

# Modeling Covariance Risk in Merton's ICAPM<sup>†</sup>

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## Abstract

We propose a new method for constructing the hedge component in Merton's ICAPM that uses a daily summary measure of economic activity to track time-varying investment opportunities. We then use nonparametric projections to compute a robust estimate of the conditional covariance between stock market returns and our daily economic activity index. We find that the new conditional covariance risk measure plays an important role in explaining time variation in the equity risk premium. Specification tests as well as out-of-sample forecasts of aggregate stock returns suggest that the new covariance risk measure performs well compared to alternative covariance measures previously proposed in the literature.

Keywords: Covariance risk factor; Merton ICAPM; Risk-return trade-off.

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# 1 Introduction

The trade-off between risk and returns is fundamental to finance. According to Merton's ICAPM (Merton (1973)), the expected excess return on the market portfolio,  $E_t[r_{t+1}]$ , should reflect its conditional variance,  $Var_{t+1|t}$ , and the conditional covariance between market returns,  $r_{t+1}$ , and economic state variables,  $x_{t+1}$ , capturing time variation in the investment opportunity set,  $Cov_{t+1|t}$ :

$$E_t[r_{t+1}] = \left( \frac{-J_{WW}W}{J_W} \right) Var_{t+1|t} + \left( \frac{-J_{Wx}W}{J_W} \right) Cov_{t+1|t}. \quad (1)$$

Here  $W$  is wealth,  $J(W, x, t)$  is the investor's indirect utility function and  $J_W$ ,  $J_{WW}$ , and  $J_{Wx}$  denote partial derivatives, so that  $(-J_{WW}W/J_W)$  measures the representative investor's relative risk aversion.

Empirical work on the risk-return relation has mostly tested the relation between the equity risk premium and the conditional variance, i.e., the first term on the right hand side of equation (1). Studies such as Campbell (1987), Breen, Glosten, and Jagannathan (1989), Turner, Startz, and Nelson (1989), Nelson (1991), Glosten, Jagannathan, and Runkle (1993), Whitelaw (1994, 2000), Harvey (2001), and Brandt and Kang (2004) find a negative trade-off. Conversely, Bali and Peng (2006), Bollerslev, Engle, and Wooldridge (1988), Harvey (1989), Scruggs (1998), Harrison and Zhang (1999), Scruggs and Glabadanidis (2003), Ghysels, Santa-Clara, and Valkanov (2005), Guo and Whitelaw (2006), Ludvigson and Ng (2007), Lundblad (2007), and Pástor, Sinha, and Swaminathan (2008) find a positive trade-off.<sup>1</sup>

A key difficulty in testing the ICAPM is that ex-ante measures of the conditional variance and covariance are unobserved. A large literature has proposed different ways to estimate the conditional variance using data on high-frequency returns as well as a variety of dynamic specifications. Far less work has been undertaken on estimating the conditional covariance term in the ICAPM. In part this reflects the lack of specific guidance from theory – the ICAPM implies that the conditional covariance term should track time-varying investment opportunities but it does not specify the identity of the state variables or how such variables map into the conditional covariance. Scruggs (1998) and Guo and Whitelaw (2006)

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<sup>1</sup>Other studies report an insignificant risk-return relation; see French, Schwert, and Stambaugh (1987), Baillie and DeGennaro (1990), Campbell and Hentschel (1992), Harrison and Zhang (1999) and Bollerslev and Zhou (2006). Bekaert and Hoerova (2014) do not find that the conditional variance predicts stock returns although they find that it is negatively correlated with future economic growth.

estimate covariance models using state variables from the literature on return predictability. Specifically, Scruggs (1998) adopts a two-factor GARCH-in-mean model to estimate covariance risk in which the nominal risk-free rate drives movements in the conditional covariance. Guo and Whitelaw (2006) assume that the conditional covariance is a linear function of a vector of observable state variables such as the relative T-bill rate and the CAY variable of Lettau and Ludvigson (2001). Bali (2008) uses a bivariate GARCH model to estimate conditional covariances and tests the ICAPM on stock portfolios formed on firm characteristics or industry membership. Bali and Engle (2010) extend this setting to a model with dynamic conditional correlations. None of these studies test the significance of the portfolios' exposure to a broad economic activity index, however.

This paper proposes a new approach for constructing the covariance risk measure that distills information from a large set of (conditioning) state variables in a parsimonious and robust manner. First, we extract a daily economic activity index from macroeconomic and financial variables observed at mixed frequencies using a dynamic (latent) factor approach similar to that proposed by Aruoba, Diebold, and Scotti (2009). We find that this economic activity measure is procyclical and significantly correlated with variables previously proposed as measures of time-varying investment opportunities such as the dividend-price ratio, interest rates, and consumption growth. We use this daily economic activity index to estimate monthly "realized covariances" between stock returns and economic activity.

The second step in our analysis estimates the conditional covariance by projecting the realized covariance on a large set of conditioning variables that predict time-varying investment opportunities. These projections are performed non-parametrically and use a new technique known as boosted regression trees. This approach, first, allows us to condition on a large set of state variables and, second, avoids imposing restrictive linearity assumptions on the relation between the conditioning variables and the covariance. This is important since theory does not impose such restrictions on this step and assumptions such as linearity can lead to a misspecified model. Indeed, empirical tests show that a linear model for the conditional covariance term is grossly misspecified.

Using our new conditional covariance measure, the third step in our analysis estimates Merton's ICAPM by performing a linear regression of stock market (excess) returns on the conditional variance

and the conditional covariance. We obtain positive and significant coefficients for both the conditional variance and covariance terms.<sup>2</sup> Moreover, at between 2 and 2.6 in the monthly data and between 3.6 and 4.3 in the quarterly data, our estimates of the representative investor's coefficient of relative risk aversion are very sensible. These estimates do not depend on the use of our new conditional covariance measure and are generated using an EGARCH specification for the conditional variance.

To understand why the coefficient on the conditional covariance term is positive, note that both expected returns and the conditional variance of returns vary countercyclically and tend to be higher during recessions when stock returns are generally low. Their higher expected returns during recessions make stocks more attractive from a hedging perspective while their higher (conditional) variance makes them less attractive. Empirically, we find that cyclical variation in the conditional variance component dominates variation in expected returns, making investment opportunities overall worse during economic downturns and better in expansions. This suggests that stocks do not provide a hedge against adverse shifts to investment opportunities and so the coefficient on the conditional covariance term should be positive.

To gain further insights into the performance of our new conditional covariance measure, the fourth step in our analysis compares it to covariance estimates from a linear model and to covariance estimates obtained using the approaches of Scruggs (1998) and Guo and Whitelaw (2006). We find evidence that an ICAPM specification based on these alternative parametric covariance measures is misspecified. In contrast, the ICAPM based on our new covariance measure does not appear to be misspecified and fits the returns data better than alternative covariance measures.

As a final test of Merton's ICAPM, we perform a pseudo out-of-sample forecast analysis that predicts stock returns by means of the recursively estimated conditional variance and covariance. We find that our new conditional covariance model performs notably better than forecasts using the covariance risk measures proposed by Scruggs (1998) and Guo and Whitelaw (2006). This indicates that the positive risk-return relation uncovered by our model is stable enough to provide more accurate out-of-sample return forecasts.

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<sup>2</sup>Lundblad (2007) and Bali (2008) note that time-series tests of the ICAPM tend to have weak power.

The remainder of the paper is organized as follows. Section 2 introduces the daily economic activity index and shows how it is related to existing proxies for time-varying investment opportunities. Section 3 explains how we construct the conditional covariance measure. Section 4 analyzes the ICAPM relation empirically and compares our approach to existing methods. Section 5 concludes.

## 2 Construction of the Daily Economic Activity Index

The state variables in the ICAPM should capture time variation in investment opportunities and so can be expected to depend on broad measures of the state of the economy. Economic variables tracking such measures are typically only available weekly, monthly or quarterly and are often published on an irregular basis. To overcome this, we construct a new economic activity measure using a methodology similar to the dynamic latent factor approach developed by Aruoba, Diebold, and Scotti (2009).

We construct our economic activity index (EAI) using a variety of variables. Investment opportunities likely depend on real economic activity which can be proxied by variables such as growth in industrial production, personal income, and GDP. Labor market variables have also been shown to be closely related to the state of the economy so we include jobless claims which are available weekly. Finally, interest rates matter directly to investment opportunities as they represent an alternative investment to stocks and have also been found to predict future economic activity and be correlated with stock market volatility.<sup>3</sup>

Our first EAI (“EAI1” henceforth) uses both real (non-financial) and financial variables. Specifically, we use weekly observations on jobless claims, which track how many people file for unemployment benefits over a given week, monthly observations on growth in real personal income less transfers and industrial production, and quarterly GDP figures. Following Merton (1973) we add to these variables a daily interest rate series in the form of the current 3-month T-bill rate, measured relative to a twelve-month moving average so as to account for local trends.

Ultimately, a smaller set of primitive shocks are likely to drive most variables, making it difficult to distinguish clearly between real and financial effects. Nevertheless, comparing the results based on an index

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<sup>3</sup>Merton (1973, p. 879) writes “one should interpret the effects of a changing interest rate ... in the way economists have generally done in the past: namely, as a single (instrumental) variable representation of shifts in the investment opportunity set.”

constructed from real and financial variables versus an index that uses only real economic variables allows us to gauge the marginal effect of conditioning on financial variables measured at the daily frequency. We therefore construct a second EAI (“EAI2”) based only on the real (non-financial) macroeconomic variables listed above, i.e., exclusive of the detrended 3-month T-bill rate. Even though none of these series is measured at the daily frequency, we can still construct a daily index series because different variables get released on different days. The weekly payroll figures start in 1960 and so both indexes are available from March 1960 to December 2012, a total of 19,299 daily observations.<sup>4</sup>

## 2.1 Methodology

Following Aruoba, Diebold, and Scotti (2009) we model the daily business cycle,  $z_t$ , as a latent variable that follows a (zero-mean) first-order autoregressive process:

$$z_t = \rho z_{t-1} + \epsilon_t. \quad (2)$$

Although  $z_t$  is unobserved, we can extract information about it through its relation with a vector of observed economic and financial state variables,  $\mathbf{y}_t$ , with  $i$ th component  $y_t^i$ . At the daily frequency the observed variables are assumed to follow processes of the form:

$$y_t^i = k_i + \beta_i z_t + \gamma_{i1} y_{t-D_i}^i + u_t^i. \quad (3)$$

The lag length for variable  $i$ ,  $D_i$ , depends on the observation frequency of variable  $i$ ; it is constant and equals seven days if the variable is observed weekly but varies over time if the variable is observed at the monthly or quarterly frequencies due to variation in the number of days in a month.

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<sup>4</sup>Following standard practice, we account for trends in the variables by modeling their changes (log first-differences) with exception of the weekly initial jobless claim numbers which, following Aruoba, Diebold, and Scotti (2009) are de-trended using a polynomial trend model. All non-daily variables are therefore “flow” variables and their temporal aggregation is handled using cumulator variables following Harvey (1990, pp. 313-318).

The model (2)-(3) can be written in state-space form as:

$$\mathbf{y}_t = \boldsymbol{\beta}\boldsymbol{\alpha}_t + \boldsymbol{\Gamma}\mathbf{w}_t + \mathbf{u}_t, \quad \mathbf{u}_t \sim (\mathbf{0}, \mathbf{H}), \quad (4)$$

$$\boldsymbol{\alpha}_{t+1} = \mathbf{T}_t\boldsymbol{\alpha}_t + \mathbf{R}\boldsymbol{\eta}_{t+1}, \quad \boldsymbol{\eta}_{t+1} \sim (\mathbf{0}, \mathbf{Q}), \quad (5)$$

where  $\boldsymbol{\alpha}_t$  is a vector of state variables that includes  $z_t$ ,  $\mathbf{w}_t$  is a vector of lagged dependent variables, and  $\mathbf{u}_t$  and  $\boldsymbol{\eta}_t$  are shocks associated with the measurement and transition equations, respectively. All matrices  $(\boldsymbol{\beta}, \boldsymbol{\Gamma}, \mathbf{R}, \mathbf{H}, \mathbf{Q})$  are constant while  $\mathbf{T}_t$  varies over time due to the temporal aggregation of the flow variables observed at the weekly, monthly and quarterly frequencies. The appendix describes the Kalman filter equations used to extract the business cycle index from (4)-(5) and explains how we deal with missing values in  $\mathbf{y}_t$ .

As an illustration, consider a model containing the following observables: detrended interest rates (daily,  $\tilde{y}_t^1$ ); initial jobless claims (weekly,  $\tilde{y}_t^2$ ); personal income (monthly,  $\tilde{y}_t^3$ ); industrial production (monthly,  $\tilde{y}_t^4$ ); and GDP (quarterly,  $\tilde{y}_t^5$ ). Following Aruoba, Diebold, and Scotti (2009), we model the weekly, monthly and quarterly variables as AR(1) processes by including their lagged values in  $\mathbf{w}_t$ . Conversely, we model the autocorrelation structure of the daily T-bill rate using an AR(1) process for the innovation to the measurement equation,  $u_t^1$ . This leads to the following model:<sup>5</sup>

$$\underbrace{\begin{pmatrix} \tilde{y}_t^1 \\ \tilde{y}_t^2 \\ \tilde{y}_t^3 \\ \tilde{y}_t^4 \\ \tilde{y}_t^5 \end{pmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} b_1 & 0 & 0 & 0 & 1 \\ 0 & b_2 & 0 & 0 & 0 \\ 0 & 0 & b_3 & 0 & 0 \\ 0 & 0 & b_4 & 0 & 0 \\ 0 & 0 & 0 & b_5 & 0 \end{pmatrix}}_{\boldsymbol{\beta}} \underbrace{\begin{pmatrix} z_t \\ z_{t-W} \\ z_{t-M} \\ z_{t-Q} \\ u_t^1 \end{pmatrix}}_{\boldsymbol{\alpha}_t} + \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ \gamma_2 & 0 & 0 & 0 \\ 0 & \gamma_3 & 0 & 0 \\ 0 & 0 & \gamma_4 & 0 \\ 0 & 0 & 0 & \gamma_5 \end{pmatrix}}_{\boldsymbol{\Gamma}} \underbrace{\begin{pmatrix} \tilde{y}_{t-W}^2 \\ \tilde{y}_{t-M}^3 \\ \tilde{y}_{t-M}^4 \\ \tilde{y}_{t-Q}^5 \end{pmatrix}}_{\mathbf{w}_t} + \underbrace{\begin{pmatrix} 0 \\ u_t^2 \\ u_t^3 \\ u_t^4 \\ u_t^5 \end{pmatrix}}_{\mathbf{u}_t}$$

$$\underbrace{\begin{pmatrix} z_{t+1} \\ z_{t-W+1} \\ z_{t-M+1} \\ z_{t-Q+1} \\ u_{t+1}^1 \end{pmatrix}}_{\boldsymbol{\alpha}_{t+1}} = \underbrace{\begin{pmatrix} \rho & 0 & 0 & 0 & 0 \\ \rho & 1 \text{ or } 0 & 0 & 0 & 0 \\ \rho & 0 & 1 \text{ or } 0 & 0 & 0 \\ \rho & 0 & 0 & 1 \text{ or } 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 \end{pmatrix}}_{\mathbf{T}_t} \underbrace{\begin{pmatrix} z_t \\ z_{t-W} \\ z_{t-M} \\ z_{t-Q} \\ u_t^1 \end{pmatrix}}_{\boldsymbol{\alpha}_t} + \underbrace{\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\mathbf{R}} \underbrace{\begin{pmatrix} \varepsilon_{t+1} \\ \zeta_{t+1} \end{pmatrix}}_{\boldsymbol{\eta}_{t+1}}.$$

<sup>5</sup>Temporal aggregation implies that element (2,2) of the  $\mathbf{T}_t$  matrix equals one on the first day of each week and is zero otherwise. Element (3,3) of the  $\mathbf{T}_t$  matrix equals one on the first day of each month and is zero otherwise. Finally, element (4,4) of the  $\mathbf{T}_t$  matrix equals one on the first day of each quarter and is zero otherwise. The formulation of the model is similar to that used for the updated ADS index published by the Philadelphia Federal Reserve.

Similarly, the covariance matrices for the innovations to this model take the form:

$$\begin{pmatrix} \mathbf{u}_t \\ \boldsymbol{\eta}_t \end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & 0 & 0 \\ H_{21} & H_{22} & H_{23} & H_{24} & H_{25} & 0 & 0 \\ H_{31} & H_{32} & H_{33} & H_{34} & H_{35} & 0 & 0 \\ H_{41} & H_{42} & H_{43} & H_{44} & H_{45} & 0 & 0 \\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{11} & Q_{12} \\ 0 & 0 & 0 & 0 & 0 & Q_{21} & Q_{22} \end{pmatrix} \right)$$

$$\mathbf{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 1 - \rho^2 & 0 \\ 0 & \sigma_1^2 \end{pmatrix}.$$

## 2.2 Properties of the Economic Activity Index

Figure 1 plots the two daily economic activity index series while Panel A in Table 1 reports summary statistics. Both EAI series closely track the economic business cycle with dips during recessions and marked increases as the economy emerges from a downturn. The mean of the EAI is -1.20 in recessions and 0.2 in expansions. Both EAI series are persistent, with a first-order autocorrelation of 0.97, left skewed and with fat tails, suggesting that bad news tend to move the indices more than good news. Since the two EAI series are very similar (with a daily correlation of 0.993), in most of the analysis we refer to them jointly as the EAI and base our discussion on EAI1. However, in some cases the two EAI measures yield sufficiently different results to be of economic interest and we highlight these cases below.

To help further interpret the EAI measure, we correlate it with a range of variables conventionally used to track time-variation in investment opportunities. Merton (1973) models such variation through changes to the mean and variance-covariances of asset returns. To capture variation in expected returns, we consider the monthly correlation between the EAI and the dividend-price ratio, the T-bill rate measured relative to a twelve-month trailing average, the term spread (measured as the difference in yields on 10-year and 3-month Treasury bonds), consumption deviations from its long-term trend (“consumption growth”), stock market returns and the CAY variable used by Lettau and Ludvigson (2001) as a predictor of expected



returns. To capture variation in the volatility of returns, we consider the default spread (measured as the spread between yields on BAA and AAA rated bond portfolios) and the realized variance. Following studies such as Campbell and Shiller (1988) and Keim and Stambaugh (1986), the dividend price ratio is generally thought to be positively correlated with expected returns, as is the CAY variable. The T-bill rate, on the other hand, has been found to be negatively correlated with the equity risk premium, see Ang and Bekaert (2007) and Campbell (1987). The default spread is commonly used to measure risk in the economy and is usually found to be positively correlated with expected returns, whereas the term spread has been found to be positively correlated with future economic activity.

Panel B of Table 1 shows that the EAI has a significantly negative (-0.52) correlation with the dividend-price ratio. Conversely, it is positively correlated with both the de-trended T-bill rate (correlation of 0.45) and consumption growth (correlation of 0.23).<sup>6</sup> These correlations are all statistically significant and hold irrespective of whether daily interest rates are used to construct the EAI. In contrast, the EAI is only weakly correlated with monthly stock returns (correlation of 0.01), the term spread (-0.08), and CAY (0.07). The strongly negative correlations between the EAI and the realized variance (correlation of -0.30) and the default spread (-0.53) suggest that the EAI tends to be lower during times with highly volatile returns and a high default spread.

Summarizing time variation in the investment opportunity set is not an easy task as witnessed by the fact that the finance literature has proposed numerous state variables for capturing variation in expected returns and the conditional variance. One way to summarize common variation in such measures is by extracting principal components (PCs) from the cross-section of variables believed to be capturing time-varying investment opportunities. We therefore correlate the EAI with the first and second PCs extracted from the variables listed in Panel B of Table 1. Plots of the first two PCs against the EAI—shown in Figure 2—reveal a positive and highly significant correlation between the EAI and both the first and second PCs with correlations of 0.53 and 0.28, respectively. Both PCs load strongly negatively on the

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<sup>6</sup>Because daily consumption data is not available, we consider instead the correlation between changes to the EAI and log-growth in real personal non-durable consumption (rather than the deviation from trend used in Panel B of Table 1) at the monthly, quarterly, semi-annual and annual horizons. We find that the correlation between changes to the EAI and real non-durable consumption growth is uniformly positive and increases with the horizon: it is 7.7% at the monthly horizon, 15.4% at the quarterly horizon, 31.3% at the semi-annual horizon and 39.7% at the annual horizon. Correlations with real durable consumption growth are similar.

dividend-price ratio and the realized variance. Moreover, the realized variance explains a larger fraction of the variation in the first PC than does the dividend-price ratio, whereas the reverse holds for the second PC. Risk measures such as the default spread receive a greater weight in (and explains a greater fraction of) the first PC while the opposite holds for stock returns. These observations suggest that the first, and more important, PC broadly captures variation in risk whereas the second PC puts more weight on variation in expected returns.

The variables in Panel B of Table 1 are selected based on their ability to predict future mean returns, as tested in a large literature summarized in Goyal and Welch (2008), or the volatility of returns. This suggests a more direct predictive test of whether the EAI captures time variation in investment opportunities measured either by expected returns or by their conditional variance. Specifically, we simply regress returns or the realized variance over the following month, quarter or year on the current EAI measure. Panel C in Table 1 shows that the EAI is uncorrelated with stock returns over the following month and quarter but is negatively correlated with stock returns over the following year. Moreover, the EAI is strongly negatively correlated with stock market variance at the monthly, quarterly and annual horizons.

We conclude that the EAI measure is significantly correlated with a range of proxies for time-varying investment opportunities previously studied in the finance literature. As such it appears to provide a parsimonious summary of daily changes in investment opportunities.

### 2.3 Realized Covariance

Using daily changes in the EAI as a proxy for time-variation in investment opportunities, we next construct a proxy for monthly “realized covariances” between stock returns and changes in the EAI:<sup>7</sup>

$$\widehat{cov}_t = \sum_{d=1}^{N_t} \Delta EAI_{d,t} \times r_{d,t}, \quad (6)$$

where  $\Delta EAI_{d,t}$  is the change in the EAI on day  $d$  during month  $t$ ,  $r_{d,t}$  is the corresponding daily stock market return on the value-weighted CRSP index and  $N_t$  is the number of trading days in month  $t$ . We

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<sup>7</sup>Given the very high persistence in the  $EAI$ , the change,  $\Delta EAI$ , essentially captures innovations to this index.

scale  $\widehat{cov}_t$  by the (unconditional) standard deviations of the two variables to make it easier to interpret. Given the very volatile nature of stock returns, unsurprisingly the realized covariance series (shown in Figure 3) is quite spiky at the monthly horizon (top window)—possibly due to measurement error—although it also displays systematic variation linked to the business cycle: The realized covariance measure has an overall mean of 0.11, but the mean in recessions is 0.25 versus a mean of 0.08 during expansions.<sup>8</sup> At the quarterly and annual horizons (middle and bottom plots) the index is notably smoother and less affected by individual daily observations.

A key advantage of the realized covariance measure is that it gives a “target” variable to model by means of observable state variables used to track time-varying investment opportunities. Absent such a proxy, we would need parametric modeling assumptions on how the conditional covariance is related to a list of candidate state variables, raising the danger of introducing model misspecification biases.

### 3 Construction of the Conditional Covariance Measure

The second step in our empirical analysis constructs the conditional covariance measure. We do so using a novel approach based on boosted regression trees (BRTs) which avoid imposing restrictive (linearity) assumptions on the unknown form of the projection of the realized covariance on economic state variables, while allowing for a large number of conditioning variables as advocated by Ludvigson and Ng (2007). We next describe our approach in detail and contrast it with a conventional linear approach.

#### 3.1 Boosted Regression Trees

Regression trees can be used to model how a dependent variable (here, the realized covariance),  $y_{t+1}$ , depends on a vector of predictor (state) variables,  $x_t$ , for  $t = 1, 2, \dots, T$ . Regression trees are characterized by the predictor variables they use to split the sample space and by their split points. For each split point we simply form two disjoint states,  $S_1, S_2$  and model the dependent variable as a constant,  $c_j$ , within each state,  $S_j$ ,  $j = 1, 2$ . The value fitted by a regression tree,  $T(x_t, \Theta)$ , with two nodes and parameters

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<sup>8</sup>The monthly plot winsorizes the October 2008 observation which generated a very large negative realized covariance. This was the result of extremely large negative (almost -18%) and very volatile returns during this month, accompanied by an improvement in the EAI during the first week of October followed by a deterioration in the index thereafter.

$\Theta = \{S_j, c_j\}_{j=1}^2$  can be written as a simple step function

$$\mathcal{T}(x_t, \Theta) = \sum_{j=1}^2 c_j I\{x_t \in S_j\}, \quad (7)$$

where the indicator variable  $I\{x_t \in S_j\}$  equals one if  $x_t \in S_j$  and is zero otherwise.

Under the objective of minimizing the sum of squared errors, the estimated constant,  $\hat{c}_j$ , becomes the simple average of  $y_{t+1}$  in state  $S_j$ :

$$\hat{c}_j = \frac{\sum_{t=1}^T y_{t+1} I\{x_t \in S_j\}}{\sum_{t=1}^T I\{x_t \in S_j\}}. \quad (8)$$

Regression trees are very flexible and can capture local features of the data that linear models overlook. Moreover, they can handle large-dimensional data without being as sensitive to outliers as linear models. This is relevant here because the identity of the best predictor variables is unknown and so must be determined empirically.

A single tree generally is too simple a model. Boosting uses the idea that combining a series of simple models can lead to more accurate forecasts than those available from a single model. Boosted regression trees (BRTs) are simply the sum of individual regression trees:

$$f_B(x_t) = \sum_{b=1}^B \mathcal{T}_b(x_t; \Theta_b), \quad (9)$$

where  $\mathcal{T}_b(x_t, \Theta_b)$  is the regression tree used in the  $b$ -th boosting iteration and  $B$  is the number of boosting iterations. Given the previous model,  $f_{B-1}(x_t)$ , the subsequent boosting iteration finds parameters  $\Theta_B = \{S_{j,B}, c_{j,B}\}_{j=1}^2$  for the next tree to solve

$$\hat{\Theta}_B = \arg \min_{\Theta_B} \sum_{t=0}^{T-1} [e_{t+1, B-1} - \mathcal{T}_B(x_t, \Theta_B)]^2, \quad (10)$$

where  $e_{t+1, B-1} = y_{t+1} - f_{B-1}(x_t)$  is the forecast error remaining after  $B - 1$  boosting iterations. The solution to (10) is the regression tree that most reduces the average of the squared residuals  $\sum_{t=1}^T e_{t+1, B-1}^2$  and  $\hat{c}_{j,B}$  is the mean of the residuals in the  $j$ th state. As the number of boosting iterations increases, the

area covered by individual states shrinks and the fit becomes better.

In summary, the BRT algorithm selects, by exhaustive search, the predictor variable whose sample space is going to be split, the optimal splitting point, and the constant value for the dependent variable in each region. The selected variable, the splitting points and the constant values are all chosen optimally to reduce the model’s residuals by the greatest amount; see (Hastie, Tibshirani, and Friedman (2009)).<sup>9</sup>

To reduce the risk of overfitting we adopt three common refinements to the basic regression tree methodology, namely (i) shrinkage, (ii) subsampling, and (iii) minimization of absolute errors. Specifically, following Friedman (2001) we use a small shrinkage parameter,  $\lambda = 0.001$ , that reduces the amount by which each boosting iteration contributes to the overall fit:

$$f_B(x_t) = f_{B-1}(x_t) + \lambda \sum_{j=1}^2 c_{j,B} I\{x_t \in S_{j,B}\}. \quad (11)$$

Each tree is fitted on a randomly drawn subset of the training data, whose length is set at one-half of the full sample, the default value most commonly used. Again this reduces the risk of overfitting. Finally, we minimize mean absolute errors to reduce the weight on extreme observations.

Our empirical analysis uses a range of state variables from Goyal and Welch (2008) designed to track time-varying investment opportunities. Specifically, we include the log dividend-price ratio, log earnings-price ratio, de-trended 3-month T-bill rate, yield on long-term government bonds, long-term returns, the term spread (10-year minus 3-month treasury yield), the default spread (yield spread between BAA and AAA-rated corporate bonds) and the inflation rate measured by the rate of change in the consumer price index. Additional details on data sources and the construction of these variables are provided by Goyal and Welch (2008). We add to this list the lagged realized covariance, for a total of nine predictors, all of which are appropriately lagged so they are known at time  $t$  for purposes of forecasting the realized covariance in period  $t + 1$ .

If each of the nine state variables used to model the conditional covariance had a linear effect on the conditional covariance, there would not be any need to use BRTs. To illustrate that nonlinearities are

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<sup>9</sup>Our estimations follow the stochastic gradient boosting approach of Friedman (2001, 2002) and employs  $B = 5,000$  boosting iterations. Robustness analysis revealed that the results are not sensitive to this choice.

important, Figure 4 provides so-called partial dependence plots. These show how each of the predictor variables maps into the conditional covariance plotted on the vertical axis. The BRT rules out interaction effects between the predictors so the marginal effect of one variable on the conditional covariance does not depend on the values taken by the other variables.

The plots in Figure 4 suggest marked non-linearities in the mapping from the predictors to the conditional covariance. For example, while the conditional covariance is predicted to be negative for negative values of the detrended T-bill rate ( $RREL_t$ ), it is predicted to be positive or close to zero for positive values of this variable. A simple threshold regression with nodes at -0.005 and 0 confirms the impression from this plot:

$$\begin{aligned} cov_{t+1} = & \alpha + \underset{(0.181)}{35.34} RREL_t \times I_{\{RREL_t < -0.005\}} + \underset{(0.080)}{94.54} RREL_t \times I_{\{-0.005 \leq RREL_t < 0\}} \\ & - \underset{(0.126)}{51.14} RREL_t \times I_{\{RREL_t \geq 0\}} + \gamma' x_t + \varepsilon_{t+1}. \end{aligned} \quad (12)$$

Here  $x_t$  are the remaining (eight) variables in the covariance specification. For negative values of  $RREL_t$  up to -0.005, there is a modestly increasing but insignificant relation between  $cov_{t+1}$  and  $RREL_t$ . This part resembles the flat portion of the plot. For values of  $RREL_t$  between -0.005 and zero we find a steep and positive relation between  $RREL_t$  and  $cov_{t+1}$ . Finally, for positive values of  $RREL_t$  we find a negative but insignificant relation between  $RREL_t$  and  $cov_{t+1}$ . A test of the null that the coefficients on  $RREL_t$  are identical across the three regions generates a  $p$ -value of 0.093.

The effect of most covariates on the conditional covariance is muted at very high or very low values of the predictors (the flat spots in Figure 4), showing no additional effect when moving from a very low (high) to a modestly low (high) value of each state variable. This makes the BRT approach more robust to outliers than linear models for which extreme values of  $x$  would generally have a stronger effect.<sup>10</sup>

A comparison of the range of fitted values across the nine predictor variables in Figure 4 suggests that the conditional covariance is most sensitive to variation in the lagged covariance, long term return,

<sup>10</sup>We test this feature by including an indicator variable that equals one if a particular predictor ( $x$ -variable) is in the top or bottom 10% of its range. We find that this indicator, when interacted with the  $x$ -variable to control for shifts in the slope, is significant at the 5% level for three of the predictor variables in a linear covariance model, namely long-term returns, the log dividend-price ratio, and the log earnings-price ratio.

detrended interest rate and the log dividend-price ratio. Variables with notably smaller marginal effects on the conditional covariance include past inflation and the long-term yield.

As a more formal way to test the BRT versus the linear covariance specification, we undertake a Ramsey RESET specification test that regresses the residuals from models fitted to the realized covariance series on the squared value of the nine predictors. If the functional form of the model is correct, then the squared value of the predictors (or any other transformation of these) should not be correlated with the residuals and this can be tested through a Wald test. The linear covariance model is always rejected by this test at the 5% level, whereas the BRT model fitted to the realized covariance based on EAI2 is not rejected at the 10% level. The presence of the outlier in October 2008 leads the BRT covariance model based on EAI1 to be rejected although, when this single outlier is removed, the specification test no longer rejects at the 10% level.

The top window in Figure 5 plots conditional covariance estimates generated by either a linear specification or the BRT model, in both cases using the nine predictors listed earlier. The two series are very different and the linear model generates more volatile and extreme estimates of the conditional covariance than the BRT model. Differences between the two estimates are particularly large during the recession in the early eighties and during the more recent global financial crisis.

### 3.2 Interpreting the Conditional Covariance Estimates

Whether the conditional covariance between stock market returns and changes to the EAI varies pro- or countercyclically is ultimately an empirical question. Counter-cyclical movements in the conditional covariance can be induced either by a stronger (positive) correlation between changes to the EAI and stock market returns during recessions or by a higher variance of the EAI and/or stock returns. Stock market volatility is known to follow a countercyclical pattern (Schwert (1989)). Similarly, economic uncertainty as measured, e.g., by the conditional volatility of macroeconomic variables, tends to be higher during recessions (Veldkamp (2005); Van Nieuwerburgh and Veldkamp (2006); Jurado, Ludvigson, and Ng (2014)), leading us to expect that the volatility of changes to the EAI is higher at such times.<sup>11</sup> Both fac-

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<sup>11</sup>Ferson and Merrick Jr (1987) also find evidence of business cycle related shifts in the parameters governing the joint distribution of consumption growth and stock returns.

tors induce countercyclical patterns in the conditional covariance. Turning to the correlation component, whether news about the state of the economy is more or less informative about future investment opportunities is an empirical question that may, in equilibrium, also depend on the market price of investment opportunity set risk.

Empirically, we find that movements in the conditional covariance are countercyclical. Specifically, the conditional covariance is 5.2% in recessions and 3.6% in expansions. Since the conditional correlations are almost identical in recessions and expansions (2.3% versus 2.1%), this difference is driven by the two conditional variances which are 22.6% (recession) versus 17.6% (expansions) for the EAI and 5.4% versus 4.5% for stock returns.

## 4 Estimating and Testing the ICAPM

Merton's ICAPM implies that time-variation in expected stock returns are linearly related to the conditional variance of stock market returns as well as variation in the conditional covariance. We next test this implication. To do so we also need an estimate of the conditional variance. In contrast to the case for the conditional covariance, few state variables other than the past variance and (signed) returns appear to possess much predictive power over the conditional variance. Following Glosten, Jagannathan, and Runkle (1993), we therefore estimate the following flexible EGARCH model (with  $p$ -values in parentheses):<sup>12</sup>

$$r_{t+1} = \underset{(0.001)}{0.04} + \underset{(0.029)}{0.01} dp_t - \underset{(0.001)}{1.58} Tbill_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, var_{t+1|t}) \quad (13)$$

$$\log(var_{t+1|t}) = \underset{(0.001)}{-0.23} + \underset{(0.001)}{0.96} \log(var_{t|t-1}) + \underset{(0.001)}{0.23} \left[ \frac{|\varepsilon_t|}{\sqrt{var_{t|t-1}}} - E \left( \frac{|\varepsilon_t|}{\sqrt{var_{t|t-1}}} \right) \right] - \underset{(0.001)}{0.08} \left( \frac{\varepsilon_t}{\sqrt{var_{t|t-1}}} \right) + \underset{(0.138)}{3.04} Tbill_t.$$

Here  $Tbill_t$  is the 3-month T-bill rate and  $dp_t$  is the dividend-price ratio, both of which are allowed to affect the conditional mean. The inclusion of these linear terms in the conditional mean in (13), while

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<sup>12</sup>Unlike Glosten et al. (1993) we model the logarithm of the conditional variance to ensure that the conditional variance is non-negative. Because we find little evidence that the state variables used to generate the BRT estimate of the conditional covariance are powerful in forecasting the variance of stock returns once information on the past variance and (signed) returns is included, we use the flexible EGARCH estimates rather than a boosted regression tree to model the conditional variance of returns. This is consistent with empirical studies such as Paye (2012) which find that lagged volatility is by far the most important predictor of future volatility. In addition, we find that outliers (jumps) in daily returns tend to make BRT estimates based on the realized variance quite noisy. An attractive robustness feature of our realized covariance measure is that the economic activity index rarely gets affected by outliers since jumps in one variable (daily returns) generally do not carry over to jumps in the other variable (daily changes in the EAI).



clearly an approximation, serves the purpose of extracting a more precise estimate of the conditional variance but only has a marginal effect on such estimates or on the resulting ICAPM analysis.<sup>13</sup>

This variance specification allows for rich dynamics as negative and positive shocks are allowed to have different effects on the conditional variance. Following Glosten et al. (1993), the conditional variance can also be influenced by the T-bill rate. This specification of the variance dynamics is comparable to estimates commonly used in the literature on volatility modeling, see, e.g., Bali (2008). The bottom window in Figure 5 shows a time-series graph of the conditional variance. This is clearly countercyclical as it increases during most recessions and comes down during expansions. Moreover, the time series of this conditional variance has a correlation of only 0.09 with the BRT conditional covariance series.

We test the ICAPM by regressing excess returns on the value-weighted CRSP index ( $r_{t+1}$ ) on the conditional variance ( $Var_{t+1|t}$ ) and conditional covariance ( $Cov_{t+1|t}$ ) measures:

$$r_{t+1} = \alpha + \beta_1 var_{t+1|t} + \beta_2 cov_{t+1|t} + \varepsilon_{t+1}. \quad (14)$$

Table 2 presents estimates for this model. Results in Panel A use EGARCH conditional variance estimates and BRT conditional covariance estimates. Consistent with the ICAPM, the coefficient estimate on the variance term is positive and statistically significant at the 10% level in the monthly data. Moreover, at a little higher than two, our model yields a sensible estimate of the representative investor's coefficient of relative risk aversion and is consistent with the range of values reported by Bali (2008) and Bali and Engle (2010). Similarly, the conditional covariance term obtains a positive coefficient with a  $p$ -value around 1, suggesting that time-varying investment opportunities are important in explaining the risk premium on stocks. The ICAPM's predictive  $R^2$  is 3.2% which is quite high compared to values typically found in studies of monthly return predictability, see, e.g., Goyal and Welch (2008).

To get a sense of the relative contributions of the variance and covariance terms, Panel A in Table 2 also shows the coefficient estimates and  $R^2$ -values when only one of these terms is included in (14). For the specification that only includes the conditional variance, we estimate a  $\beta_1$ -coefficient of 2.5 with

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<sup>13</sup>This is also true when we implement a two-step procedure that first uses the BRT approach to fit the mean of stock returns, then estimates an EGARCH model to the residual variation in returns and, finally, includes the resulting conditional variance estimate in the ICAPM.

a  $p$ -value of 4% and an  $R^2$  of 0.6%. For the model that only includes the conditional covariance, the estimate of  $\beta_2$  remains the same (0.11) with a  $p$ -value less than 1% and an  $R^2$  of 2.8%. These results suggest that the conditional covariance explains roughly three times as much of the variation in monthly stock returns as the conditional variance. Interestingly, although the EAI1 and EAI2 measures are very strongly correlated at the daily frequency, the  $R^2$  estimates in Table 2 suggest that the interest rate information used to construct EAI1 (but not EAI2) contains a small but persistent component that is correlated with subsequent returns. Indeed, the residuals from regressing EAI1 on EAI2 have a first-order serial correlation of 0.62 at the monthly horizon.<sup>14</sup>

In sharp contrast, if the conditional covariance measure is constructed using linear regression in place of our nonparametric projections, Panel B in Table 2 shows that the ICAPM coefficient estimate for  $\beta_2$  falls to 0.0, becomes insignificant ( $p$ -value of 0.47) and the resulting  $R^2$  obtained for the conditional covariance declines to essentially zero. The contrast between the estimated slope on the conditional covariance term in Panel A (BRT estimate) and Panel B (linear estimate) highlights the importance of using a flexible and robust estimation approach to construct the conditional covariance term in the ICAPM.

We next construct the realized covariance measure from quarterly data, thus reducing the effect of daily outliers, and use this to estimate quarterly conditional covariances. The ICAPM results, reported in Panel C in Table 2, continue to be strong. Specifically, the ICAPM estimate of  $\beta_1$  is 3.65 for the quarterly data, while the coefficient on both the variance and covariance terms continue to be significant at the 10% level (jointly) or at the 1% level when considered separately. The quarterly results for the EAI2 measure are slightly stronger than those using the EAI1 measure.

Figure 6 plots expected excess returns implied by (14) at the monthly (top window) and quarterly (bottom window) frequencies using the two economic activity indices. In both cases the time series for the conditional risk premium are very similar regardless of whether we construct the conditional covariance

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<sup>14</sup>To further address this point, we conduct an encompassing test that first computes the residuals from the ICAPM that uses EAI1 to compute the conditional covariance. We then regress these residuals on the conditional covariance measure based on EAI2 to see if this second covariance measure explains variations in future stock returns left unexplained by the covariance measure that uses EAI1. Finally, we reverse this regression to see if the return residuals from the ICAPM that uses EAI2 to compute the conditional covariance are predicted by variations in the covariance based on EAI1. We find that the model that uses the EAI1 covariance measure encompasses the model that uses the EAI2 measure to construct the conditional covariance, but not the reverse. This suggests that the interest rate information used by the EAI1 measure does make a significant difference when it comes to explaining time-varying expected returns.

from the EAI that uses only real economic variables or the EAI that also includes daily interest rates. As expected, the quarterly series is notably smoother, however.

#### 4.1 Economic Interpretation of findings

To interpret these results, particularly the positive sign on the covariance term in (14), it is useful to consider the ICAPM with stochastic volatility developed by Campbell, Giglio, Polk, and Turley (2014). Campbell et al. (2014) assume that investors have Epstein-Zin preferences and that the state variables in the economy,  $\mathbf{x}_t$ , follow a VAR process with time-varying volatility driven by the conditional variance of stock returns,  $\sigma_t^2$ . Specifically, the VAR takes the form

$$\mathbf{x}_{t+1} = \bar{\mathbf{x}} + \mathbf{\Gamma}(\mathbf{x}_t - \bar{\mathbf{x}}) + \sigma_t \mathbf{u}_{t+1}. \quad (15)$$

Here  $r_{t+1}$  and  $\sigma_{t+1}^2$  are the first and second elements of  $\mathbf{x}_{t+1}$ , respectively,  $\bar{\mathbf{x}}$  and  $\mathbf{\Gamma}$  are vectors and matrices of constant parameters and  $\mathbf{u}_{t+1}$  is a vector of shocks with constant variance-covariance matrix  $\mathbf{\Sigma}$  and  $\mathbf{\Sigma}_{11} = 1$ . This specification makes the stochastic volatility process affine.

Using this setting, Campbell et al. (2014) show that the expected excess return (adjusted for a convexity term) takes the approximate form

$$\begin{aligned} E_t[r_{t+1}] - r_{ft} + \frac{1}{2} \text{var}_t(r_{t+1}) &= \gamma \text{var}_t(r_{t+1}) + (\gamma - 1) \text{cov}_t \left( r_{t+1}, [E_{t+1} - E_t] \sum_{j=1}^{\infty} \rho^j r_{t+j+1} \right) \\ &\quad - \frac{\omega}{2} \text{cov}_t \left( r_{t+1}, [E_{t+1} - E_t] \sum_{j=1}^{\infty} \rho^j \sigma_{t+j}^2 \right), \end{aligned} \quad (16)$$

where  $\omega > 0$  is a positive scalar that is an increasing function of  $\gamma$  (for  $\gamma > 1$ ) and  $\gamma$  is the investor's coefficient of relative risk aversion.

Assuming that  $\gamma > 1$ , assets whose returns have a negative covariance with revisions to expected future returns (the first covariance term in (16)) will earn a lower risk premium. Such assets' returns tend to be high in states with bad news about expected future returns and so these assets provide a hedge.

Conversely, assets whose returns have a negative covariance with (conditional) variance shocks (the second covariance term in (16)) will earn a higher risk premium. Such assets pay low returns in states with bad news (higher variance) and thus do not provide a hedge against adverse shocks to investment opportunities.

Thus, if stocks hedge against shocks to future discount rates,  $cov_t \left( r_{t+1}, [E_{t+1} - E_t] \sum_{j=1}^{\infty} \rho^j r_{t+j+1} \right) < 0$ , they are more desirable and should command a lower risk premium. However, if stocks become more risky during bad economic states with low returns,  $cov_t \left( r_{t+1}, [E_{t+1} - E_t] \sum_{j=1}^{\infty} \rho^j \sigma_{t+j}^2 \right) < 0$ , they should command a higher risk premium.

We estimate the VAR specification in (15) as follows. Using monthly data from 1960-2012, we use weighted least squares to estimate a first-order VAR for the state variables  $\mathbf{x}_{t+1} = [r_{t+1} \text{ } RVAR_{t+1} \text{ } EP_{t+1} \text{ } TMS_{t+1} \text{ } DEFSPR_{t+1}]$ , where  $r_{t+1}$  is the excess return on the CRSP value-weighted index,  $RVAR_{t+1}$  is the realized variance computed from daily stock returns,  $EP_{t+1}$  is the log earnings-price ratio,  $TMS_{t+1}$  is the term spread, and  $DEFSPR_{t+1}$  is the default spread as defined earlier in Subsection 3.1.

The VAR specification used by Campbell et al. (2014) is different from the models considered earlier in our analysis such as (13). Despite such differences, their framework can be used to compute an estimate that approximates the relative importance of the expected return and conditional variance terms. Empirically, we find that low values of the EAI measure are associated with high values of both expected future returns and conditional variances. The positive coefficient on the conditional covariance term in the ICAPM therefore suggests that the conditional variance term dominates the expected return component. Consistent with this we find that for values of  $\gamma$  within the range we estimate empirically, the conditional variance term in (16) dominates the expected return component and the conditional variance effect is particularly strong during recessions, suggesting that investment opportunities become less attractive during times with low values of the EAI. Stated differently, provided that investors are not too risk tolerant, the conditional equity risk premium is more sensitive to variation in the conditional variance than to variation in expected returns.

Campbell et al. (2014) substitute consumption out of their analysis. However, the ICAPM and consumption-based models are equivalent representations of the same basic economic setup which suggests an alternative way to get intuition for the positive sign on the covariance term in the ICAPM. Suppose

that the derivative of the marginal utility of consumption with respect to the EAI is high when the EAI is low so reductions in economic activity increase the marginal utility of consumption. Further, suppose that stock market returns and the EAI are positively correlated as we find holds empirically.<sup>15</sup> Then we would expect the coefficient on the conditional covariance term in the ICAPM to be positive. Assets whose returns are higher in states with higher values of the EAI tend to perform better in states with high consumption growth given the positive correlation between consumption growth and the EAI (Table 1, Panel B). Conversely, such assets tend to have lower returns when the EAI and consumption growth is low and the marginal utility of consumption is high. Such assets must command a higher risk premium.

To obtain an explicit expression for how covariance risk relates to expected returns in a model that features both the conditional variance and conditional covariance, assume that investors are endowed with Epstein-Zin preferences. In such a setting, the expected excess return (adjusted for a convexity term) becomes (e.g., Campbell (2003))

$$E_t[r_{t+1}] - r_{ft} + \frac{1}{2}Var_t(r_{t+1}) = (1 - \theta)Var_t(r_{t+1}) + \frac{\theta}{\psi}Cov_t(\Delta c_{t+1}, r_{t+1}), \quad (17)$$

where  $\theta = (1 - \gamma)/(1 - 1/\psi)$ ,  $\gamma$  is a risk aversion parameter and  $\psi$  is the intertemporal elasticity of substitution parameter. Under power utility  $\gamma = 1/\psi$  and so  $\theta = 1$ . In this case the conditional variance term in (17) disappears. More broadly, when  $\gamma \neq 1/\psi$ , both the variance and covariance terms matter. Specifically, when  $\gamma > 1$  and  $\psi < 1$  (as many studies suggest, e.g., Mankiw (1981), Campbell and Mankiw (1989), Yogo (2004), and the review article by Campbell (2003)),  $\theta > 0$  and the effect of covariance risk on expected excess returns will be positive, consistent with our empirical finding.

## 4.2 Tests of the ICAPM Specification

To test more formally if the ICAPM in (14) is correctly specified, we report a range of diagnostic tests. If the ICAPM is correctly specified, the difference between the realized return and the predicted (fitted) return (i.e., the forecast error) should be uncorrelated with any function of the conditional variance or covariance regressors in (14). Such functions can be approximated by means of a polynomial in these

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<sup>15</sup>At the annual horizon, the correlation between the EAI and stock market returns is 0.63.

regressors. To test this implication we adopt the Ramsey RESET specification test described earlier. This projects the regression residual on the squared value of the regressors and uses a Wald test to test the null that they are jointly insignificant, as implied by a correct specification. The results from this test, provided in Panel A of Table 3, show no significant evidence against the ICAPM based on our conditional covariance measure.

An implication of the ICAPM (14) is that mean returns increase monotonically in both the conditional variance and the conditional covariance. To test this implication, we use the approach in Patton and Timmermann (2010). To see how this works for the conditional covariance, suppose we sort pairs of monthly observations into  $g = 1, \dots, G$  bins,  $\{r_{t+1}^g, \widehat{cov}_{t+1|t}^g\}$  ranked by the conditional covariance estimate. A monotonic mean-covariance relation implies that, as we move from periods with low conditional covariance to periods with high conditional covariance, mean returns should rise. Specifically, the null hypothesis is that the expected return increases when ranked by the associated value of  $\widehat{cov}_{t+1|t}^g$ :

$$H_0 : E \left[ r_{t+1}^g | \widehat{cov}_{t+1|t}^g \right] \geq E \left[ r_{t+1}^{g-1} | \widehat{cov}_{t+1|t}^{g-1} \right], \text{ for } g = 2, \dots, G. \quad (18)$$

Because  $\widehat{cov}_{t+1|t}^g > \widehat{cov}_{t+1|t}^{g-1}$ , this hypothesis says that the expected return associated with observations where the conditional covariance is high exceeds the expected return associated with observations with lower conditional covariance. The null that the conditional mean increases monotonically in the conditional covariance is rejected if there is sufficient evidence against it so that low  $p$ -values suggest a rejection of the ICAPM.<sup>16</sup>

For robustness, we perform the test on different numbers of bins,  $G$ , with 50 or 100 observations per bin. Panel B in Table 3 shows that the null hypothesis of a monotonically increasing relation between either the conditional variance and expected returns or between the conditional covariance and expected returns is not rejected. Hence, investors receive higher compensation for bearing increased variance risk as well as increased risk of unfavorable shifts in the investment opportunity set, i.e., covariance risk.

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<sup>16</sup>The test statistic has a distribution that, under the null, is a weighted sum of chi-squared variables whose critical values can be computed via Monte Carlo simulation.

### 4.3 Alternative Estimates of Covariance Risk

To evaluate the contribution of our new conditional covariance risk measure, it is important to compare it to existing measures from the literature. Previous studies such as Scruggs (1998) and Guo and Whitelaw (2006) also use estimates of covariance risk to test the ICAPM. In the absence of a proxy for realized covariance risk, these studies make model-dependent assumptions to identify the covariance risk component. Guo and Whitelaw (2006) assume that the covariance is a linear function of conditioning state variables. Specifically, let  $\mathbf{X}_{t+1} = (CAY_{t+1}, RREL_{t+1})'$  be a vector comprising the *CAY* variable of Lettau and Ludvigson (2001), and the de-trended T-bill rate, *RREL*.  $\mathbf{Z}_{t+1} = (v_{t+1}^2, CAY_{t+1}, RREL_{t+1})'$  adds the squared volatility,  $v_{t+1}^2$  to  $\mathbf{X}_{t+1}$ . Guo and Whitelaw assume that  $\mathbf{X}_{t+1}$  and  $\mathbf{Z}_{t+1}$  follow *VAR* processes

$$\begin{aligned}\mathbf{X}_{t+1} &= \mathbf{A}_0 + \mathbf{A}_1 \mathbf{X}_t + \epsilon_{X_{t+1}}, \\ \mathbf{Z}_{t+1} &= \mathbf{B}_0 + \mathbf{B}_1 \mathbf{Z}_t + \epsilon_{Z_{t+1}}.\end{aligned}\tag{19}$$

Using Merton's ICAPM along with the assumption that the hedge component,  $cov_{t+1|t}$ , is a linear function of  $\mathbf{X}_t$ , i.e.,  $\beta_2 cov_{t+1|t} = \phi_0 + \phi_1 \mathbf{X}_t$ , Guo and Whitelaw (2006) derive an equation for market excess returns

$$r_{t+1} = \phi_0 + \beta_1 \left[ \omega_0 + \omega_1 \mathbf{Z}_t - \rho \omega_1 (\mathbf{I} - \rho \mathbf{B}_1)^{-1} \epsilon_{Z_{t+1}} \right] + \phi_1 \left[ \mathbf{X}_t - \rho (\mathbf{I} - \rho \mathbf{A}_1)^{-1} \epsilon_{X_{t+1}} \right] + \epsilon_{t+1},$$

where  $\omega_0$  and  $\omega_1$  are functions of the coefficients in the  $\mathbf{B}$  matrices in (19) and  $\rho$  is a constant log-linearization term.

Scruggs (1998) models the conditional covariance through an EGARCH model for stock market returns which uses the T-bill rate to capture time-varying investment opportunities. Scruggs' univariate EGARCH-X-in-mean specification is most closely related to our approach

$$\begin{aligned}r_{t+1} &= \lambda_0 + \lambda_1 var_{t+1|t} + \lambda_2 Tbill_t + \epsilon_{t+1}, \\ \log(var_{t+1|t}) &= \theta_0 + \theta_1 \log(var_{t|t-1}) + \theta_2 \left( \frac{|\epsilon_t|}{\sqrt{var_{t|t-1}}} - E \left[ \frac{|\epsilon_t|}{\sqrt{var_{t|t-1}}} \right] \right) + \theta_3 \left( \frac{\epsilon_t}{\sqrt{var_{t|t-1}}} \right) + \theta_4 Tbill_t.\end{aligned}\tag{20}$$

The RESET specification test for the Guo-Whitelaw model (shown in Panel A of Table 3) suggests that this model is grossly misspecified. The Scruggs specification generates a  $p$ -value of 0.08, suggesting mild evidence that this model is misspecified. Monotonicity is rejected for the Scruggs model for the variance term (using bins with 50 observations) but not for the covariance term. Similarly, monotonicity tests reject the null of a monotonically increasing relation between the Guo-Whitelaw variance term and the covariance term based on RREL when 100 observations are included in each bin for the monotonicity test.

The BRT approach uses more instruments (nine) than the small set used by Scruggs or Guo and Whitelaw. To address the importance of this, we use the BRT approach based only on CAY and RREL to construct the conditional covariance measure. These are the two variables used by Guo and Whitelaw to construct their covariance measure (RREL is similar to the T-bill rate used in Scruggs' GARCH model). When we re-estimate the ICAPM using the conditional covariance measure based on this reduced set of instruments, the covariance term remains significant at the 5% level ( $p$ -value of 0.03) but the  $R^2$  declines from 3.2% to 1.2%, for the full model that includes both conditional variance and covariance terms. Moreover, when only the covariance term based on these two instruments is used to predict excess returns, the  $p$ -value on this term is 0.08 (previously below 0.01) and the  $R^2$  is 0.4% (previously 2.8%). These results suggest that using a relatively large set of instruments makes a difference.

#### 4.4 Out-of-sample forecast comparisons

To further evaluate our new conditional covariance measure and compare it against alternative approaches, we finally report the outcome of a pseudo out-of-sample forecast experiment.<sup>17</sup> This serves two purposes. First, out-of-sample forecasts can be used to assess the effect of overfitting. Second, the results of the analysis serve as an indication of the stability of the risk-return relation. Clark and McCracken (2005) argue that good in-sample forecasting results accompanied by a failure to produce good out-of-sample forecasts are indicative of model instability.

We use the first 10 years of the sample to construct initial estimates of the conditional variance and

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<sup>17</sup>We use the term “pseudo” out-of-sample to account for the fact that the forecasts are not generated in real time and there are various choices in such experiments (such as the sample split between the estimation and evaluation sample) which can affect the outcome.



covariances followed by a 20-year training sample used to estimate the ICAPM parameters. We then generate recursive forecasts of excess returns for the remainder of the period, 1990-2012, adding new data points as they become available.

Table 4 shows results from this exercise using the out-of-sample  $R^2$ -measure ( $R_{OoS}^2$ ) of Campbell and Thompson (2008). This is computed relative to the prevailing mean model so positive values indicate more precise forecasts than those from the prevailing mean, while negative values suggest less precise forecasts. At the monthly horizon (Panel A) the ICAPM that includes both conditional variance and covariance terms generates a slightly negative but insignificant  $R_{OoS}^2$ -value. A univariate model that only includes the conditional covariance measure generates an  $R_{OoS}^2$ -value of 1.00% (significant at the 10% level), while the model that uses only the EGARCH conditional variance generates an  $R_{OoS}^2$ -value of -1.06%. Hence, although the conditional variance is a significantly positive predictor of expected returns in-sample, it has insufficient power to improve equity premium forecasts out-of-sample.

When we use a linear model for the conditional covariance, the out-of-sample forecasting performance deteriorates substantially and the  $R_{OoS}^2$  becomes negative (-2.67% for EAI1 and -1.46% for EAI2).

Both the Guo-Whitelaw and Scruggs models generate large negative  $R_{OoS}^2$ -values, suggesting worse performance than the ICAPM and particularly than the forecasts based on our new covariance measure.

Quarterly results shown in Panel B of Table 4 suggest that the two ICAPM specifications based on the EAI1 and EAI2 measures produce good out-of-sample forecasts with positive  $R_{OoS}^2$ -values that are significant at either the 5% or 10% levels.

#### 4.5 Robustness to the conditioning information set

Following the analysis in Ludvigson and Ng (2007), we consider using a much larger information set. Specifically, suppose that a large set of state variables  $z_{it}, i = 1, \dots, N$  is generated by a factor model of the form  $z_{it} = \lambda_i' f_t + e_{it}$ , where  $f_t$  is a vector of common factors,  $\lambda_i$  is a set of factor loadings, and  $e_{it}$  is an idiosyncratic error. Using common factors as predictor variables rather than the  $N$  individual regressors achieves a substantial reduction in the dimension of the information set. We follow Ludvigson and Ng (2007) and extract factors through the principal components method. Their extended data contain 132

macroeconomic time series and 147 financial time series for a total of 279 series over the period 1959-2011. By considering this large set of predictor variables, we address a potentially important source of model misspecification caused by omitted variables. Following Jurado, Ludvigson, and Ng (2014) we use 12 factors extracted from the 279 economic time series and one factor extracted from their squared values. We add these 13 factors to the 9 baseline regressors in the construction of the conditional covariance.

Panel D in Table 2 shows that the ICAPM results continue to be strong for the conditional covariance estimates that include the factors in the conditioning information set. For EAI1 the effect of the conditional covariance continues to be strongly positive and significant at the 1% level, while for EAI2 the conditional covariance term generates a  $p$ -value of 0.06.

Our results are also robust to changes in the composition of the variables used to extract the EAI measure. For example, adding consumption growth to the list of economic variables does not alter the conclusions. Nor does using a daily term spread variable in place of the detrended T-bill rate alter the results—most notably, we continue to find that both the conditional variance and covariance terms obtain positive and significant coefficients in the ICAPM regression.

## 5 Conclusion

Merton's ICAPM is a cornerstone in the finance profession's understanding of the trade-off between risk and returns. A central part of the model is that expected returns depend not only on the conditional variance but also on a conditional covariance term or hedge factor. While many empirical studies have tested for a positive, linear relation between expected returns and the conditional variance, fewer studies have attempted to construct measures of conditional covariance risk, notable exceptions being Scruggs (1998), Guo and Whitelaw (2006) and Bali (2008).

Our paper develops a new daily economic activity index that summarizes news about the state of the economy obtained from mixed-frequency data and is shown to be strongly correlated with existing measures of time-varying investment opportunities. Using this daily index and daily stock returns we construct a proxy for realized covariances. We use nonparametric projections of this proxy on a large set of state variables previously linked to time-varying investment opportunities to construct an estimate of

the conditional covariance. We find economically strong and statistically significant evidence of a strongly positive relation between this conditional covariance term and expected returns. Our results thus suggest that information on real economic activity helps explain time variation in expected stock returns.

## References

- ANG, A., AND G. BEKAERT (2007): “Stock Return Predictability: Is it There?,” *Review of Financial Studies*, 20(3), 651.
- ARUOBA, S., F. DIEBOLD, AND C. SCOTTI (2009): “Real-Time Measurement of Business Conditions,” *Journal of Business & Economic Statistics*, 27(4).
- BAILLIE, R. T., AND R. P. DEGENNARO (1990): “Stock returns and volatility,” *Journal of financial and quantitative analysis*, 25(2), 203–214.
- BALI, T. G. (2008): “The intertemporal relation between expected returns and risk,” *Journal of Financial Economics*, 87(1), 101–131.
- BALI, T. G., AND R. F. ENGLE (2010): “The intertemporal capital asset pricing model with dynamic conditional correlations,” *Journal of Monetary Economics*, 57(4), 377–390.
- BALI, T. G., AND L. PENG (2006): “Is there a risk–return trade-off? Evidence from high-frequency data,” *Journal of Applied Econometrics*, 21(8), 1169–1198.
- BEKAERT, G., AND M. HOEROVA (2014): “The VIX, the variance premium and stock market volatility,” *Journal of Econometrics*, 183(2), 181–192.
- BOLLERSLEV, T., R. F. ENGLE, AND J. M. WOOLDRIDGE (1988): “A Capital Asset Pricing Model with Time-Varying Covariances,” *Journal of Political Economy*, 96(1), 116–31.
- BOLLERSLEV, T., AND H. ZHOU (2006): “Volatility puzzles: a simple framework for gauging return–volatility regressions,” *Journal of Econometrics*, 131(1), 123–150.

- BRANDT, M. W., AND Q. KANG (2004): “On the relationship between the conditional mean and volatility of stock returns: A latent VAR approach,” *Journal of Financial Economics*, 72(2), 217–257.
- BREEN, W., L. GLOSTEN, AND R. JAGANNATHAN (1989): “Economic significance of predictable variations in stock index returns,” *Journal of Finance*, 44(5), 1177–89.
- CAMPBELL, J., AND R. SHILLER (1988): “The dividend-price ratio and expectations of future dividends and discount factors,” *Review of Financial Studies*, 1(3), 195–228.
- CAMPBELL, J., AND S. THOMPSON (2008): “Predicting excess stock returns out of sample: Can anything beat the historical average?,” *Review of Financial Studies*, 21(4), 1509–1531.
- CAMPBELL, J. Y. (1987): “Stock returns and the term structure,” *Journal of Financial Economics*, 18(2), 373–399.
- CAMPBELL, J. Y. (2003): “Consumption-based asset pricing,” *Handbook of the Economics of Finance*, 1, 803–887.
- CAMPBELL, J. Y., S. GIGLIO, C. POLK, AND R. TURLEY (2014): “An intertemporal capm with stochastic volatility,” *Working Paper*.
- CAMPBELL, J. Y., AND L. HENTSCHEL (1992): “No news is good news: An asymmetric model of changing volatility in stock returns,” *Journal of financial Economics*, 31(3), 281–318.
- CAMPBELL, J. Y., AND N. G. MANKIW (1989): “Consumption, income and interest rates: Reinterpreting the time series evidence,” in *NBER Macroeconomics Annual 1989, Volume 4*, pp. 185–246. MIT Press.
- CLARK, T. E., AND M. W. MCCracken (2005): “The power of tests of predictive ability in the presence of structural breaks,” *Journal of Econometrics*, 124(1), 1–31.
- FERSON, W. E., AND J. J. MERRICK JR (1987): “Non-stationarity and stage-of-the-business-cycle effects in consumption-based asset pricing relations,” *Journal of Financial Economics*, 18(1), 127–146.

- FRENCH, K. R., G. W. SCHWERT, AND R. F. STAMBAUGH (1987): “Expected stock returns and volatility,” *Journal of Financial Economics*, 19(1), 3–29.
- FRIEDMAN, J. (2002): “Stochastic gradient boosting,” *Computational Statistics and Data Analysis*, 38(4), 367–378.
- FRIEDMAN, J. H. (2001): “Greedy function approximation: A gradient boosting machine,” *Annals of Statistics*, 29, 1189–1232.
- GHYSELS, E., P. SANTA-CLARA, AND R. VALKANOV (2005): “There is a Risk-Return Tradeoff After All,” *Journal of Financial Economics*, 76(3), 509–548.
- GLOSTEN, L., R. JAGANNATHAN, AND D. RUNKLE (1993): “On the relation between the expected value and the volatility of the nominal excess return on stocks,” *Journal of Finance*, 48(5), 1779–1801.
- GOYAL, A., AND I. WELCH (2008): “A Comprehensive Look at The Empirical Performance of Equity Premium Prediction,” *Review of Financial Studies*, 21(4), 1455–1508.
- GUO, H., AND R. WHITE LAW (2006): “Uncovering the risk return relation in the stock market,” *Journal of Finance*, 2(61), 1433–1463.
- HARRISON, P., AND H. ZHANG (1999): “An investigation of the risk and return relation at long horizons,” *Review of Economics and Statistics*, 81(3), 399–408.
- HARVEY, A. C. (1990): *Forecasting, structural time series models and the Kalman filter*. Cambridge university press.
- HARVEY, C. (2001): “The specification of conditional expectations,” *Journal of Empirical Finance*, 8(5), 573–637.
- HARVEY, C. R. (1989): “Time-varying conditional covariances in tests of asset pricing models,” *Journal of Financial Economics*, 24(2), 289–317.

- HASTIE, T., R. TIBSHIRANI, AND J. FRIEDMAN (2009): *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Springer.
- JURADO, K., S. C. LUDVIGSON, AND S. NG (2014): “Measuring uncertainty,” *American Economic Review (Forthcoming)*.
- KEIM, D., AND R. STAMBAUGH (1986): “Predicting returns in the stock and bond markets,” *Journal of Financial Economics*, 17(2), 357–390.
- LETTAU, M., AND S. LUDVIGSON (2001): “Consumption, aggregate wealth, and expected stock returns,” *the Journal of Finance*, 56(3), 815–849.
- LUDVIGSON, S., AND S. NG (2007): “The empirical risk-return relation: A factor analysis approach,” *Journal of Financial Economics*, 83(1), 171–222.
- LUNDBLAD, C. (2007): “The risk return tradeoff in the long run: 1836–2003,” *Journal of Financial Economics*, 85(1), 123–150.
- MANKIW, N. G. (1981): “The permanent income hypothesis and the real interest rate,” *Economics Letters*, 7(4), 307–311.
- MERTON, R. (1973): “An intertemporal capital asset pricing model,” *Econometrica*, 5(41), 867–887.
- NELSON, D. B. (1991): “Conditional heteroskedasticity in asset returns: A new approach,” *Econometrica*, pp. 347–370.
- PÁSTOR, L., M. SINHA, AND B. SWAMINATHAN (2008): “Estimating the intertemporal risk–return tradeoff using the implied cost of capital,” *The Journal of Finance*, 63(6), 2859–2897.
- PATTON, A. J., AND A. TIMMERMANN (2010): “Monotonicity in asset returns: New tests with applications to the term structure, the CAPM, and portfolio sorts,” *Journal of Financial Economics*, 98(3), 605–625.

- PAYE, B. S. (2012): “Déjà vol’: Predictive regressions for aggregate stock market volatility using macroeconomic variables,” *Journal of Financial Economics*, 106(3), 527 – 546.
- SCHWERT, G. W. (1989): “Why Does Stock Market Volatility Change over Time?,” *Journal of Finance*, 44(5), 1115–53.
- SCRUGGS, J. T. (1998): “Resolving the Puzzling Intertemporal Relation between the Market Risk Premium and Conditional Market Variance: A Two-Factor Approach,” *The Journal of Finance*, 53(2), 575–603.
- SCRUGGS, J. T., AND P. GLABADANIDIS (2003): “Risk premia and the dynamic covariance between stock and bond returns,” *Journal of Financial and Quantitative Analysis*, 38(02), 295–316.
- TURNER, C. M., R. STARTZ, AND C. R. NELSON (1989): “A Markov model of heteroskedasticity, risk, and learning in the stock market,” *Journal of Financial Economics*, 25(1), 3–22.
- VAN NIEUWERBURGH, S., AND L. VELDKAMP (2006): “Learning asymmetries in real business cycles,” *Journal of Monetary Economics*, 53(4), 753–772.
- VELDKAMP, L. L. (2005): “Slow boom, sudden crash,” *Journal of Economic Theory*, 124(2), 230–257.
- WHITELAW, R. (2000): “Stock market risk and return: an equilibrium approach,” *Review of Financial Studies*, 13(3), 521–547.
- WHITELAW, R. F. (1994): “Time Variations and Covariations in the Expectation and Volatility of Stock Market Returns,” *Journal of Finance*, 49(2), 515–41.
- YOGO, M. (2004): “Estimating the elasticity of intertemporal substitution when instruments are weak,” *Review of Economics and Statistics*, 86(3), 797–810.

## Appendix: Kalman filter extraction of the economic activity index

Let  $\mathcal{Y}_t \equiv \{\mathbf{y}_1, \dots, \mathbf{y}_t\}$  denote the current information set and, using the notation from (4)-(5), define the conditional expectation of the unobserved state given current and lagged information as  $\boldsymbol{\alpha}_{t|t} \equiv E(\boldsymbol{\alpha}_t | \mathcal{Y}_t)$ ,  $\boldsymbol{\alpha}_{t|t-1} \equiv E(\boldsymbol{\alpha}_t | \mathcal{Y}_{t-1})$ . Similarly, define the conditional variance estimates for the unobserved state  $\mathbf{P}_{t|t} \equiv \text{var}(\boldsymbol{\alpha}_t | \mathcal{Y}_t)$ ,  $\mathbf{P}_{t|t-1} \equiv \text{var}(\boldsymbol{\alpha}_t | \mathcal{Y}_{t-1})$ . Finally, let the expectation error be given by  $\mathbf{u}_t = \mathbf{y}_t - \boldsymbol{\beta}\boldsymbol{\alpha}_{t|t-1} - \boldsymbol{\Gamma}\mathbf{w}_t$ , while  $\mathbf{F}_t = \boldsymbol{\beta}\mathbf{P}_{t|t}\boldsymbol{\beta}' + \mathbf{H}$ .

Given these definitions, the following Kalman Filter equations are used to extract and update estimates of the latent variable that tracks the state of the economy:

$$\begin{aligned}\boldsymbol{\alpha}_{t|t} &= \boldsymbol{\alpha}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{Z}'\mathbf{F}_t^{-1}\mathbf{u}_t \\ \mathbf{P}_{t|t} &= \mathbf{P}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{Z}'\mathbf{F}_t^{-1}\mathbf{Z}\mathbf{P}_{t|t-1} \\ \boldsymbol{\alpha}_{t+1|t} &= \mathbf{T}_t\boldsymbol{\alpha}_{t|t} \\ \mathbf{P}_{t+1|t} &= \mathbf{T}_t\mathbf{P}_{t|t}\mathbf{T}_t' + \mathbf{R}\mathbf{Q}\mathbf{R}'\end{aligned}$$

The Kalman filter is well suited for handling missing data. If all elements of  $\mathbf{y}_t$  are missing, we can skip the updating step and the recursion becomes

$$\begin{aligned}\boldsymbol{\alpha}_{t+1|t} &= \mathbf{T}_t\boldsymbol{\alpha}_{t|t} \\ \mathbf{P}_{t+1|t} &= \mathbf{T}_t\mathbf{P}_{t|t}\mathbf{T}_t' + \mathbf{R}\mathbf{Q}\mathbf{R}'\end{aligned}$$

If only some (but not all) of the elements are missing, we modify the observation equation as follows:

$$\mathbf{y}_t^* = \boldsymbol{\beta}^*\boldsymbol{\alpha}_{t|t} + \boldsymbol{\Gamma}_t^*\mathbf{w}_t + \mathbf{u}_t^*, \quad \mathbf{u}_t^* \sim \mathcal{N}(\mathbf{0}, \mathbf{H}_t^*),$$

where  $\mathbf{y}_t^* = \mathbf{W}_t\mathbf{y}_t$ ,  $\boldsymbol{\beta}^* = \mathbf{W}_t\boldsymbol{\beta}$ ,  $\boldsymbol{\Gamma}_t^* = \mathbf{W}_t\boldsymbol{\Gamma}$ ,  $\mathbf{u}_t^* = \mathbf{W}_t\mathbf{u}_t$  and  $\mathbf{H}_t^* = \mathbf{W}_t\mathbf{H}\mathbf{W}_t'$  and  $\mathbf{W}_t$  is a matrix whose  $N_t^*$  rows are the rows of the identity matrix  $\mathbf{I}_N$  corresponding to the observed elements of  $\mathbf{y}_t$ . Model parameters are estimated using the prediction error decomposition.



**Table 1: Statistical and Economic Properties of the Economic Activity Indices**

<b>Panel A. Summary Statistics</b>						
	<u>EAI1</u>			<u>EAI2</u>		
Mean	0.00			0.01		
St.Dev	0.70			0.69		
Skewness	-1.03			-1.00		
Kurtosis	4.90			4.82		
AR(1)	0.97			0.97		
Exp_mean	0.21			0.21		
Rec_mean	-1.20			-1.18		

<b>Panel B. Correlation with Variables Tracking Time-Varying Investment Opportunities</b>						
	<u>EAI1</u>			<u>EAI2</u>		
	Corr	p-value	$R^2$	Corr	p-value	$R^2$
Dividend-Price Ratio	-0.52	0.00	27.5%	-0.52	0.00	27.2%
De-trended T-bill	0.45	0.00	20.0%	0.45	0.00	19.9%
Consumption Growth	0.23	0.05	5.30%	0.23	0.05	5.40%
Term Spread	-0.08	0.39	0.7%	-0.08	0.38	0.7%
Default Spread	-0.53	0.00	27.6%	-0.53	0.00	27.6%
CAY	0.07	0.73	0.43%	0.07	0.74	0.46%
Stock Returns	0.01	0.91	0.00%	0.01	0.92	0.00%
Realized Variance	-0.30	0.06	8.9%	-0.30	0.05	8.8%
Principal Component 1	0.53	0.00	28.5%	0.53	0.00	28.4%
Principal Component 2	0.28	0.01	7.6%	0.27	0.01	7.5%

<b>Panel C. Predictive Power of Economic Indices</b>						
	<u>EAI1</u>			<u>EAI2</u>		
	Coeff	p-value	$R^2$	Coeff	p-value	$R^2$
Monthly Stock Returns	-0.002	0.63	0.1%	-0.002	0.62	0.1%
Monthly Stock Market Variance	-0.002	0.06	7.4%	-0.002	0.06	7.3%
Quarterly Stock Returns	-0.010	0.26	0.6%	-0.009	0.28	0.6%
Quarterly Stock Market Variance	-0.004	0.00	8.7%	-0.004	0.00	8.6%
Annual Stock Returns	-0.075	0.02	10.1%	-0.076	0.02	9.9%
Annual Stock Market Variance	-0.009	0.07	6.7%	-0.010	0.07	6.8%

We construct two daily economic activity indices. The first measure (EAI1) extracts the index from data on daily interest rates, weekly initial jobless claims, monthly growth in real personal income and industrial production and quarterly GDP, while the second index (EAI2) removes the daily interest rate series from the list. Panel A provides summary statistics for these indices. Panel B correlates the indices with monthly observations on alternative measures of time-varying investment opportunities such as the dividend-price ratio, the detrended 3-month T-bill rate, consumption growth, term and default spread variables, the CAY variable of Lettau and Ludvigson (2001) (interpolated to obtain monthly series from quarterly figures), stock returns, realized variances estimated using daily stock returns and the first two principal components extracted from these series. We also show the  $R^2$  from univariate regressions of each variable on the economic activity indices. Finally, Panel C reports the outcome of predictive regressions of 1, 3 and 12-month stock returns and realized variances on the lagged economic activity indices. In all cases the sample runs from 1960 through 2012.

**Table 2: ICAPM Regression Estimates**

Panel A. Baseline Results							
EAI1				EAI2			
Constant	Variance	Covariance	$R^2$	Constant	Variance	Covariance	$R^2$
-0.00 (0.18)	2.01 (0.07)	0.11 (0.00)	3.2%	0.00 (0.15)	2.61 (0.01)	0.09 (0.01)	1.7%
0.00 (0.73)	2.48 (0.04)		0.6%	0.00 (0.73)	2.48 (0.04)		0.6%
0.00 (0.86)		0.11 (0.00)	2.8%	0.00 (0.51)		0.08 (0.01)	1.0%
Panel B. Results Using Linear Covariance Estimates							
EAI1				EAI2			
Constant	Variance	Covariance	$R^2$	Constant	Variance	Covariance	$R^2$
0.00 (0.73)	2.49 (0.06)	0.00 (0.47)	0.6%	0.00 (0.82)	2.38 (0.03)	0.00 (0.37)	0.6%
0.00 (0.73)	2.48 (0.04)		0.6%	0.00 (0.73)	2.48 (0.04)		0.6%
0.01 (0.03)		0.00 (0.45)	0.0%	0.01 (0.02)		-0.01 (0.77)	0.1%
Panel C. Quarterly Results							
EAI1				EAI2			
Constant	Variance	Covariance	$R^2$	Constant	Variance	Covariance	$R^2$
-0.02 (0.04)	3.65 (0.00)	0.04 (0.07)	5.6%	-0.03 (0.03)	3.64 (0.00)	0.07 (0.02)	6.7%
-0.02 (0.06)	4.26 (0.00)		4.7%	-0.02 (0.06)	4.26 (0.00)		4.7%
0.00 (0.64)		0.07 (0.00)	2.5%	0.00 (0.60)		0.08 (0.00)	3.5%
Panel D. Results with Factor-based BRT Estimates							
EAI1				EAI2			
Constant	Variance	Covariance	$R^2$	Constant	Variance	Covariance	$R^2$
-0.01 (0.09)	2.59 (0.04)	0.13 (0.00)	3.8%	0.00 (0.21)	2.74 (0.02)	0.07 (0.06)	1.4%
0.00 (0.74)	2.45 (0.07)		0.6%	0.00 (0.74)	2.45 (0.07)		0.6%
0.00 (0.99)		0.13 (0.00)	3.2%	0.00 (0.45)		0.07 (0.08)	0.7%

This table reports least squares estimates of the coefficients of the ICAPM along with heteroskedasticity and autocorrelation consistent p-values in brackets. We also show estimates for univariate models that include either the conditional variance or the conditional covariance term, but not both. The conditional covariance estimates reported in Panels A and B are constructed by using boosted regression trees (BRT) to nonparametrically project realized covariances on a set of conditioning state variables. The conditional variance is constructed using an EGARCH model that uses the dividend-price ratio and the T-bill rate as covariates in the mean equation and the T-bill rate as covariate in the variance equation. For comparison, Panel B uses a linear model to construct the conditional covariance measure. Panel C reports results when the ICAPM is fitted to observations sampled at the quarterly horizon. Panel D expands the set of conditioning variables using 12 common factors extracted from 279 economic time series and one factor from their squared values, as reported by Jurado, Ludvigsson and Ng (2014). Estimates in Panels A-C use data from 1960 to 2012, while the factor results in Panel D use data from 1960 through 2011:11.

**Table 3: Specification Tests for the ICAPM****Panel A. Ramsey RESET Tests**

	Wald-Test	p-value
Conditional Covariance: EAI1	0.09	0.92
Conditional Covariance: EAI2	0.21	0.81
Guo-Whitelaw	14.65	0.00
Scruggs	2.51	0.08

**Panel B. Monotonicity Tests**

Measure of Risk	50 Obs. per group	100 Obs. per group
Variance	0.409	0.527
Covariance : EAI1	0.858	0.996
Covariance : EAI2	0.437	0.981
Scruggs - Variance	0.000	0.174
Scruggs - Covariance	0.267	0.411
Guo-Whitelaw - Variance	0.722	0.075
Guo-Whitelaw - RREL	0.181	0.042
Guo-Whitelaw - CAY	0.998	0.984

Panel A reports the outcome of Ramsey RESET specification tests. The test first obtains the residuals from a linear ICAPM regression of stock returns on the conditional variance and covariance term obtained from the model listed in the corresponding row. If the linear ICAPM is correctly specified, these residuals should be uncorrelated with squares (or other transformations) of the conditional variance and covariance. We test this implication by regressing the ICAPM residuals on the squared values of the conditional variance and covariance and report a Wald test for their joint significance (p-value reported in the second column). Panel B reports the outcome of a test of a monotonically increasing relation between different conditional variance or covariance measures on the one hand and mean returns on the other. The null is that the mapping from the conditional variance and covariance terms into expected returns is increasing, so high p-values are consistent with the ICAPM, whereas low p-values suggest that the relationship is non-monotonic and therefore nonlinear, in contradiction of the ICAPM. For the Guo-Whitelaw approach, we provide separate tests for the individual instruments (detrended T-bill rate, RREL, and CAY). The test is based on forming bins of observations ranked by the magnitude of the conditional variance or covariance. Each bin contains either 50 (left column) or 100 observations (right column). The test statistic is based on the test described in Patton and Timmermann (2010).

**Table 4. Out-of-Sample Performance of Return Forecasts  
Generated by ICAPM Specifications**

**Panel A. Monthly Results**

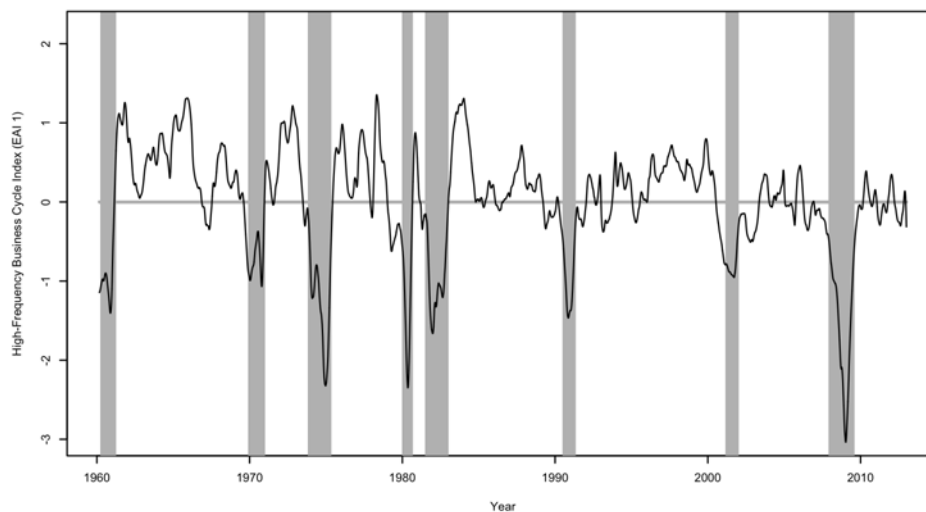
	OOS_R	McCracken p-value
ICAPM-(EAI1)	-0.07	(>0.10)
ICAPM-(EAI2)	-0.37	(>0.10)
Covariance (EAI1)	1.00	(<0.10)
Covariance (EAI2)	0.36	(<0.05)
EGARCH Variance	-1.06	(>0.10)
ICAPM-(EAI1) (Linear)	-2.67	(>0.10)
ICAPM-(EAI2) (Linear)	-1.46	(>0.10)
Guo-Whitelaw	-2.29	(>0.10)
Scruggs	-1.25	(>0.10)

**Panel B. Quarterly Results**

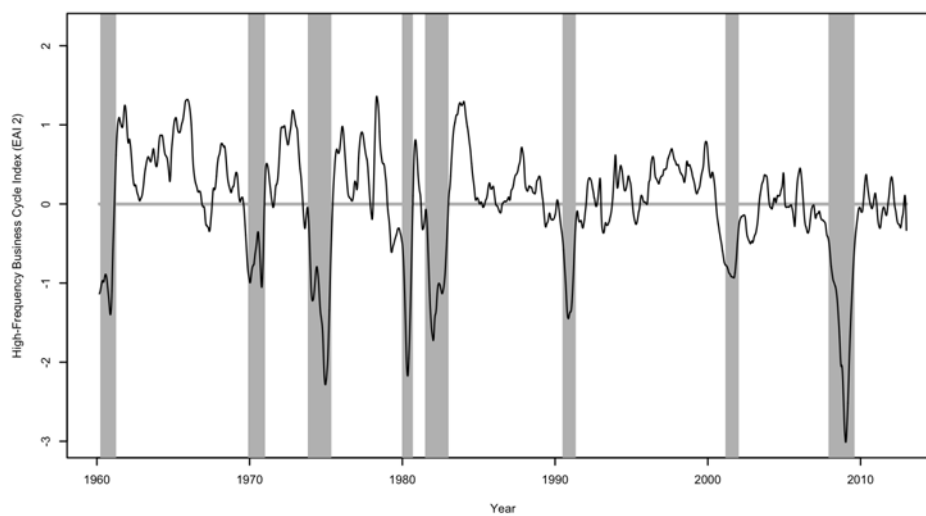
	OOS_R	McCracken p-value
ICAPM-(EAI1)	1.68	(<0.10)
ICAPM-(EAI2)	2.59	(<0.05)
Covariance (EAI1)	1.62	(<0.10)
Covariance (EAI2)	2.43	(<0.05)
EGARCH Variance	0.78	(<0.05)

This table reports the out-of-sample  $R^2$  measure of Campbell and Thompson (2008) along with McCracken (2007) p-values. Results are based on recursive one-step-ahead forecasts of monthly (panel A) or quarterly (panel B) stock returns. We use the first 10 years of the sample to obtain conditional variance and covariance estimates followed by a 20-year warm-up period to estimate the parameters of the ICAPM. The remaining sample (1990-2012) is used to evaluate the forecasts. Forecasts are generated using estimates based only on information up to the time of the prediction and parameter estimates are updated recursively as new information arrives, using an expanding estimation window. The Campbell-Thompson  $R^2$  value is computed relative to forecasts from a prevailing mean (constant expected return) model, with positive values suggesting that the model listed in the row generates more accurate return forecasts than this benchmark, while negative values suggest the opposite.

## Plots of the Daily Economic Activity Index Series



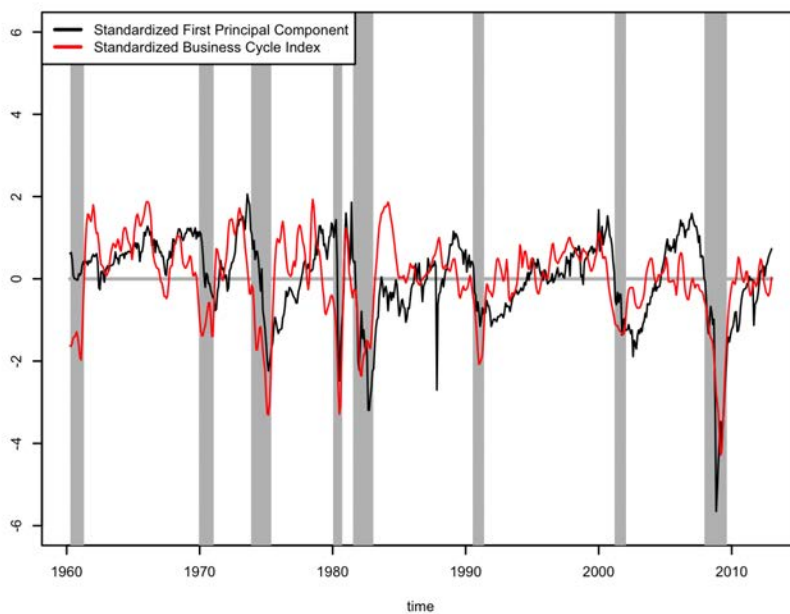
(a) EAI1



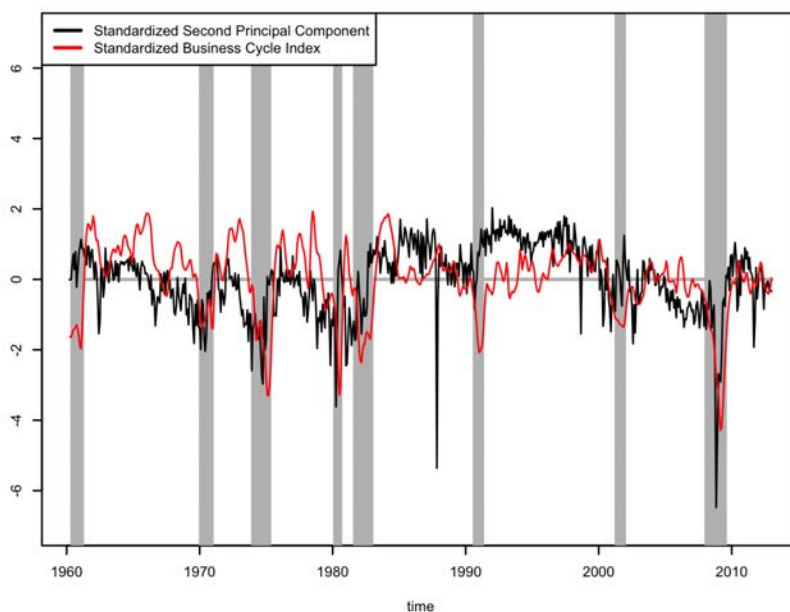
(b) EAI2

Figure 1: The plots show values for our two daily economic activity index series. The first measure (EAI1) extracts the index from data on daily interest rates, weekly initial jobless claims, monthly growth in real personal income and industrial production and quarterly GDP, while the second index (EAI2) removes daily interest rates from the list. Grey areas show recession periods as tracked by the NBER recession indicator.

## Principal Components Extracted from Variables Tracking Time-Varying Investment Opportunities Versus the Economic Activity Index



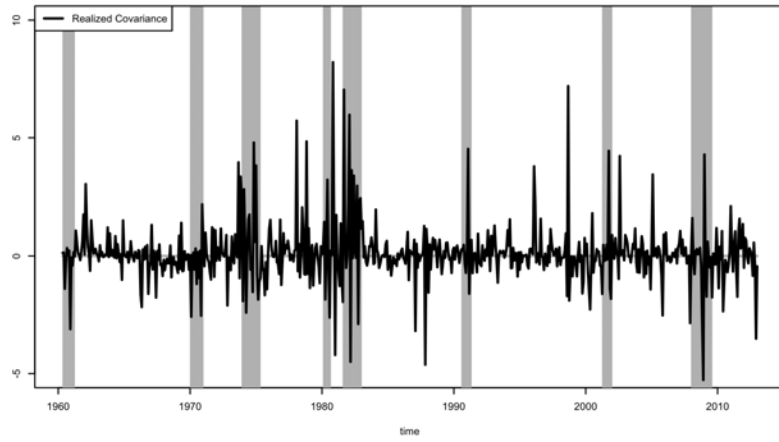
(a) First Principal Component



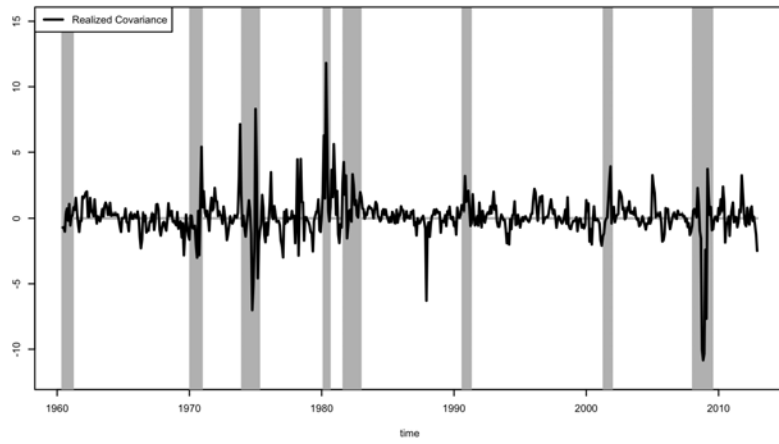
(b) Second Principal Component

Figure 2: We extract the first two principal components from the set of variables tracking time-varying investment opportunities listed in Panel B of Table 1. We then plot the first principal component (top window) and second principal component (bottom window) against the economic activity index EAI1. For comparability, the series have been standardized so they lie on a similar scale.

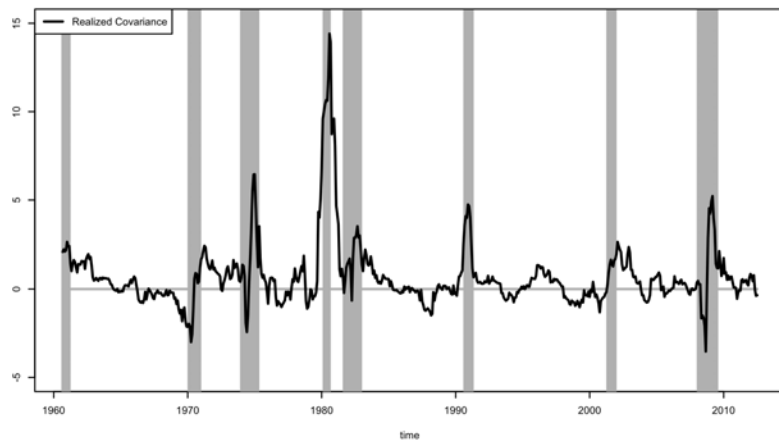
## Realized Covariance Series



(a) Monthly



(b) Quarterly



(c) Annual

Figure 3: The plots show monthly (top window), quarterly (middle window) and annual (bottom window) values of the realized covariance between daily changes to the economic activity index and daily stock returns. The monthly plot in the top window has winsorized the October 2008 observation.

## Partial Dependence Plots

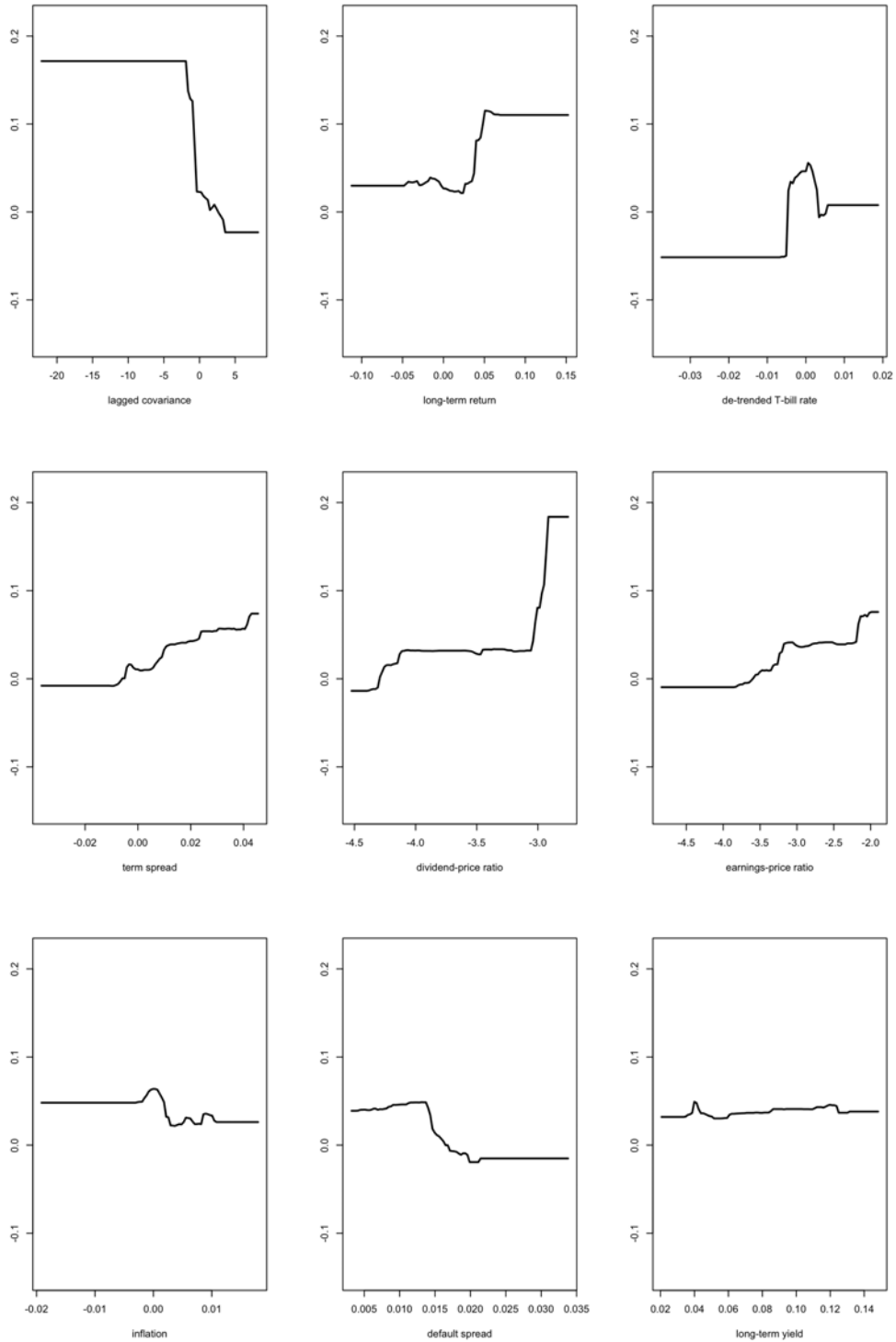


Figure 4: We use boosted regression trees to nonlinearly project the realized covariance on nine conditioning variables. Each panel plots the conditional covariance on the vertical axis against the predictor variable listed on the x-axis. The support of the x-axis corresponds to the actual values taken by each predictor. Thus, each plot shows how the marginal effect of each predictor on the conditional covariance changes with the range of each predictor variable.



## Time-Series Plots of the Conditional Variance and Covariance

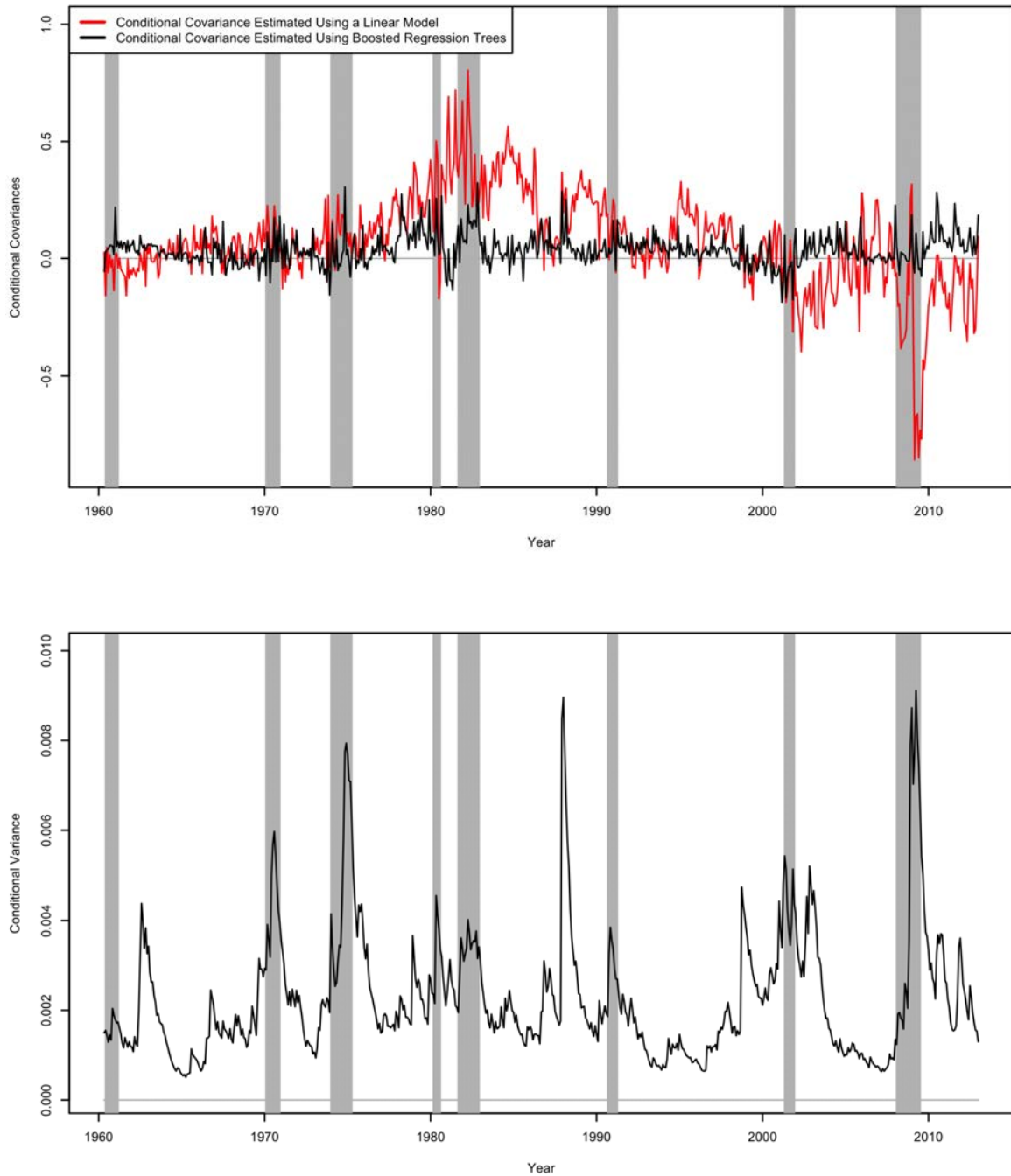
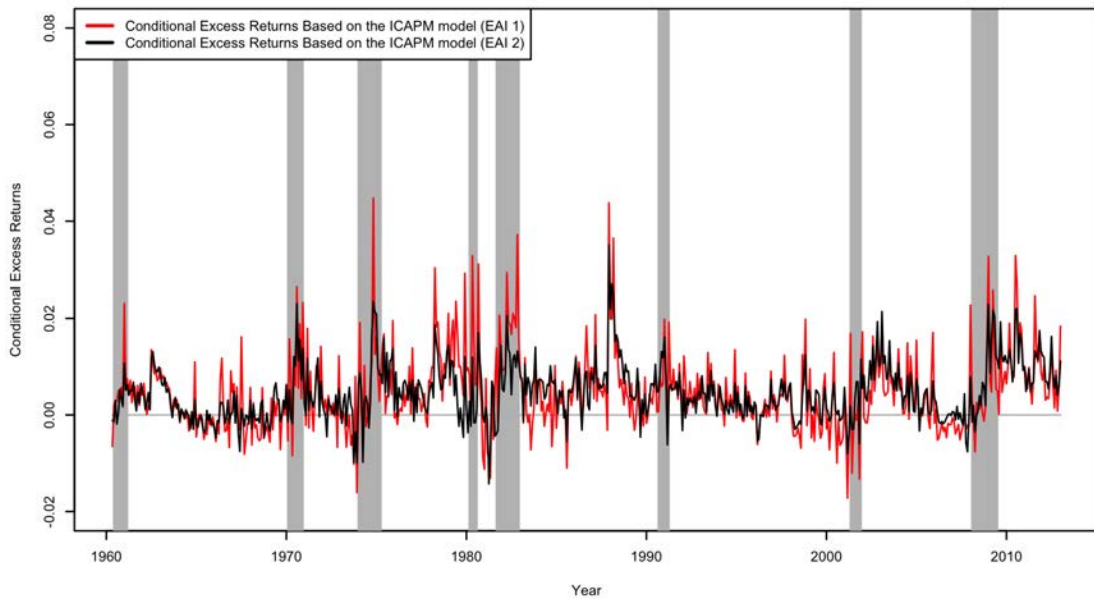
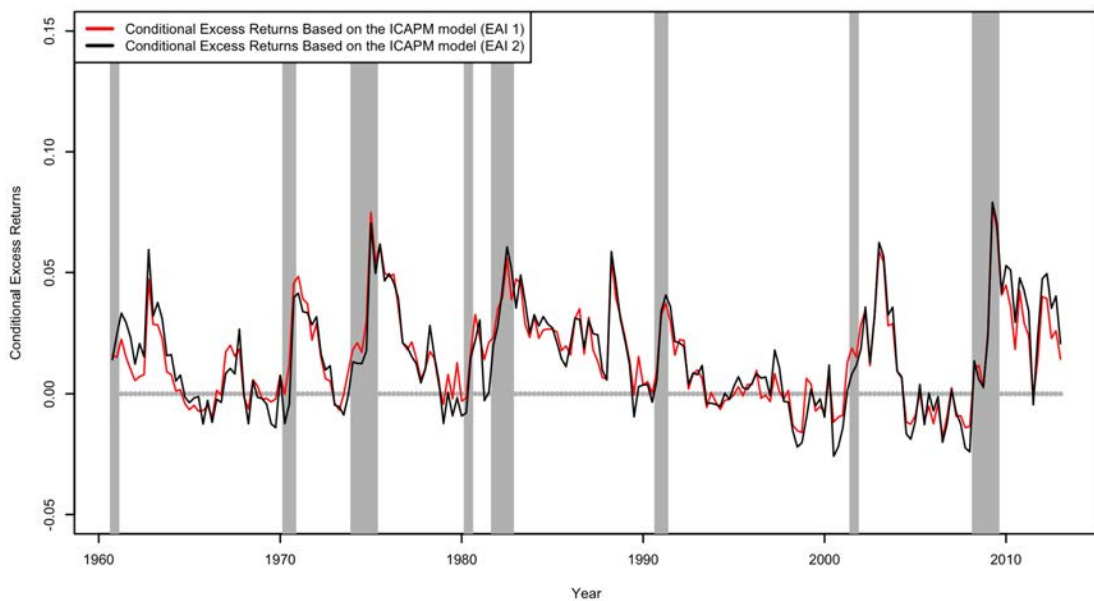


Figure 5: The top window plots conditional covariance estimates generated by fitting either a linear model or a boosted regression tree to the monthly realized covariance series. Both of these covariance models use nine predictors. The bottom window shows the conditional variance generated from an EGARCH model that uses the dividend-price ratio and the T-bill rate as covariates in the mean equation and the T-bill rate as covariate in the variance equation.

## Expected Returns Implied by the ICAPM



(a) Monthly Frequency



(b) Quarterly Frequency

Figure 6: The figures plot monthly (top window) and quarterly (bottom window) time series of expected excess returns (conditional risk premia) implied by the ICAPM specification that uses our nonparametric conditional covariance estimate. EAI1 and EAI2 represent different ways of constructing the underlying economic activity index (EAI1 extracts the index from data on daily interest rates, weekly initial jobless claims, monthly growth in real personal income and industrial production and quarterly GDP, while EAI2 removes the daily interest rate series from the list).