast horizons than at short forecast horizons. Second, there is a systematic relationship between the cross-sectional dispersion and "average" beliefs, with differences in opinion about GDP growth varying countercyclically. Third, there is considerable persistence through time in individual forecasters' views relative to that of the median forecaster and persistence tends to be higher at the longer forecast horizons.

3 The Term Structure of Cross-sectional Dispersion

Survey data on economic forecasts has been the subject of a large literature−see Pesaran and Weale (2006) for a recent review−and many studies have found this type of data to be of high quality, e.g., Romer and Romer (2000) and Ang, Bekaert and Wei (2007). The focus of this literature has, however, mainly been on measuring the precision of average survey expectations as opposed to understanding why and by how much forecasters disagree.

Studying dispersion in beliefs at different forecast horizons turns out to provide important clues on why forecasters disagree. In fact, the importance of heterogeneity in priors can be identified primarily from the long end of the term structure of cross-sectional dispersion, while the importance of heterogeneity in signals is primarily identified from the short end of the term structure. Intuition for this comes from considering a simple AR(1) example: in such a case, the h-period forecast is simply the present state times the AR(1) coefficient raised to the appropriate power, ϕ^h. Using parameter values similar to those obtained in our empirical analysis, less than one-third of the current signal carries over after 24 months. Hence, any difference between agents' signals about the current state is not going to be very important for the long-horizon forecasts, and so disagreement in long-term forecasts must largely reflect different beliefs about the long-run mean. To formalize this intuition in a more general setting, we next turn to our model.

3.1 A model for disagreement between forecasters

We are interested in how the disagreement among forecasters about an "event" measured at a fixed time period, t, (e.g., GDP growth in 2011) changes as the forecast horizon, h, is reduced, a so-called fixed-event forecast, see Nordhaus (1987) and Clements (1997). This setup with a time-varying forecast horizon matches the focus in some theoretical models (e.g. Amador and Weill (2009)), that study how heterogeneity among agents evolves leading up to the revelation of the true value of a predicted variable.

We study how agents update their forecasts of some variable measured, e.g., at the annual frequency, when they receive news on this variable more frequently, e.g., on a monthly basis. To this end, let y_t denote the single-period variable (e.g., monthly log-first differences of GDP or a price index tracking inflation), while the rolling sum of the 12 most recent single-period observations of y is denoted z_t :

$$z_t = \sum_{j=0}^{11} y_{t-j}. \tag{1}$$

That is, y_t is the monthly variable (e.g., monthly GDP growth) and z_t is the corresponding annual variable. Our use of a variable tracking monthly changes in GDP (y_t) is simply a modelling device: US GDP figures are currently only available quarterly, but economic forecasters can be assumed to employ higher frequency data when constructing their monthly forecasts of GDP. Giannone, et al. (2008), for example, propose methods to incorporate into macroeconomic forecasts news about the economy between formal announcement dates. When we take our model to data, we focus, naturally, on those aspects of the model that have empirical counterparts. Since we shall be concerned with flow variables that forecasters gradually learn about as new information arrives prior to and during the period of their measurement, the fact that part of the outcome may be known prior to the end of the measurement period (the "event date") means that the timing of the forecasts has to be carefully considered.

We assume that agents choose their forecasts to minimize the expected value of the squared forecast error, $e_{t|t-h} \equiv z_t - \hat{z}_{t|t-h}$, where z_t is the predicted variable, $\hat{z}_{t|t-h}$ is the forecast computed at time $t-h$, t is again the event date and h is the forecast horizon. Under this loss function, the optimal $h-$period forecast is simply the conditional expectation of z_t given information at time $t-h, \mathcal{F}_{t-h}$:

$$\hat{z}^*_{t|t-h} = E[z_t|\mathcal{F}_{t-h}]. \tag{2}$$

To track the evolution in the predicted variable, we follow Patton and Timmermann (2010) and use a simple reduced-form model that, in common with popular macroeconomic models, decomposes y_t into a persistent first-order autoregressive component, x_t, and a temporary component, u_t:

$$\begin{aligned} y_t &= x_t + u_t \qquad (3) \\ x_t &= \phi x_{t-1} + \varepsilon_t, \qquad -1 < \phi < 1 \\ u_t &\sim iid\ (0, \sigma_u^2),\ \varepsilon_t \sim iid\ (0, \sigma_\varepsilon^2),\ E[u_t \varepsilon_s] = 0 \quad \text{for all } t, s. \end{aligned}$$

Here ϕ measures the persistence of x_t, while u_t and ε_t are innovations that are both serially uncorrelated and mutually uncorrelated. Without loss of generality, we assume that the unconditional mean of x_t, and thus y_t and z_t, is zero.

The advantage of using this highly parsimonious model is that it picks up the stylized fact that variables such as GDP growth and inflation clearly contain a persistent component. Unlike more structural approaches, it avoids having to take a stand on which particular variables agents use to compute their forecasts, a decision which in practice can be very complicated, see Stock and Watson (2002, 2006). The model can easily be extended to account for higher order dynamics, although given the relatively short time series we will consider, this is unlikely to be feasible in our empirical application.[2]

The model in (3) represents the data generating process for the macroeconomic variable being forecasted. To understand cross-sectional dispersion in beliefs, we next introduce heterogeneity across forecasters. We shall model disagreement between forecasters as arising from two possible

[2] For similar reasons we also ignore heteroskedasticity in the underlying data generating process, although we do not view this as being too important over the sample period studied here.

sources: differences in information signals observed by individual forecasters, or differences in their prior beliefs about, or econometric models for, long-run average levels. We define the cross-sectional dispersion among forecasters as

$$d^2_{t|t-h} \equiv \frac{1}{N_{t-h}} \sum_{i=1}^{N_{t-h}} \left(\hat{z}_{i,t|t-h} - \bar{z}_{t|t-h}\right)^2, \tag{4}$$

where $\bar{z}_{t|t-h} \equiv (1/N_{t-h}) \sum_{i=1}^{N_{t-h}} \hat{z}_{i,t|t-h}$ is the consensus forecast of z_t at time $t-h$, $\hat{z}_{i,t|t-h}$ is forecaster i's prediction of z_t at time $t-h$ and N_{t-h} is the number of forecasters at time $t-h$. Notice that $d^2_{t|t-h}$ is a measure of subjective uncertainty reflected in agents' perceptions as distinct from objective measures of risk derived, e.g., from structural or time-series forecasting models.

To capture heterogeneity in forecasters' information, we assume that each forecaster observes a different signal of the current value of y_t, denoted $\tilde{y}_{i,t}$. This assumption replicates the fact that different agents employ slightly different high-frequency variables for forming their forecast of GDP growth and inflation. Of course, many of the variables they examine will be common to all forecasters, such as government announcements of GDP growth, inflation and other key macroeconomic series, and so the signals the forecasters observe will, potentially, be highly correlated. The structure we assume is:

$$\begin{aligned} \tilde{y}_{i,t} &= y_t + \eta_t + \nu_{i,t} \\ \eta_t &\sim iid\left(0, \sigma^2_\eta\right) \ \forall\ t \\ \nu_{i,t} &\sim iid\left(0, \sigma^2_\nu\right) \ \forall\ t, i \\ E\left[\nu_{i,t}\eta_s\right] &= 0 \ \text{ for all } t, s, i. \end{aligned} \tag{5}$$

Individual forecasters' measurements of y_t are contaminated with a common source of noise, denoted η_t, representing factors such as measurement errors, and independent idiosyncratic noise, denoted $\nu_{i,t}$. Participants in the survey we use are not formally able to observe each others' forecasts for the current period but they do observe previous survey forecasts.[3] For this reason, we include a second measurement variable, $\tilde{y}_{t-1}$, which is the measured value of y_{t-1} contaminated with only the common noise:

$$\tilde{y}_{t-1} = y_{t-1} + \eta_{t-1}. \tag{6}$$

[3]As the participants in our survey are professional forecasters they may be able to observe each others' current forecasts through published versions of their forecasts, for example investment bank newsletters or recommendations. If this is possible, then we would expect to find σ_ν close to zero.

From this, the individual forecaster is able to compute an optimal forecast from the variables observable to him:

$$\hat{z}^{*}_{i,t|t-h} \equiv E\left[z_t|\mathcal{F}_{i|t-h}\right], \qquad \mathcal{F}_{i|t-h} = \{\tilde{y}_{i|t-h-j}, \tilde{y}_{t-h-1-j}\}_{j=0}^{t-h}. \tag{7}$$

Differences in signals about the predicted variable alone are unlikely to explain the observed degree of dispersion in the forecasts. The simplest way to verify this is to consider dispersion for very long horizons: as $h \to \infty$ the optimal forecasts converge towards the unconditional mean of the predicted variable. Since we assume that all forecasters use the same (true) model to update their expectations about z this implies that dispersion should asymptote to zero as $h \to \infty$. As Figures 1-2 reveal, this implication is in stark contrast with our data, which suggests instead that the cross-sectional dispersion converges to a constant but non-zero level as the forecast horizon grows. Thus there must be a second source of dispersion beyond differences in signals.[4]

We therefore consider a second source of dispersion by assuming that each forecaster comes with prior beliefs about the unconditional mean of z_t, denoted μ_i. We assume that forecaster i shrinks the optimal forecast based on his information set $\mathcal{F}_{i|t-h}$ towards his prior belief about the unconditional mean of z_t. The degree of shrinkage is governed by a parameter $\kappa^2 \geq 0$, with low values of κ^2 implying a small weight on the data-based forecast $\hat{z}^{*}_{i,t|t-h}$ (i.e., a large degree of shrinkage towards the prior belief) and large values of κ^2 implying a high weight on $\hat{z}^{*}_{i,t|t-h}$. As $\kappa^2 \to 0$, the forecaster places all weight on his prior beliefs and none on the data; as $\kappa^2 \to \infty$ the forecaster places no weight on his prior beliefs:

$$\begin{aligned} \hat{z}_{i,t|t-h} &= \omega_h \mu_i + (1-\omega_h) E\left[z_t|\mathcal{F}_{i|t-h}\right], \\ \omega_h &= \frac{E\left[e^2_{i,t|t-h}\right]}{\kappa^2 + E\left[e^2_{i,t|t-h}\right]} \\ e_{i,t|t-h} &\equiv z_t - E\left[z_t|\mathcal{F}_{i|t-h}\right]. \end{aligned} \tag{8}$$

We allow the weights placed on the prior and the conditional expectation, $E\left[z_t|\mathcal{F}_{i|t-h}\right]$, to vary across the forecast horizons in a manner consistent with standard forecast combinations: as $\hat{z}^{*}_{i,t|t-h} \equiv E\left[z_t|\mathcal{F}_{i|t-h}\right]$ becomes more accurate (i.e., as $E\left[e^2_{i,t|t-h}\right]$ decreases) the weight attached to that forecast increases. This weighting scheme lets agents put more weight on the more precise signals in

[4] While 24 months may not seem like a long forecast horizon, Lahiri and Sheng (2008b) report evidence that the 24-month and 10-year survey forecasts of real GDP growth and inflation are in fact very similar.

their short-term forecasts and less weight on signals at longer horizons. As pointed out by Lahiri and Sheng (2008a,b), the "anchoring" of long-run forecasts is a consequence of Bayesian updating.[5] Also, note that

$$\omega_h \to \frac{V[z_t]}{\kappa^2 + V[z_t]} \text{ as } h \to \infty.$$

Hence the weight on the prior in the long-run forecast can be quite large if κ^2 is small relative to $V[z_t]$. For analytical tractability, and for better finite sample identification of κ^2, we impose that κ^2 is the same across all forecasters.

Our analysis assumes that forecasters know both the form and the parameters of the data generating process for z_t but do not observe this variable. Instead they only observe $\tilde{y}_{it}$ and $\tilde{y}_{t-1}$ which are noisy estimates of $[y_t, y_{t-1}]'$. In common with many macroeconomic studies (e.g., Woodford (2003)), we further assume that agents use the Kalman filter to optimally predict (forecast, "nowcast" and "backcast") the values of y_t needed for the forecast of $z_t \equiv \Sigma_{j=0}^{11} y_{t-j}$.[6] Thus the learning problem faced by forecasters in our model relates to the latent state of the economy (measured by x_t and y_t), but not to the parameters of the model. This simplification is necessitated by our relatively short time series of data. Details on the state-space representation of the model and the forecasters' updating equations are provided in a technical appendix.

A possible interpretation of the heterogeneity in beliefs represented above by μ_i is that it captures differences in econometric models for long-run growth or inflation (for example, models with or without cointegrating relationships imposed), or it captures differences in sample periods used for the computation of their forecasts (due to, for example, differences in beliefs about the dates of structural breaks). Given the short time-series dimension of our data we are unable to distinguish between these competing interpretations.

The shrinkage of agents' forecasts towards time-invariant long-run levels, μ_i, can alternatively be motivated by uncertainty about the value of the information signals received by agents. If agents know the interpretation of signals, under very mild conditions they will eventually hold

[5] We refrain from adopting a formal Bayesian framework for the individual forecasters since individual forecasters frequently enter and exit during our sample. This makes it impossible to capture how a single forecaster updates his/her views using Bayesian updating rules. The weighting scheme we employ has an intuitive Bayesian interpretation as a combination of the prior and the data to obtain the posterior.

[6] The assumption that forecasters make efficient use of the most recent information is most appropriate for professional forecasters such as those we consider in our empirical analysis, but is less likely to hold for households which may only update their views infrequently, see Carroll (2003).

identical beliefs. A standard Bayesian model would therefore require all disagreement to eventually be driven by differences in private signals. However, as shown by Acemoglu et al. (2007), if agents are uncertain about the interpretation of the signals, they need not agree even after observing an infinite sequence of identical signals. This is important since Figures 1-3 show no evidence that agents' beliefs converge even after 18 years of observations in our sample.[7]

4 Estimation of the Model

The cross-sectional dispersion implied by our model is defined by

$$\delta_h^2 \equiv \frac{1}{N} \sum_{i=1}^{N} E\left[\left(\hat{z}_{i,t|t-h} - \bar{z}_{t|t-h}\right)^2\right]. \quad (9)$$

We use the simulated method of moments (SMM), (Gourieroux and Monfort (1996a); Hall (2005)), to match the cross-sectional dispersion implied by our model, δ_h^2, with its sample equivalent in the data given in equation (4). Unfortunately, a closed-form expression for δ_h^2 is not available and so we resort to simulations to evaluate δ_h^2. In brief, we do this by simulating the state variables for T observations, and then generating a different $\tilde{y}_{it}$ series for each of the N forecasters. For each forecaster we obtain the optimal Kalman filter forecast and then combine this with the forecaster's prior to obtain the final forecast using equation (8). We then compute the cross-sectional variance of the individual forecasts to obtain $d_{t|t-h}^2$ and average these across time to obtain δ_h^2.

Our model also yields predictions for the root mean-squared error (RMSE) of the consensus forecast, which we match to the data to pin down the parameters of the data generating process, $\left(\sigma_u^2, \sigma_\varepsilon^2, \phi\right)$. Details on these moments are presented in the technical appendix. Given our model for the term structure of dispersion in beliefs and the RMSE of the consensus forecast, all that remains is to specify a residual term for the model. Since the dispersion is measured by the cross-sectional variance, it is sensible to allow the innovation term to be heteroskedastic, with variance related to the level of the dispersion. This form of heteroskedasticity, where the cross-sectional dispersion increases with the level of the predicted variable, has been documented empirically for inflation

[7] Agents' beliefs may also fail to converge because of non-stationarities, cf. Kurz (1994). Another source of dispersion in agents' beliefs which we do not consider here is differences in the forecasters' objectives (loss function). Capistran and Timmermann (2009) consider this possibility to explain differences among agents' forecasts of US inflation measured at a given horizon and find that this can explain some of the dispersion in forecasts.

data by, e.g., Grier and Perry (1998) and Capistran and Timmermann (2009). We use the following model:

$$d^2_{t|t-h} = \delta^2_h \cdot \lambda_{t|t-h},$$
$$E\left[\lambda_{t|t-h}\right] = 1, \quad V\left[\lambda_{t|t-h}\right] = \sigma^2_\lambda \tag{10}$$

where $d^2_{t|t-h}$ is the observed value of the cross-sectional dispersion. In particular, we assume that the residual, $\lambda_{t|t-h}$, is log-normally distributed with unit mean:

$$\lambda_{t|t-h} \sim iid \ \log N\left(-\frac{1}{2}\sigma^2_\lambda, \sigma^2_\lambda\right).$$

In addition to the term structures of consensus MSE-values and cross-sectional dispersion (each yielding up to 24 moment conditions) we also include moments implied by the term structure of dispersion variances to help estimate σ^2_λ. The parameters of our model are obtained by solving the following expression:

$$\hat{\boldsymbol{\theta}}_T \equiv \arg\min_{\boldsymbol{\theta}} \ \mathbf{g}_T(\boldsymbol{\theta})' \mathbf{g}_T(\boldsymbol{\theta}), \tag{11}$$

where $\boldsymbol{\theta} \equiv \left[\sigma^2_u, \sigma^2_\varepsilon, \phi, \sigma^2_\eta, \sigma^2_\nu, \kappa^2, \sigma^2_\mu, \sigma^2_\lambda\right]'$, and, for $h = 1, 2, ..., 24$,

$$\mathbf{g}_{T,h}(\boldsymbol{\theta}) \equiv \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} e^2_{t|t-h} - MSE_h(\boldsymbol{\theta}) \\ d^2_{t|t-h} - \delta^2_h(\boldsymbol{\theta}) \\ \left(d^2_{t|t-h} - \delta^2_h(\theta)\right)^2 - \delta^4_h(\boldsymbol{\theta})\left(\exp\left(\sigma^2_\lambda\right) - 1\right) \end{bmatrix}. \tag{12}$$

In total our model generates 72 moment conditions and contains 8 unknown parameters. In practice we use only six forecast horizons ($h = 1, 3, 6, 12, 18, 24$) in the estimation, rather than the full set of 24, in response to studies of the finite-sample properties of GMM estimates (Tauchen, (1986)) which find that using many more moment conditions than required for identification leads to poor approximations from the asymptotic theory, particularly when the moments are highly correlated, as in our application.[8] We use the identity matrix as the weighting matrix in our initial SMM estimation, and then use the efficient weight matrix for the final parameter estimates and tests.

To obtain the covariance matrix of the moments in (12), used to compute standard errors and the test of over-identifying restrictions, we use the model-implied covariance matrix of the moments

[8] We have also estimated the models presented in this paper using the full set of 24 moment conditions and the results were qualitatively similar.

based on the estimated parameters. This matrix is not available in closed-form and so we simulate 50 non-overlapping years of data to estimate it, imposing that the innovations to these processes are normally distributed, and using the expressions given in the Appendix to obtain the Kalman filter forecasts.[9] As noted above, a closed-form expression for δ_h^2 is not available and so we use simulations to obtain an estimate of it. For each evaluation of the objective function, we simulate 50 non-overlapping years of data for 30 forecasters to estimate δ_h^2.[10] The priors for each of the 30 forecasters, μ_i, are simulated as $iid\ N\left(0, \sigma_\mu^2\right)$.[11] We multiply the estimated δ_h^2 series by $\lambda_{t|t-h}$, defined in equation (10) and from this we obtain 'measured' values of dispersion, $d_{t|t-h}^2 = \delta_h^2 \cdot \lambda_{t|t-h}$, and the squared dispersion residual, $\lambda_{t|t-h}^2$, which are used in the second and third set of moment conditions in (12), respectively. From these, combined with the MSE-values, we compute the sample covariance matrix of the moments.

5 Empirical Results on Forecast Disagreement

We next turn to our empirical results from the econometric analysis of the cross-sectional dispersion in the survey forecasts of GDP growth and inflation. We use revised data to measure the realized value of the target variable (GDP growth or inflation), but note that this is strongly correlated (correlation of 0.90) with the first release of the real-time series, the data recommended by Corradi, Fernandez and Swanson (2007). Our model in Section 3 assumed that the target variable is the December-on-December change in real GDP or the consumer price index, which can conveniently be written as the sum of the month-on-month changes in the log-levels of these series, as in equation (1). The Consensus Economics survey formally defines the target variable slightly differently to this but the impact of this difference on the model fit is negligible.[12]

[9] We examined the sensitivity of this estimate to changes in the size of the simulation and to re-simulating the model, and found that when 50 non-overlapping years of data are used, changes in the estimated covariance matrix are negligible.

[10] The actual number of forecasters in each survey exhibited some variation across t and h, but in the simulations we set $N = 30$ for all t, h for simplicity. Simulation variability for this choice of N and T is small, particularly relative to the values of the time-series variation in $d_{t|t-h}^2$ that we observe in the data.

[11] As a normalization we assume that $N^{-1}\sum_{i=1}^{N}\mu_i = 0$ since we cannot separately identify $N^{-1}\sum_{i=1}^{N}\mu_i$ and $\sigma_\mu^2 \equiv N^{-1}\sum_{i=1}^{N}\mu_i^2$ from our data on forecast dispersions. This normalization is reasonable if we think that the number of "optimistic" forecasters is approximately equal to the number of "pessimistic" forecasters.

[12] Generalizing the model to accommodate the exact definition of the target variable in the Consensus Economics survey involves lengthy algebra and complicates the description of the model, see Patton and Timmermann (2010)

To gain intuition for how the parameters of our constant-dispersion model are identified, notice that of those parameters, three $\left(\phi, \sigma_u^2, \sigma_\varepsilon^2\right)$ characterize the data generating process in (3). These parameters are mostly, though not solely, identified by the moments pertaining to the RMSE-values of the average forecast. In contrast, $\sigma_\eta^2, \sigma_\mu^2, \sigma_\nu^2$ and κ^2 are primarily determined by the moments capturing the term structure of cross-sectional dispersion. Figure 4 shows how the model-implied cross-sectional dispersion in beliefs varies across different horizons as each of the four parameters take on low, medium and high values.[13] The plots suggest that the parameters have very different effects on the term structure. Increases in σ_η^2 increase the dispersion at short horizons, but have little effect on long-horizon dispersion. Increases in σ_v^2 have a smaller but similar effect. Variations in σ_μ^2 lead to big shifts in the long-run dispersion in beliefs, but have little effect on short-run disagreements, while conversely variations in κ^2 lead to small variations in the long-run dispersion but imply large changes in the short-run dispersion. Thus the parameters generally have very different effects on different portions of the term structure, which helps identify their values.

Figure 5 plots the term structure of cross-sectional dispersion for GDP growth and inflation, i.e. the cross-sectional standard deviation of forecasts, averaged across the full sample, 1991-2008, listed against the forecast horizon. The cross-sectional dispersion of output growth is only slowly reduced for horizons in excess of 12 months, but declines rapidly for $h < 12$ months from 0.38 at the 12-month horizon to 0.07 at the 1-month horizon. For inflation, again there is a systematic reduction in the dispersion as the forecast horizon shrinks. The cross-sectional dispersion declines from 0.44 at the 24-month horizon to 0.32 at the 12-month horizon and 0.08 at the 1-month horizon.

It is interesting to contrast the pattern in Figure 5 with the cross-sectional dispersion implied by the analysis in Amador and Weill (2009). In their model, more precise public information leads agents to rely less on private information and so slows down learning, crowding out valuable private information. Provided that prior beliefs are sufficiently dispersed, initially agents put increasingly more weight on their private information, leading to a convex segment of the aggregate learning curve. Subsequently, the learning curve becomes concave due to the fact that the true state is eventually revealed. Hence information diffuses over time along an S-shaped curve, while the cross-

for details.

[13] The "medium" value of each of these parameters (except for σ_ν) corresponds to the fitted value for GDP growth (reported in Table 3 below), and the high/low values are the fitted values ± 2 times the standard errors. The fitted value for σ_ν was very near zero, and so for that parameter we use that as the "low" value and obtain medium and high values as the fitted value $+2$ and $+4$ standard errors.

sectional dispersion in beliefs converges towards zero along a hump-shaped curve, i.e. it starts low, then increases monotonically, reaches a peak before decreasing towards zero. While our data is consistent with the reduced dispersion found at short forecast horizons, the very high dispersion observed at the longest horizons is clearly at odds with this type of learning model.

5.1 Parameter estimates and hypothesis tests

Table 3 reports parameter estimates for the model based on the moments in (12). For both GDP growth and inflation the estimates of σ_μ and κ suggest considerable heterogeneity across forecasters in our panel. Conversely, the estimates of σ_ν suggest that differences in individual signals may not be important.[14] Indeed, for GDP growth the test statistics for σ_ν and σ_μ are 0 and 12.98, respectively, while for inflation the test statistics are 0.001 and 2.97. Thus for both series we fail to reject the null that $\sigma_\nu = 0$, while we are able to reject the null that $\sigma_\mu = 0$ at the 5% level. Heterogeneity in signals about GDP growth and inflation therefore does *not* appear to be a significant source of disagreement among professional forecasters, whereas heterogeneity in beliefs about the long-run levels of GDP growth and inflation is strongly significant.

Our tests of the over-identifying restrictions indicate that the model provides a good fit to the GDP growth consensus forecast and forecast dispersion, with the J-test p-value being 0.77. Moreover, the top panel of Figure 5 confirms that the model provides a close fit to the empirical term structure of forecast dispersions. This panel also shows that the model with σ_ν set to zero provides an almost identical fit to the model with this parameter freely estimated, consistent with the test results for this hypothesis. Thus differences in individual information about GDP growth, modelled by ν_{it}, do not appear important for explaining forecast dispersion; the most important features are the differences in prior beliefs about long-run GDP growth and the accuracy of Kalman filter-based forecasts (as they affect the weight given to the prior relative to the Kalman filter forecast).

In sharp contrast, the model for inflation forecasts and dispersions is rejected by the test of

[14] Testing the null that σ_ν (or σ_μ) is zero against it being strictly positive is complicated by the fact that zero is the boundary of the support for this parameter, which means that standard t-tests are not applicable. In such cases the squared t-statistic no longer has an asymptotic χ^2_1 distribution under the null, rather it will be distributed as a mixture of a χ^2_1 and a χ^2_0, see, e.g., Gourieroux and Monfort (1996b, Chapter 21), and the 90% and 95% critical values for this distribution are 1.64 and 2.71.

over-identifying restrictions (see the row labeled 'J p-val' in Table 3). The model fits dispersion well for horizons greater than 12 months, but for horizons less than 4 months the observed dispersion is above what is predicted by our model. Given the functional form specified for the weight attached to the prior belief about long-run inflation versus the Kalman filter-based forecast, the model predicts that each forecaster will place 95.0% and 99.1% weight on the Kalman filter-based forecast for horizons of $h = 3$ and $h = 1$ month, and since the Kalman filter forecasts are very similar across forecasters at short horizons our model predicts that dispersion will be low.

Observed inflation forecast dispersion is high relative both to the predictions of our model, and to observed forecast errors: observed dispersion (in standard deviations) for horizons $h = 3$ and $h = 1$ are 0.11 and 0.07, compared with the RMSE of the consensus forecast at these horizons of 0.19 and 0.08. Contrast this with the corresponding figures for the GDP forecasts, with dispersions of 0.13 and 0.07 and RMSE-values of 0.34 and 0.30. Thus, the dispersion of inflation forecasts is around 69% as large as the RMSE of the consensus forecast for short horizons ($1 \leq h \leq 6$), whereas the dispersion of GDP growth forecasts is around 35% as large as the RMSE of the consensus forecast.

To further illustrate this point, Figure 6 plots the observed ratio of dispersion to RMSE, along with the predicted ratios, for horizons ranging from 24 months to 1 month, for both GDP growth and inflation. The upper panel of this plot reveals that our model is able to capture the basic shape of this function for GDP growth, while the lower panel shows how this ratio diverges for short horizons, especially the one-month horizon, and is not described well by our model. Patton and Timmermann (2010) show that this model fits the RMSE term structure well, and so the divergence of the observed data from our model is not due to a poor model for the RMSE. The upward sloping function for the dispersion-to-RMSE ratio is difficult to explain within the confines of our model, or indeed any model assuming a quadratic penalty for forecast errors and efficient use of information, and thus poses a puzzle.

6 Time-varying dispersion

There is a growing amount of theoretical and empirical work on the relationship between the uncertainty facing economic agents and the economic environment. van Nieuwerburgh and Veldkamp (2006), Veldkamp (2006) and Veldkamp and Wolfers (2007) propose endogenous information models where agents' participation in economic activity leads to more precise information about unobserved economic state variables such as (aggregate) technology shocks. In these models the number of signals observed by agents is proportional to the economy's activity level so more information is gathered in a good state of the economy than in a bad state. Recessions are therefore times of greater uncertainty which in turn means that dispersion among agents' forecasts can be expected to be wider during such periods. Similarly, Mackowiak and Wiederholt (2009) show that an increase in the variance of nominal aggregate demand leads firms to pay more attention to aggregate activity and less to idiosyncratic conditions. This could lead to a decrease in the cross-sectional dispersion in beliefs about the aggregate nominal demand. Thus, changing volatility in the variance of nominal aggregate demand−e.g., around turning points of the business cycle−can lead to time-varying cross-sectional dispersion.

To address such issues, we generalize our model to allow forecast dispersion to vary over the business cycle. There are of course many variables that vary with the business cycle that we could use in our empirical model for time-varying dispersion. We simply employ the default spread (the difference in average yields of corporate bonds rated by Moody's as BAA vs. AAA), which is known to be strongly counter-cyclical, increasing during economic downturns. Over our sample period the default spread ranges from 55 basis points in 1995, 1997 and 2000, to 338 basis points in December 2008.

6.1 Time-varying differences in beliefs

As a simple, robust way to explore the relation between disagreements among forecasters and the state of the economy, we first estimate a pooled regression of the logarithm of the cross-sectional dispersion on the logarithm of the default spread and separate horizon dummies, α_h, i.e.,

$$\log(d^2_{t|t-h}) = \alpha_h + \beta_{SPR} \log(S_{t-h}) + \varepsilon_{t-h},\ t = 1, 2, ..., T;\ h = 1, 2, ..., 24, \tag{13}$$

where S_{t-h} is the default spread in month $t-h$. This approach is robust in the sense that it does not impose our model, but conversely also does not reveal the source of any cyclical variations in belief dispersion. Panel A in Table 4 reports the estimated coefficients. For GDP growth the estimate of β_{SPR} is 0.43 with a t-statistic around four. For inflation the estimate of β_{SPR} is 0.78 and the t-statistic exceeds nine. Thus dispersion in output growth and inflation expectations is significantly higher during economic recessions than during upturns.[15]

We next explore how counter-cyclical movements in the cross-sectional dispersion can be introduced in the context of our model. The most natural way to allow the default spread to influence dispersion in our model is through the variance of the individual signals received by the forecasters, σ_ν^2, or through κ^2 which determines how much weight forecasters put on their data-based forecast relative to their long-run model forecast. Given that the former variable explained very little of the (unconditional) dispersion term structure, we focus on the latter channel. We specify our model as

$$\log(\kappa_t^2) = \beta_0^\kappa + \beta_1^\kappa \log(S_t). \tag{14}$$

In this model, if $\beta_1^\kappa < 0$, then increases in the default spread coincide with periods where forecasters put less emphasis on their signals and more weight on their long-run models. Since we estimate the main source of differences in beliefs to be attributable to differences in models or priors, a negative value of β_1^κ would indicate that periods with increased default spreads coincide with periods with greater dispersion.

Leaving the rest of the model unchanged, the model with time-varying dispersion is estimated in a similar way to the model with constant dispersion, with the following modifications. We use the stationary bootstrap of Politis and Romano (1994), with average block length of 12 months, to "stretch" the default spread time series, S_t, to be 50 years in length for the simulation. This maintains, asymptotically, the properties of this process and allows us to simulate longer time series than we have in our data set. Following this step the remainder of the simulation is the same as for the constant dispersion case above, noting that the combination weights applied to the Kalman-filter forecast and the "prior" are now time-varying as κ_t^2 is time-varying. In the estimation stage we need to compute the value of $\delta_h^2\left(\kappa_t^2\right)$, so that we can compute the dispersion residual. In the constant dispersion model, this is simply the mean of $d_{t|t-h}^2$, but in the time-varying dispersion model this depends on κ_t^2. It is not computationally feasible to simulate $\delta_h^2\left(\kappa_t^2\right)$ for each unique value of κ_t^2 in our sample, and so we estimate it for κ_t^2 equal to its sample minimum, maximum and its $[0.25, 0.5, 0.75]$ sample quantiles, and then use a cubic spline to interpolate this function, obtaining $\tilde{\delta}_h^2\left(\kappa_t^2\right)$. We check the accuracy of this approximation for values in between these nodes and the errors are very small. We then use $\tilde{\delta}_h^2\left(\kappa_t^2\right)$, and the data, to compute the dispersion

[15] These findings are consistent with the work of Döpke and Fritsche (2006) for a panel of forecasters for Germany over a different sample period.

residuals and use these in the SMM estimation of the parameters of the model.

Empirical results for this model are presented in Panel B of Table 4. Consistent with the work of Veldkamp (2006) and van Nieuwerburgh and Veldkamp (2006), for GDP growth the negative sign of $\hat{\beta}_1^{\kappa}$ implies that when spreads are high, forecasters rely less on (common) information and disagree more. Moreover, the estimate $\hat{\beta}_1^{\kappa}$ is significant at the 10% level and, as indicated by the last row, the model is not rejected. For inflation forecasts, in contrast, this parameter estimate is positive but not significantly different from zero and the model is rejected.

To see the implications of our analysis for the time series of cross-sectional dispersion, Figure 7 plots the actual dispersion versus the fitted (model-implied) dispersion at horizons of $h = 24$ and 12 months. For GDP growth the model implies considerable variation in disagreement among forecasters with dispersions increasing markedly during economic downturns as tracked by the default spread. Moreover, the model nicely tracks time-variations in the cross-sectional dispersion of GDP growth with large positive correlations between the model-implied and actual dispersion. Unsurprisingly, given the poor fit of the inflation model, for the inflation series the model fails to similarly match time-variations in the actual dispersion.

7 Conclusion

This paper analyzed the degree of heterogeneity in forecasters' opinions, the nature and source of differences in opinions, and how such differences evolve over the economic cycle. We found that differences in agents' forecasts of macroeconomic variables such as GDP growth and inflation tend to be much greater at long forecast horizons up to two years compared with short horizons of a few months. Moreover, such differences in opinion tend to persist through time. To understand these patterns, we developed a simple, parsimonious model for the cross-sectional dispersion among forecasters. Our analysis allows for heterogeneity in forecasters' information signals and in their prior beliefs or models.

Our analysis reveals several puzzling features that may be difficult to explain with simple and popular forecasting models of the type used by macroeconomists. First, our empirical results suggest that heterogeneity in forecasters' information signals is not a major factor in explaining the cross-sectional dispersion in forecasts of GDP growth and inflation: heterogeneity in priors or models is more important. Second, the dispersion of forecasts does not appear to fall through time, suggesting that beliefs do not converge. Third, we find that forecasters' views of inflation at short horizons display "excess dispersion" that cannot be matched by our model and seems far greater than one would expect from differences in either prior beliefs or information signals. Fourth, and finally, our analysis shows that differences in opinions about GDP growth or inflation move strongly counter-cyclically, increasing during bad states of the world, although such variations again do not appear to be driven by heterogeneity in signals.

Technical Appendix

This appendix provides details of how we derive the moments used in the empirical estimation in Section 3. We first introduce the state and measurement equations underlying the model from Section 3 cast in state space form and then show how the forecasters' updating equations can be solved.

A.1. State and Measurement Equations

Our model involves unobserved variables and so we cast it in state space form, using notation similar to that in Hamilton (1994). To account for the way the target variable is constructed, $z_t \equiv \Sigma_{j=0}^{11} y_{t-j}$, we augment the state equation with eleven lags of y_t so the target variable can be written as a linear combination of the state variable. The state equation is

$$\begin{bmatrix} x_t \\ y_t \\ y_{t-1} \\ \vdots \\ y_{t-11} \end{bmatrix} = \begin{bmatrix} \phi & 0 & 0 & \cdots & 0 \\ \phi & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{t-12} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t + u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{15}$$

which we write as

$$\boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \mathbf{v}_t. \tag{16}$$

The measurement equation involves two variables: the estimate of y_t incorporating both common and idiosyncratic measurement error, and the estimate of y_{t-1} incorporating just common measurement error. In a minor abuse of notation relative to our discussion of this model in Section 3, we will call the former $\tilde{y}_{it}^*$ and the latter $\tilde{y}_{c,t-1}$, so that we may stack them into a vector $\tilde{\mathbf{y}}_{it}$:

$$\begin{bmatrix} \tilde{y}_{it}^* \\ \tilde{y}_{c,t-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ y_{t-1} \\ \vdots \\ y_{t-11} \end{bmatrix} + \begin{bmatrix} \eta_t + \nu_{it} \\ \varphi_{t-1} \end{bmatrix} \tag{17}$$

which we write as

$$\tilde{\mathbf{y}}_{it} = \mathbf{H}'\boldsymbol{\xi}_t + \mathbf{w}_{it},$$

We introduce the measurement error φ_{t-1}, distinct from η_t but with the same distribution, so that the vector $\mathbf{w}_{it}$ remains serially uncorrelated which simplifies the model.

The various shocks in the state and measurement equations are distributed as:

$$(u_t, \varepsilon_t, \eta_t, \varphi_t, \nu_{1t}, ..., \nu_{NT})' \sim iid\ N\left(\mathbf{0}, diag\left\{\left(\sigma_u^2, \sigma_\varepsilon^2, \sigma_\eta^2, \sigma_\eta^2, \sigma_\nu^2, ..., \sigma_\nu^2\right)\right\}\right)$$

where $diag\{\mathbf{a}\}$ is a square diagonal matrix with the vector $\mathbf{a}$ on the main diagonal. Then $\mathbf{v}_t \sim iid\ N(0, \mathbf{Q})$, with

$$\mathbf{Q} = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_\varepsilon^2 & 0 & \cdots & 0 \\ \sigma_\varepsilon^2 & \sigma_\varepsilon^2 + \sigma_u^2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

And finally $\mathbf{w}_{it} \sim iid\ N(0, \mathbf{R})$, with

$$\mathbf{R} = \begin{bmatrix} \sigma_\eta^2 + \sigma_\nu^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix}.$$

Notice that by extending the state variable to include lags of $\mathbf{y}_t$ we do not need to treat forecasts, nowcasts and backcasts separately; they can all be treated simultaneously as "forecasts" of the state vector $\boldsymbol{\xi}_t$. This simplifies the algebra considerably.

A.2. The Forecasters' Updating Process

Our empirical data provides us with estimates of forecast uncertainty at different forecast horizons measured both in the form of the root mean squared forecast error (RMSE) of the "average" or consensus forecast or in the form of the cross-sectional standard deviation of the forecasts (i.e., the dispersion). We next characterize how the forecasters update their beliefs and derive the model-implied counterparts of these two measures of uncertainty and disagreement.

Let

$$\begin{aligned} \mathcal{F}_{i,t} &= \sigma\left(\tilde{\mathbf{y}}_{it}, \tilde{\mathbf{y}}_{i,t-1}, ..., \tilde{\mathbf{y}}_{i,1}\right) \\ \hat{\boldsymbol{\xi}}_{i,t|t-h} &\equiv E\left[\boldsymbol{\xi}_t | \mathcal{F}_{i|t-h}\right], \quad h \geq 0, \end{aligned}$$

where the expectation is obtained using standard Kalman filtering methods.

We assume that the forecasters have been using the Kalman filter long enough that all updating matrices, defined below, are at their steady-state values. This is done simply to remove any "start of sample" effects that may or may not be present in our actual data. Following Hamilton (1994):

$$\begin{aligned} \mathbf{P}_{i,t+1|t} &\equiv E\left[\left(\boldsymbol{\xi}_{t+1}-\hat{\boldsymbol{\xi}}_{i,t+1|t}\right)\left(\boldsymbol{\xi}_{t+1}-\hat{\boldsymbol{\xi}}_{i,t+1|t}\right)'\right] \\ &= \left(\mathbf{F}-\mathbf{K}_{i,t}\right)\mathbf{P}_{i,t|t-1}\left(\mathbf{F}'-\mathbf{K}'_{i,t}\right)+\mathbf{K}_{i,t}\mathbf{R}\mathbf{K}'_{i,t}+\mathbf{Q} \\ &\rightarrow \mathbf{P}_1^*. \end{aligned} \tag{18}$$

Note that although the individual forecasters receive different signals, and thus generate different forecasts $\hat{\boldsymbol{\xi}}_{i,t+1|t}$, all signals have the same covariance structure and so will converge to the same matrix, $\mathbf{P}_1^*$. Similarly,[16]

$$\begin{aligned} \mathbf{K}_{i,t} &\equiv \mathbf{F}\mathbf{P}_{i,t|t-1}\left(\mathbf{P}_{i,t|t-1}+\mathbf{R}\right)^{-1} \rightarrow \mathbf{K}^*, \\ \mathbf{P}_{i,t|t} &\equiv E\left[\left(\boldsymbol{\xi}_t-\hat{\boldsymbol{\xi}}_{i,t|t}\right)\left(\boldsymbol{\xi}_t-\hat{\boldsymbol{\xi}}_{i,t|t}\right)'\right] \\ &= \mathbf{P}_{i,t|t-1}-\mathbf{P}_{i,t|t-1}\left(\mathbf{P}_{i,t|t-1}+\mathbf{R}\right)^{-1}\mathbf{P}_{i,t|t-1} \\ &\rightarrow \mathbf{P}_1^*-\mathbf{P}_1^*\left(\mathbf{P}_1^*+\mathbf{R}\right)^{-1}\mathbf{P}_1^* \equiv \mathbf{P}_0^*. \end{aligned} \tag{19}$$

To estimate the matrices $\mathbf{P}_1^*$, $\mathbf{P}_0^*$, and $\mathbf{K}^*$, we simulate 100 non-overlapping years of data and update $\mathbf{P}_{i,t|t-1}$, $\mathbf{P}_{i,t|t}$ and $\mathbf{K}_{i,t}$ using the above equations. We use these matrices at the end of the 100^{th} year as estimates of $\mathbf{P}_1^*$, $\mathbf{P}_0^*$, and $\mathbf{K}^*$. Multi-step prediction error uses

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{i,t+h|t} &= \mathbf{F}^h\hat{\boldsymbol{\xi}}_{i,t|t}, \\ \text{so } \mathbf{P}_{i,t+h|t} &\equiv E\left[\left(\boldsymbol{\xi}_{t+h}-\hat{\boldsymbol{\xi}}_{i,t+h|t}\right)\left(\boldsymbol{\xi}_{t+h}-\hat{\boldsymbol{\xi}}_{i,t+h|t}\right)'\right] \\ &= \mathbf{F}^h\mathbf{P}_{i,t|t}\left(\mathbf{F}'\right)^h+\sum_{j=0}^{h-1}\mathbf{F}^j\mathbf{Q}\left(\mathbf{F}'\right)^j \rightarrow \mathbf{P}_h^*, \text{ for } h \geq 1. \end{aligned} \tag{20}$$

The matrices $\mathbf{P}_h^*$ for $h = 1, 2, ..., 24$ are sufficient for us to obtain the term structure of RMSE, (that is, the RMSE-values across different horizons, $h = 1, .., H$), for an individual forecaster, but the moments we include in the estimation are from the consensus forecasts, and so we need the

[16] The convergence of $\mathbf{P}_{i,t|t-1}$, $\mathbf{P}_{i,t|t}$ and $\mathbf{K}_{i,t}$ to their steady-state values relies on $|\phi| < 1$, see Hamilton (1994), Proposition 13.1, and we impose this in the estimation.

RMSE term structure for the consensus, which requires slightly more work.[17] Let

$$\bar{\boldsymbol{\xi}}_{t|t-h} \equiv \frac{1}{N}\sum_{i=1}^{N}\hat{\boldsymbol{\xi}}_{i,t|t-h} \tag{21}$$

be the consensus forecast of the state vector. We now derive the term structure of RMSE for this forecast, but first it is useful to derive the RMSE of the consensus "nowcast":

$$\begin{aligned}
\bar{\mathbf{P}}_0^* &\equiv V\left[\boldsymbol{\xi}_t - \bar{\boldsymbol{\xi}}_{t|t}\right] \\
&= V\left[\frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t}\right)\right] \\
&= \frac{1}{N^2}\sum_{i=1}^{N} V\left[\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t}\right] + \frac{2}{N^2}\sum_{i=1}^{N-1}\sum_{k=i+1}^{N} Cov\left[\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t}, \boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{k,t|t}\right] \\
&= \frac{1}{N}\mathbf{P}_0^* + \frac{N-1}{N} E\left[\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t}\right)\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{k,t|t}\right)'\right], \qquad (22)
\end{aligned}$$

using the assumption that all of our forecasters receive signals with identical distributions. It is possible to show that the current nowcast error is the following function of the previous period's nowcast error and the intervening innovations:

$$\begin{aligned}
\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t} &= \left(\mathbf{I} - \mathbf{P}_1^*\mathbf{H}\left(\mathbf{H}'\mathbf{P}_1^*\mathbf{H} + \mathbf{R}\right)^{-1}\mathbf{H}'\right)\mathbf{F}\left(\boldsymbol{\xi}_{t-1} - \hat{\boldsymbol{\xi}}_{i,t-1|t-1}\right) \\
&\quad + \left(\mathbf{I} - \mathbf{P}_1^*\mathbf{H}\left(\mathbf{H}'\mathbf{P}_1^*\mathbf{H} + \mathbf{R}\right)^{-1}\mathbf{H}'\right)\mathbf{v}_t \\
&\quad - \mathbf{P}_1^*\mathbf{H}\left(\mathbf{H}'\mathbf{P}_1^*\mathbf{H} + \mathbf{R}\right)^{-1}\mathbf{w}_{it} \\
&\equiv \mathbf{A}\left(\boldsymbol{\xi}_{t-1} - \hat{\boldsymbol{\xi}}_{i,t-1|t-1}\right) + \mathbf{B}\mathbf{v}_t + \mathbf{C}\mathbf{w}_{it}, \qquad (23)
\end{aligned}$$

where $\mathbf{v}_t$ and $\mathbf{w}_{it}$ are defined above. We use this result to derive the covariance between nowcast errors across different forecasters:

$$\begin{aligned}
\mathbf{P}_{0ik}^* &\equiv E\left[\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t}\right)\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{k,t|t}\right)'\right] \qquad (24) \\
&= E\left[\left(\mathbf{A}\left(\boldsymbol{\xi}_{t-1} - \hat{\boldsymbol{\xi}}_{i,t-1|t-1}\right) + \mathbf{B}\mathbf{v}_t + \mathbf{C}\mathbf{w}_{it}\right)\left(\mathbf{A}\left(\boldsymbol{\xi}_{t-1} - \hat{\boldsymbol{\xi}}_{k,t-1|t-1}\right) + \mathbf{B}\mathbf{v}_t + \mathbf{C}\mathbf{w}_{kt}\right)'\right] \\
&= \mathbf{A}E\left[\left(\boldsymbol{\xi}_{t-1} - \hat{\boldsymbol{\xi}}_{i,t-1|t-1}\right)\left(\boldsymbol{\xi}_{t-1} - \hat{\boldsymbol{\xi}}_{k,t-1|t-1}\right)'\right]\mathbf{A}' + \mathbf{B}\mathbf{Q}\mathbf{B}' + \mathbf{C}E\left[\mathbf{w}_{it}\mathbf{w}_{kt}'\right]\mathbf{C}',
\end{aligned}$$

with all other terms in the two nowcast errors having zero covariance. Letting

$$E\left[\mathbf{w}_{it}\mathbf{w}_{kt}'\right] = \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix} \equiv \mathbf{R}_{ik},$$

[17] Patton and Timmermann (2010) also consider the behavior of the consensus forecast error but do not analyze cross-sectional dispersion in forecasts.

we then have

$$\mathbf{P}^*_{0ik} = \mathbf{A}\mathbf{P}^*_{0ik}\mathbf{A}' + \mathbf{B}\mathbf{Q}\mathbf{B}' + \mathbf{C}\mathbf{R}_{ik}\mathbf{C}',$$

which exploits the stationarity of this process, and yields an implicit solution for the covariance of nowcast errors across forecasters, $\mathbf{P}^*_{0ik}$.[18] Thus the variance of the error of the consensus nowcast of the state vector is:

$$\bar{\mathbf{P}}^*_0 \equiv V\left[\boldsymbol{\xi}_t - \bar{\boldsymbol{\xi}}_{t|t}\right] = \frac{1}{N}\mathbf{P}^*_0 + \frac{N-1}{N}\mathbf{P}^*_{0ik}. \tag{25}$$

The variance of the consensus forecast of the state vector for $h \geq 1$ can be similarly obtained. Using the following expression for forecast errors as a function of a previous nowcast error and the intervening innovations:

$$\boldsymbol{\xi}_t - \boldsymbol{\xi}_{i,t|t-h} = \mathbf{F}^h\left(\boldsymbol{\xi}_{t-h} - \boldsymbol{\xi}_{i,t-h|t-h}\right) + \sum_{j=0}^{h-1}\mathbf{F}^j\mathbf{v}_{t-j},\ h \geq 1, \tag{26}$$

we obtain

$$\begin{aligned}
\bar{\mathbf{P}}^*_h &\equiv V\left[\boldsymbol{\xi}_t - \bar{\boldsymbol{\xi}}_{t|t-h}\right] \\
&= V\left[\frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t-h}\right)\right] \\
&= \frac{1}{N^2}\sum_{i=1}^{N} V\left[\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t-h}\right] + \frac{2}{N^2}\sum_{i=1}^{N-1}\sum_{k=i+1}^{N} Cov\left[\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t-h}, \boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{k,t|t-h}\right] \\
&= \frac{1}{N}\mathbf{P}^*_h + \frac{N-1}{N}E\left[\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t-h}\right)\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{k,t|t-h}\right)'\right]
\end{aligned} \tag{27}$$

To evaluate this expression requires knowledge of the covariance between the individual forecast

[18] Like other covariance matrices that appear in more standard Kalman filtering applications, see Hamilton (1994), Proposition 13.1 for example, it is not possible to obtain an explicit expression for $\mathbf{P}^*_{0ik}$.

errors measured at different horizons:

$$
\begin{aligned}
\mathbf{P}^*_{hik} &\equiv E\left[\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{i,t|t-h}\right)\left(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{k,t|t-h}\right)'\right] \\
&= E\left[\left(\mathbf{F}^h\left(\boldsymbol{\xi}_{t-h} - \boldsymbol{\xi}_{i,t-h|t-h}\right) + \sum_{j=0}^{h-1}\mathbf{F}^j\mathbf{v}_{t-j}\right)\left(\mathbf{F}^h\left(\boldsymbol{\xi}_{t-h} - \boldsymbol{\xi}_{k,t-h|t-h}\right) + \sum_{j=0}^{h-1}\mathbf{F}^j\mathbf{v}_{t-j}\right)'\right] \\
&= \mathbf{F}^h E\left[\left(\boldsymbol{\xi}_{t-h} - \boldsymbol{\xi}_{i,t-h|t-h}\right)\left(\boldsymbol{\xi}_{t-h} - \boldsymbol{\xi}_{k,t-h|t-h}\right)'\right]\left(\mathbf{F}^h\right)' \\
&\quad + E\left[\left(\sum_{j=0}^{h-1}\mathbf{F}^j\mathbf{v}_{t-j}\right)\left(\sum_{j=0}^{h-1}\mathbf{F}^j\mathbf{v}_{t-j}\right)'\right] \qquad (28) \\
&= \mathbf{F}^h\mathbf{P}^*_{0ik}\left(\mathbf{F}^h\right)' + \sum_{j=0}^{h-1}\mathbf{F}^j\mathbf{Q}\left(\mathbf{F}^j\right)', \quad h \geq 1. \qquad (29)
\end{aligned}
$$

With these moment matrices in place it is simple to obtain the term structure of MSE-values for the consensus forecast of the target variable. Let $\boldsymbol{\omega} \equiv [0, \boldsymbol{\iota}'_{12}]'$, where $\boldsymbol{\iota}_k$ is a $k \times 1$ vector of ones, then:

$$
V\left[z_t - \bar{z}_{t|t-h}\right] = V\left[\boldsymbol{\omega}'\left(\boldsymbol{\xi}_t - \bar{\boldsymbol{\xi}}_{t|t-h}\right)\right] = \boldsymbol{\omega}'\bar{\mathbf{P}}^*_h\boldsymbol{\omega}, \text{ for } h \geq 0. \qquad (30)
$$

The above expression yields 24 moments (the mean squared errors for the 24 forecast horizons) that can be used to estimate the parameters of the model that govern the dynamics of GDP growth and inflation.

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Table 1: Summary statistics for the consensus forecast and the dispersion of forecasts, across horizons

	GDP growth			Inflation		
Forecast Horizon	Mean Forecast	Mean Dispersion	Correlation	Mean Forecast	Mean Dispersion	Correlation
1	2.861	0.071	−0.602*	2.839	0.075	0.448
2	2.824	0.088	−0.414	2.865	0.083	0.454
3	2.788	0.127	−0.410*	2.890	0.108	0.257
4	2.810	0.140	−0.524*	2.907	0.125	0.359
5	2.785	0.153	−0.375*	2.909	0.147	0.383
6	2.824	0.197	−0.482*	2.888	0.177	0.264
7	2.827	0.222	−0.534*	2.881	0.198	0.176
8	2.817	0.252	−0.462*	2.814	0.212	−0.084
9	2.748	0.306	−0.694*	2.713	0.259	0.563*
10	2.677	0.341	−0.640*	2.648	0.287	0.522*
11	2.606	0.355	−0.799*	2.591	0.286	0.261
12	2.559	0.380	−0.688*	2.653	0.323	0.478*
13	2.569	0.388	−0.646*	2.720	0.327	0.248
14	2.558	0.407	−0.698*	2.772	0.347	0.424*
15	2.626	0.419	−0.516*	2.814	0.355	0.388
16	2.739	0.414	−0.231	2.855	0.369	0.294
17	2.806	0.391	0.066	2.841	0.385	0.034
18	2.848	0.374	−0.102	2.847	0.405	0.129
19	2.852	0.391	−0.120	2.870	0.433	0.512*
20	2.856	0.387	−0.148	2.872	0.411	−0.020
21	2.849	0.396	−0.092	2.828	0.416	0.325
22	2.869	0.396	−0.297	2.825	0.435	0.228
23	2.865	0.400	−0.305	2.836	0.430	0.461*
24	2.853	0.420	−0.300	2.865	0.436	0.386

Notes: This table presents the average consensus forecast, average cross-sectional dispersion in forecasts, and the time-series correlation between the consensus forecast and the dispersion in forecasts, for each horizon between 1 month and 24 months, computed across all years in the sample period (1991-2008). Correlation coefficients that are significantly different from zero at the 10% level (using Newey-West (1987) standard errors) are marked with an asterisk.

Table 2: Transition matrices for high/medium/low forecasters

	GDP growth				Inflation		
	Short-horizon forecasts						
	Low	Mid	High		Low	Mid	High
Low	0.451^*	0.323	$0.225^\dagger$	Low	0.491^*	0.290	$0.218^\dagger$
Mid	$0.258^\dagger$	0.433^*	0.309	Mid	$0.204^\dagger$	0.415^*	0.381^*
High	$0.106^\dagger$	$0.266^\dagger$	0.629^*	High	$0.082^\dagger$	$0.246^\dagger$	0.672^*
	Long-horizon forecasts						
	Low	Mid	High		Low	Mid	High
Low	0.583^*	0.333	$0.085^\dagger$	Low	0.603^*	$0.285^\dagger$	$0.112^\dagger$
Mid	$0.201^\dagger$	0.519^*	$0.280^\dagger$	Mid	$0.230^\dagger$	0.554^*	$0.216^\dagger$
High	$0.073^\dagger$	$0.264^\dagger$	0.663^*	High	$0.091^\dagger$	$0.219^\dagger$	0.700^*

Notes: This table presents the probability of a forecaster transitioning from a given tercile of the cross-sectional distribution of forecasts (low, middle, and high) to another tercile in the following year. The upper panel presents the average of these probabilities for all forecasts with horizon up to 12 months ("short horizon") and the lower panel presents the corresponding results for forecasts with horizon greater than 12 months ('long horizon"). Estimated probabilities that are significantly greater (lower) than 0.33 at the 10% level are marked with an asterisk (dagger).

Table 3: Parameter estimates of the joint consensus forecast and constant dispersion model

	GDP growth	**Inflation**
σ_u	0.000	0.074
	(——)	(0.062)
σ_ε	0.073	0.018
	(0.040)	(0.013)
ϕ	0.890	0.981
	(0.039)	(0.031)
σ_η	0.083	0.000
	(0.069)	(——)
σ_ν	0.000	0.486
	(2.287)	(14.264)
σ_μ	0.380	0.531
	(0.081)	(0.260)
κ	0.552	0.593
	(0.153)	(0.344)
J p-val	0.766	0.000
H_0: $\sigma_\nu = 0$	0.000	0.001
	(0.500)	(0.486)
H_0: $\sigma_\mu = 0$	12.982	2.970
	(0.000)	(0.042)

Notes: This table reports simulated method of moments (SMM) parameter estimates of the Kalman filter model of the consensus forecasts and forecast dispersions, with standard errors in parentheses. The model is estimated using six moments each from the MSE term structure for the consensus forecast and from the cross-sectional term structure of dispersion for each variable. p-values from the test of over-identifying restrictions are given in the row titled "J p-val". The final two rows present the test statistics, with p-values in parentheses, of the tests for no heterogeneity in signals ($H_0 : \sigma_\nu = 0$) and no heterogeneity in beliefs ($H_0 : \sigma_\mu = 0$).

Table 4: Parameter estimates for two models of time-varying dispersion

	GDP growth	**Inflation**
PANEL A: OLS estimates of a panel model for dispersion		
Fixed effects?	Yes	Yes
β_{SPR}	0.425	0.779
	(0.115)	(0.082)
PANEL B: SMM parameter estimates of a Kalman filter model		
σ_u	0.000	0.033
	(——)	(0.054)
σ_ε	0.042	0.023
	(0.022)	(0.014)
ϕ	0.954	0.957
	(0.026)	(0.038)
σ_η	0.073	0.000
	(0.066)	(——)
σ_ν	0.046	0.486
	(0.108)	(8.359)
σ_μ	0.682	0.530
	(0.378)	(0.322)
β_0^κ	4.452	−2.179
	(1.819)	(3.253)
β_1^κ	−4.451	1.190
	(2.284)	(4.558)
J p-val	0.983	0.000

Notes: The first two rows of this table report the results from the estimation of a panel model for log-dispersion, with horizon-specific fixed effects, as a function of the log default spread. In the interests of brevity, the individual fixed effect parameters are not reported. The remainder of the table reports simulated method of moments (SMM) parameter estimates of the Kalman filter model of the consensus forecasts and forecast dispersions, with standard errors in parentheses. p-values from the test of over-identifying restrictions are given in the row titled "J p-val". The model is estimated using six moments each from the MSE term structure for the consensus forecast and from the cross-sectional term structure of dispersion for each variable.

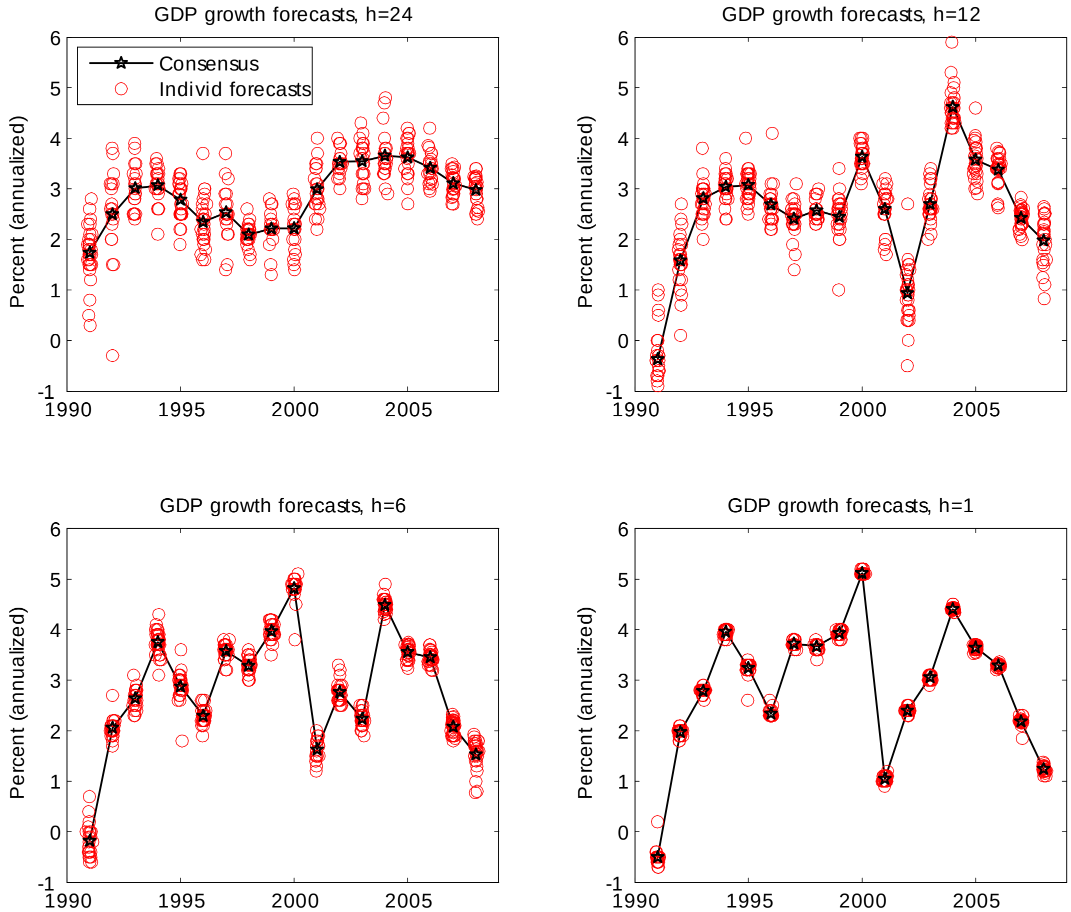


Figure 1: *Consensus and individual forecasts for GDP growth over the period 1991-2008, for four forecast horizons (24 months, 12 months, 6 months and 1 month).*

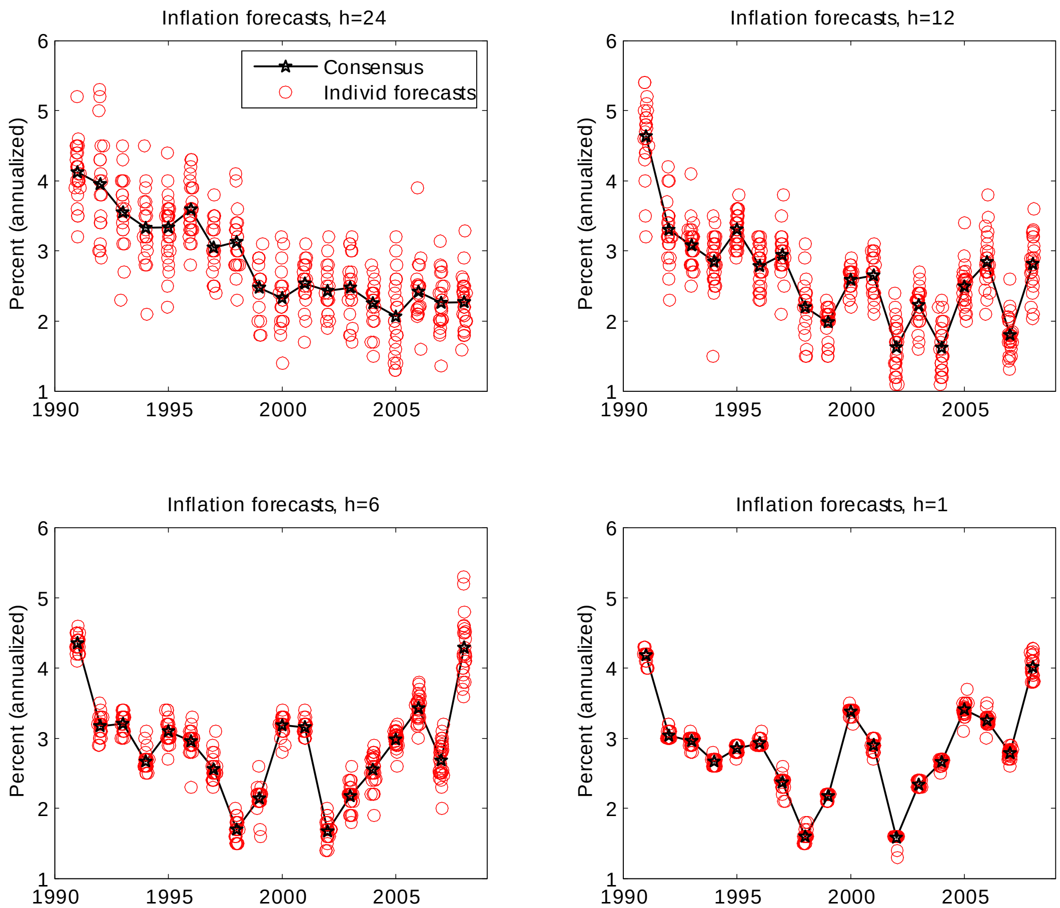


Figure 2: *Consensus and individual forecasts for inflation over the period 1991-2008, for four forecast horizons (24 months, 12 months, 6 months and 1 month).*

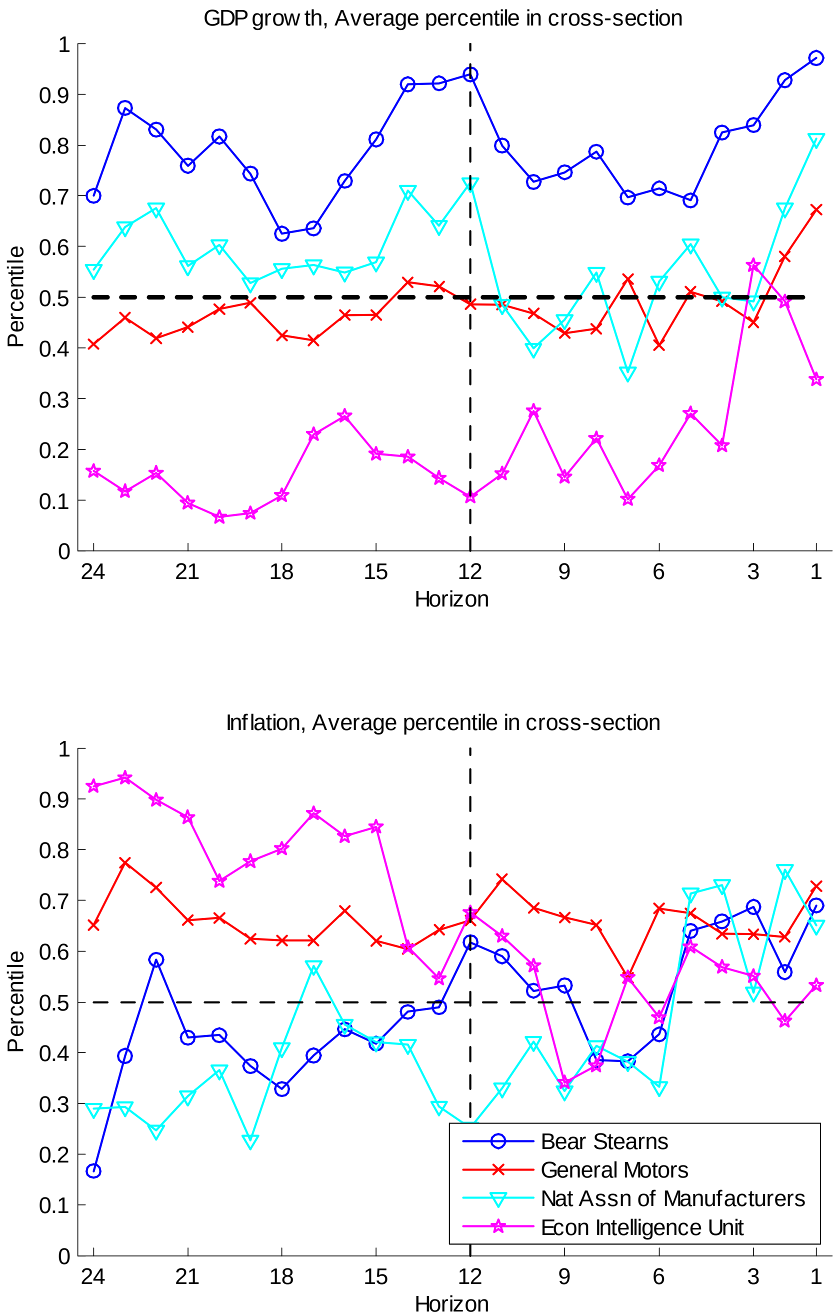


Figure 3: *Average position in the cross-sectional distribution of forecasters of four selected forecasters, for GDP growth and inflation, for each forecast horizon.*

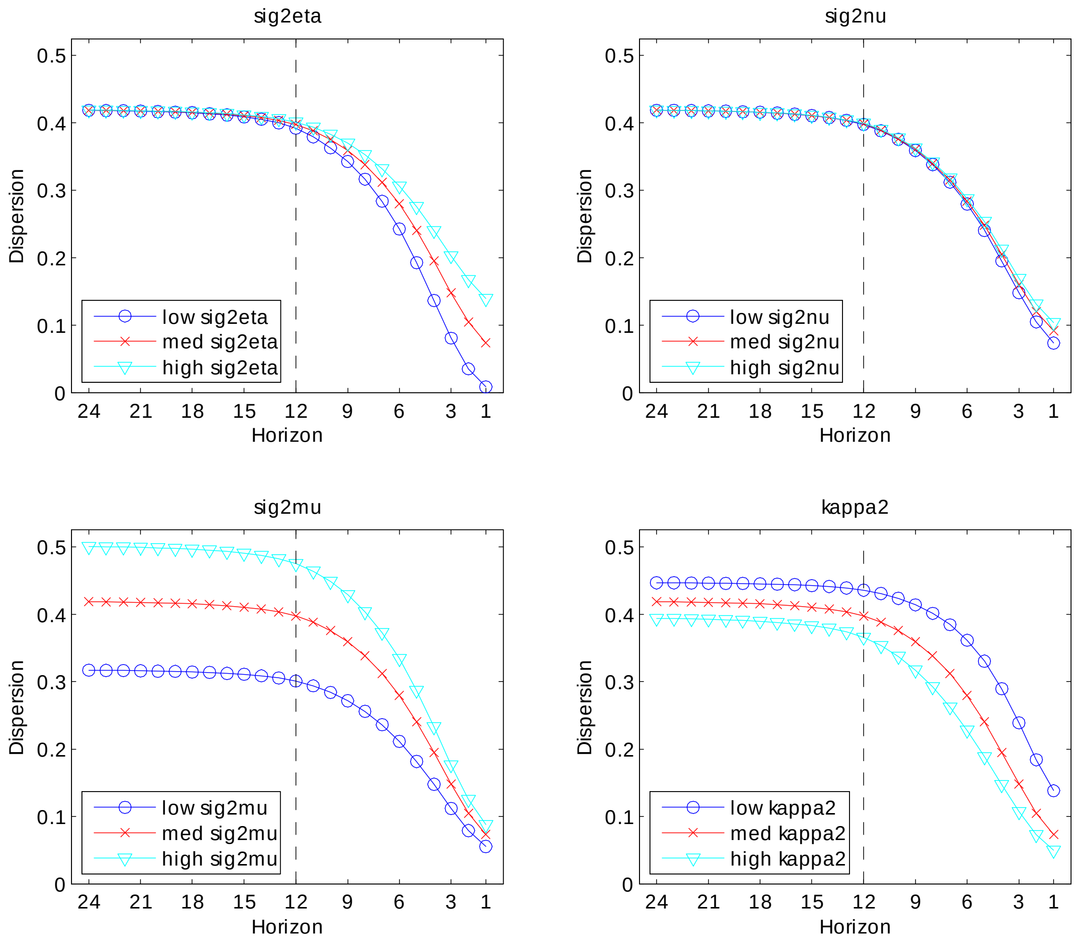


Figure 4: *Term structure of forecast dispersion when varying the variance of the common noise component (sig2eta), the variance of the idiosyncratic noise (sig2nu), the variance of the differences in prior beliefs, and the variance of perceived accuracy of prior beliefs (kappa2).*

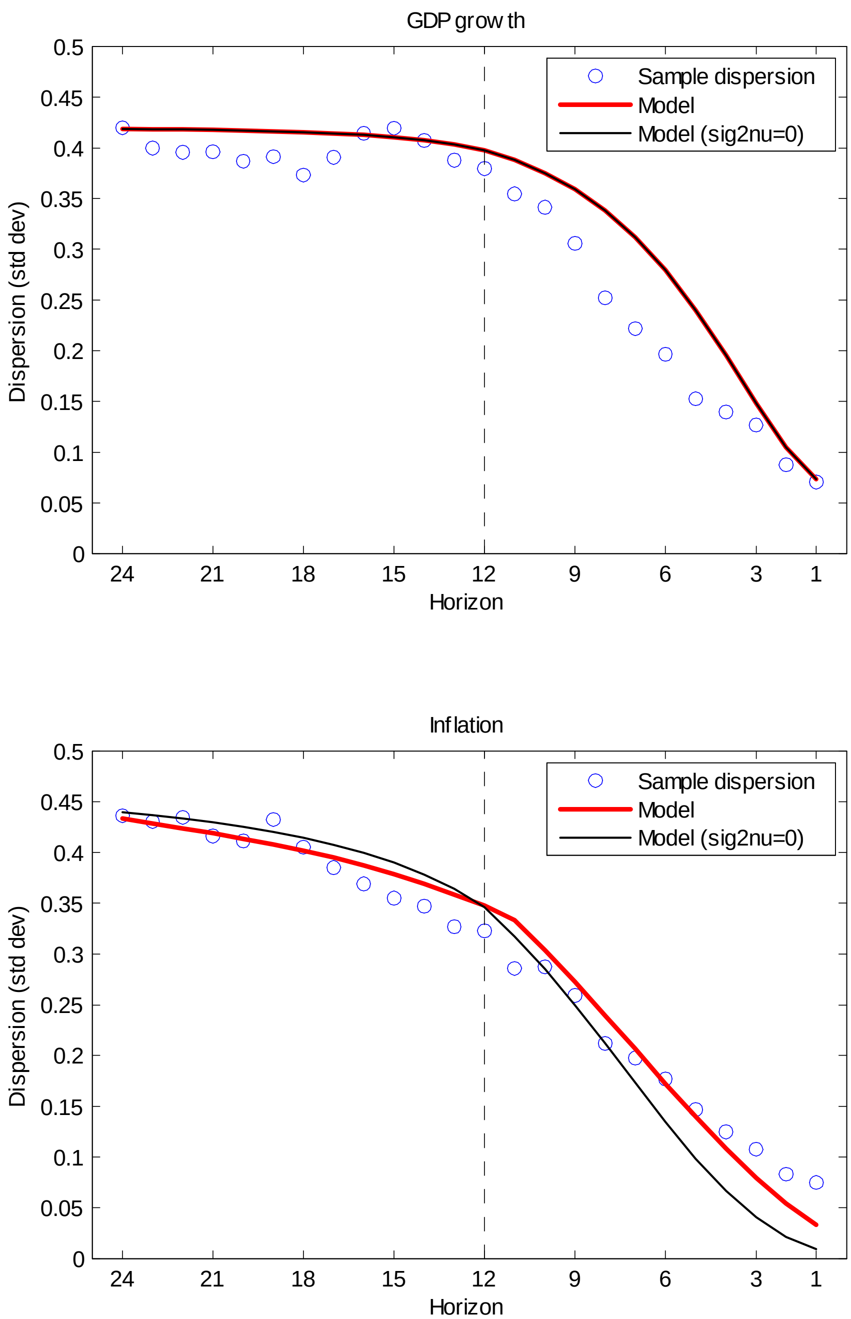
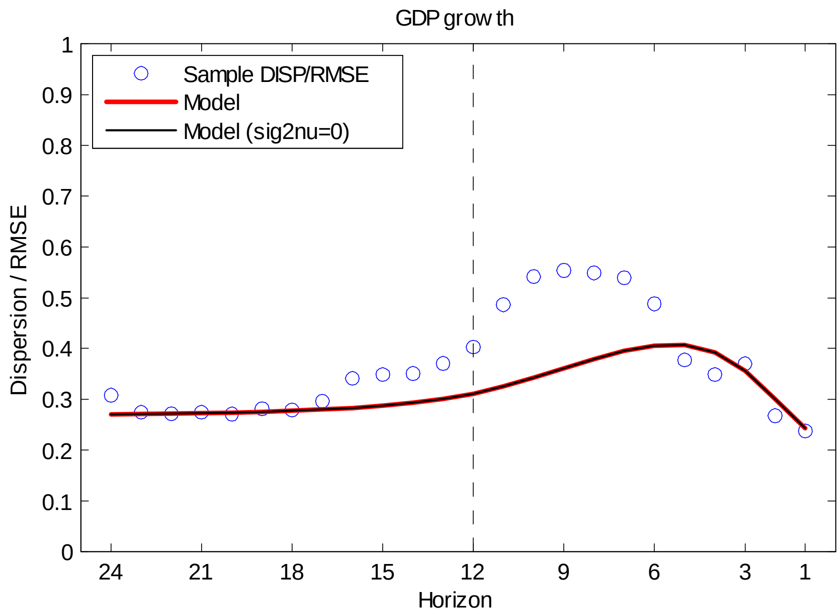


Figure 5: *Cross-sectional dispersion (standard deviation) of forecasts of GDP growth and Inflation in the U.S.*



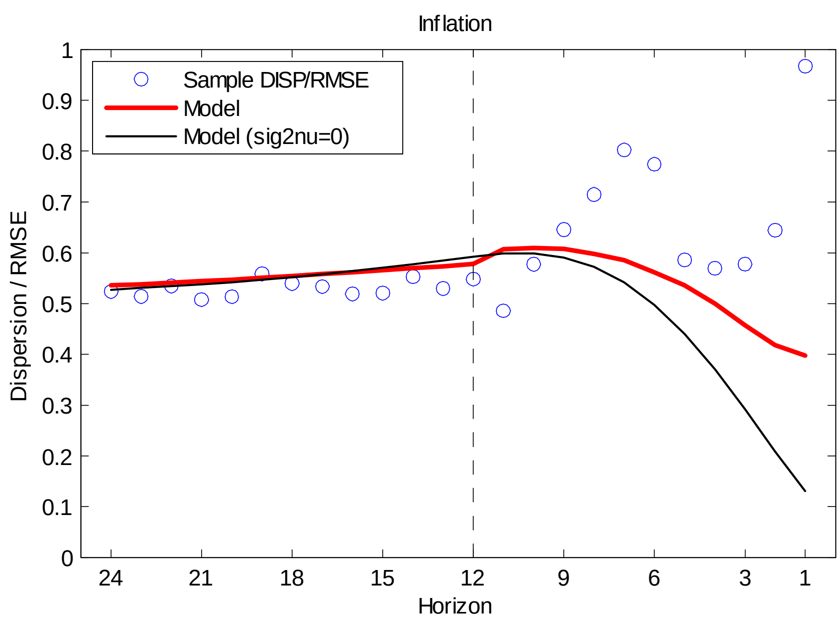


Figure 6: *Ratio of cross-sectional dispersion to root mean squared forecast errors for US GDP growth and Inflation as a function of the forecast horizon.*

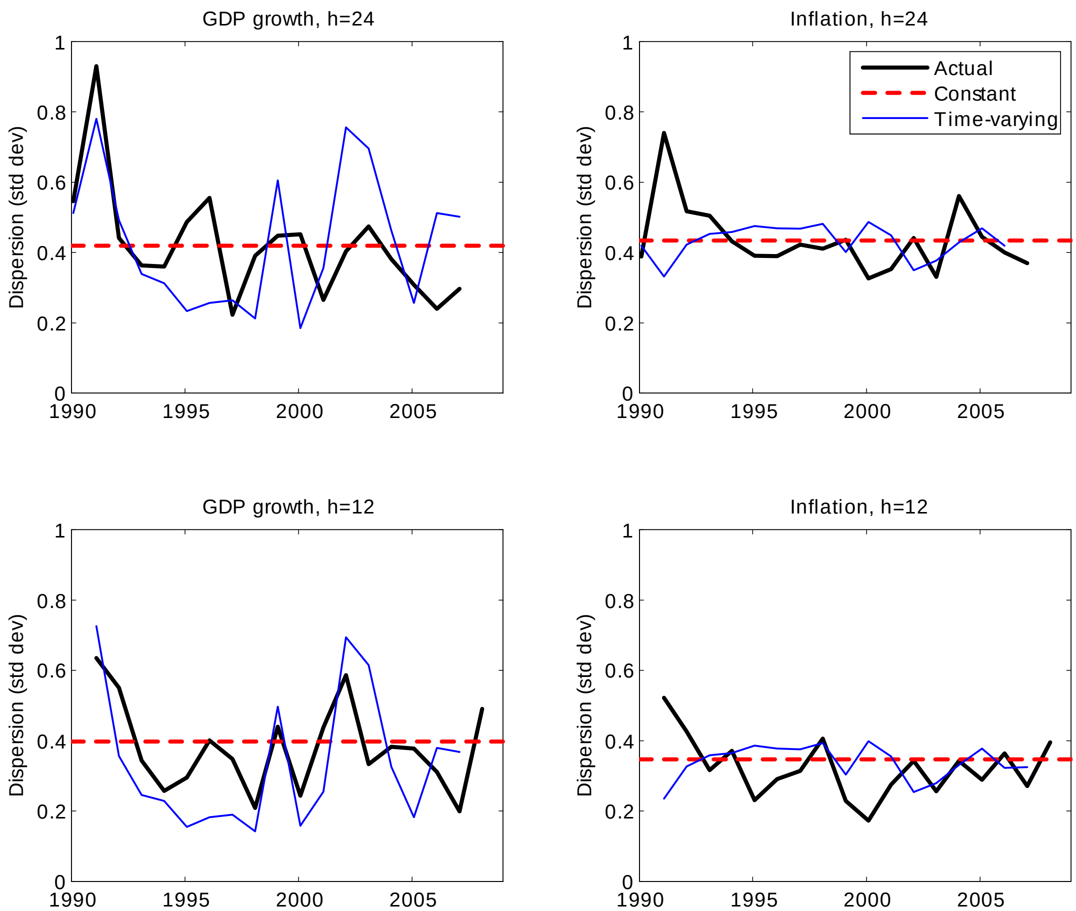


Figure 7: *Time series of observed cross-sectional dispersion of GDP growth and inflation forecasts, and model-implied time series of forecast dispersions, for forecast horizons of 12 and 24 months.*