The Impact of the Manufacturer-hired Sales Agent on a Supply Chain with Information Asymmetry

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Abstract

This paper studies the impact of a manufacturer-hired sales agent on a supply chain comprising a manufacturer and a retailer. The sales agent is working mainly at the retailer's location in order to boost the demand. We focus on a wholesale price contract, under which the retailer decides how much to order from the manufacturer. The information structure within the supply chain and the efficiency of the sales agent affect the supply chain members' expected profits. We show that, due to the agency issue between the sales agent and the manufacturer, when the retailer's demand forecast accuracy is similar to the manufacturer's and the wholesale price is fixed, the retailer's profit is decreasing in his demand forecast accuracy. We also illustrate that when the retailer's forecast accuracy is much better than the manufacturer's and the wholesale price is endogenous, his expected profit is decreasing in his forecast accuracy. Moreover, we demonstrate that having a more efficient sales agent is beneficial for the retailer when the wholesale price is fixed, whereas it is not always the case when the wholesale price is determined depending on the efficiency of the sales agent.

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1 Introduction

Upstream manufacturers often sell their products to consumers through downstream retailers. Many factors influence this relationship between a manufacturer and a retailer in a supply chain. One such factor is a sales agent as an independent entity that can substantially alter the dynamics within the supply chain. Sales agents are widely employed in the industry. According to Zoltners et al. (2001), nearly twelve percent of the total workforce in the US is employed in full-time sales occupations. Furthermore, firms' average expenditure on salesforce typically ranges between ten percent to forty percent of their sales revenues (Albers and Mantrala 2008). In addition, enhanced salesforce management often can increase sales revenue by more than ten percent (Zoltners et al. 2008).

There are different types of sales agents depending on who hires them within a supply chain. In this paper, we focus on sales agents hired by upstream manufacturers. U.S. Department of Labor (2012b) has estimated that wholesale and manufacturing sales representatives held about two million jobs in the U.S. in 2010. Specifically, in the automobile industry, sales agents are often hired by auto manufacturers to boost demand at the dealer site. For example, these sales agents are called area sales managers in Toyota. Based on the job description of these sales agents in Toyota Financial Services (2012) and U.S. Department of Labor (2012a), the manufacturer-hired sales agents increase demand by improving sales techniques of the retailer and/or marketing. In this paper, we focus on the impact of a manufacturer-hired sales agent on the supply chain, in which the primary task of the sales agent is to boost demand.

We focus on two influential factors: (i) the sales agent's efficiency and (ii) the accuracy of the available demand information at the downstream level. The first factor is how efficient the sales agents are in fostering demand. A more efficient sales agent can increase demand by the same amount as the less efficient one but using a smaller amount of effort. The second factor is the degree to which information about local demand available at the downstream level (e.g., at the retailer site) and not available to the manufacturer. Our primary research question is how the expected profits of the manufacturer and the retailer depend on these two factors. For example, do the manufacturer and the retailer always benefit from a more efficient sales agent or more informed downstream parties?

To answer this research question, we examine a supply chain in which a manufacturer employs a wholesale price contract with a retailer for a single product and hires a sales agent to boost consumer demand. In this paper, we study both a fixed wholesale price model, and an endogenous wholesale price model in which the manufacturer sets the wholesale price optimally.

We first characterize the optimal linear compensation contract for the manufacturer-hired sales agent, and then demonstrate that even though more accurate demand information at the downstream level has a direct benefit to the supply chain, it can sometimes hurt the manufacturer, the retailer, or both due to strategic interaction between the manufacturer and the sales agent. Furthermore, we show that the impact of demand forecast accuracy at the downstream level on the retailer's expected profit depends critically on whether the wholesale price is fixed or set endogenously. In addition, we illustrate that the retailer benefits from a more efficient sales agent if the wholesale price is fixed. However, under an endogenous wholesale price model, the retailer may be hurt by a more efficient sales agent.

This paper proceeds as follows: In Section 2, we review the related literature. We then present the model in Section 3. We analyze the model with exogenous wholesale price, and endogenous wholesale price in Sections 4 and 5, respectively. Furthermore, we discuss extensions of our paper in Section 6. Finally, we provide our concluding remarks in Section 7.

2 Literature Review

Our paper explores the impact of a sales agent on operational decisions within a supply chain. One of the important problems related to a sales agent is how to structure the compensation scheme, which has been studied in marketing and economics literature. Coughlan (1993) provides a comprehensive review of salesforce management and compensation. For the compensation of the sales agent, we consider a menu of linear contracts and focus on the impact of sales agent contracting on the firm's operational decisions, i.e., production and inventory choices, which has been underexplored in the literature of marketing and economics.

The operations literature related to sales agents has studied the impact of sales agent compensation on a firm's operational decisions, e.g., inventory planning. Chen (2000) proposes a sales agent compensation package to induce the agent to exert an effort in a way that smooths out consumer demand across time. In practice, in particular at IBM, the compensation scheme proposed by Gonik (1978) has been implemented. This scheme intends to provide incentives to the agent to truthfully reveal what it knows about demand, based on the premise that the sales agent has more accurate demand information, and to work hard also. Studying Gonik's scheme and comparing it with a menu of contracts, Chen (2005) shows that Gonik's solution can be dominated by a menu of linear contracts when the firm also determines the inventory level. Analyzing sales agent contracting when demand information is censored by the inventory level, Chu and Lai (2012) demonstrate that the optimal contract with the agent takes the form of a sales-quota based bonus contract, and the optimal service level under this contract is always higher than the first-best level.

The new angle that we take in this paper is to understand *the impact of a manufacturer-hired sales agent on a supply chain*, whereas the previous literature has focused mostly on the impact to a single member of a supply chain (either a manufacturer or a retailer). One exception is Chen and Xiao (2012), who study the impact of the retailer's forecasting accuracy on both the manufacturer and the retailer in the presence of a sales agent. They focus on a *retailer-hired* sales agent without inventory consideration, whereas we analyze a *manufacturer-hired* sales agent with consideration of the retailer's inventory decision.

Among the few works that study the impact of a manufacturer-hired sales agent on the supply chain, Hopp et al. (2010) is the most relevant paper to this study. They consider a supply chain with a manufacturer-hired sales agent in a setting in which demand is *deterministic*. We also study the impact of a manufacturer-hired sales agent, which corresponds to their wholesale-salesperson, but under *stochastic* demand. Considering the demand uncertainties and information asymmetry, we can then answer our research question, which cannot be answered using the prior model in Hopp et al. (2010); we specifically develop a model to investigate how the impact of the sales agent depends on the forecast accuracy of uncertain demand.

3 The Model

We consider a supply chain in which a manufacturer (she) employs a wholesale price contract with a retailer (he), and she hires a sales agent (it) to boost demand. In Section 4, we study the fixed wholesale price case (e.g., because of a prior long term contract between the manufacturer and the retailer, or due to competition among potential manufacturers). In Section 5, we extend the model to allow the manufacturer to set the wholesale price optimally. We restrict attention to a wholesale price contract between the manufacturer and the retailer, which is common and the simplest contract form employed between firms.

3.1 Demand and Information Structure

Consumer demand is random and depends on the market condition Θ, and the effort e that the sales agent makes; specifically, consumer demand D is given as $D = \Theta + e$, where market condition Θ is normally distributed with mean θ and variance σ^2 (or, accuracy $h \equiv \frac{1}{\sigma^2}$), i.e., $\Theta \sim N(\theta, \sigma^2)$. Furthermore, the sales effort e can be interpreted as the amount of work that a sales agent needs to exert to increase demand by e units.

At the retail site, information about regional demand is available due to the direct contact with consumers and the potential availability of past sales data (see, e.g., Lee et al. 2000). Since this information is often not available upstream, the retailer as well as the sales agent, who works closely with the retailer to boost demand at the retail site, have an informational advantage about regional demand compared to the manufacturer. For example, according to the job description of the sales agent in Toyota, sales agents spend 75% of their work hours at the dealer site (Toyota Financial Services 2012). Consequently, the retailer's demand forecast is likely to be shared with the sales agent. To model this informational advantage, we assume that the sales agent and the retailer observe a private signal of demand, Ψ, that the manufacturer cannot obtain; specifically, the retailer and the sales agent observe a noisy signal that is normally distributed with a mean equal to the market condition, i.e., $\Psi|(\Theta = \hat{\theta}) \sim N(\hat{\theta}, \widetilde{\sigma}^2)$. We define *the accuracy of this signal for the downstream parties* as $\widetilde{h} \equiv \frac{1}{\widetilde{\sigma}^2}$.[1] Lastly, all parameter values and the information structure are common knowledge, and ψ is the only private information in our model.

3.2 The Sales Agent's Cost and Payoff

We assume an increasing and convex cost function for the sales agent's effort, reflecting the increasing marginal cost of effort; specifically, the cost of effort is $C(e) = \frac{1}{2k}e^2$, with $k > 0$ (see, e.g., Chen and Xiao 2012). Note that k can be interpreted as the *efficiency*, or the competency, of the sales

[1]We acknowledge that our model and the results are limited to the normal distribution for signals.

agent. If k is higher, the sales agent can boost demand at a lower cost. Without loss of generality, we normalize the sales agent's reservation payoff to zero.

The effort of the sales agent includes improving sales techniques of the retailer by providing him (*i*) more detailed product information, and (*ii*) marketing recommendations such as how to lay out advertising materials within the retail location (Toyota Financial Services 2012). Note that the sales agent's effort directly involves or requires permission of the retailer. As a result, this effort becomes observable by the retailer.[2]

For the compensation scheme of the sales agent, we focus on the form of a linear commission in the observed outcome combined with a constant salary. This form of payment is (*i*) widely used in practice (U.S. Department of Labor 2012b) as well as in research (see, e.g., Chen 2005) for sales agent compensation schemes, (*ii*) easy to implement, and (*iii*) an optimal contract form in a broad range of circumstances, as McAfee and McMillan (1987) have shown.

We consider the sales agent's compensation to be contingent on the manufacturer's demand (i.e., on the order quantity of the retailer). Note that the sales agent's effort is usually not contractible, which precludes the manufacturer from making the sales agent's compensation directly contingent on the effort.[3] Specifically, in this paper, a sales agent's compensation can be written as $P(q) = \alpha q + \beta$, where q is the retailer's order quantity; α is the commission rate, and β is the constant salary. Moreover, the manufacturer may offer a menu of linear contracts, $\{P_\psi(q) = \alpha_\psi q + \beta_\psi : \forall \psi\}$, which will be self-selected by the sales agent, where ψ is the realization of the noisy signal Ψ. One way to implement a menu of contracts is to employ a nonlinear compensation scheme, which has been observed in practice (see, e.g., Oyer 2000). Furthermore, the sales agent has the option not to engage in a contract.

3.3 The Retailer's and the Manufacturer's Profit Functions

We consider a single-period problem in which the manufacturer produces each unit of product at a constant unit-production cost c, and sells it to the retailer at a wholesale price w. The product is sold in the consumer market at retail price p and has zero salvage value. The insights in the

[2]However, in Section A.2 of the online supplement, we provide formal analysis of the case in which the effort of the sales agent is not observable by the retailer. We show that, in such a case, the sales agent does not exert any effort, and hence it does not have any impact on the supply chain.

[3]This assumption is also consistent with the field study of Hopp et al. (2010).

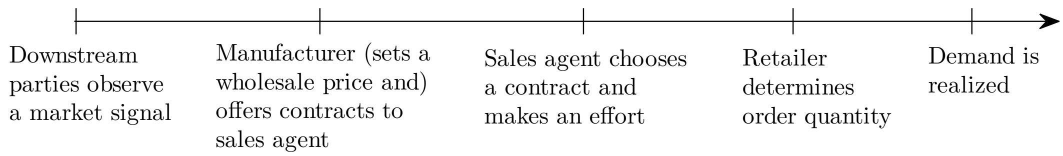

Figure 1: The sequence of events.

paper are preserved under a positive salvage value as long as it is less than the wholesale price. We assume $p > w > c > 0$.

The retailer's profit under order quantity q and realized demand x is $\Pi_R = p \min\{x, q\} - w\, q = (p - w)\, q - p\,(q - x)^+$, where $y^+ \equiv \max\{y, 0\}$. Similarly, the manufacturer's profit can be written as $\Pi_M = (w - c)\, q - P_\psi(q)$, where $P_\psi(q)$ is the compensation to the sales agent when she observes signal ψ and the retailer orders q. We refer to the sum of the retailer's and the manufacturer's profits as the supply chain's profit.

3.4 Sequence of Events

As illustrated in Figure 1, at the beginning, the downstream parties, i.e., the retailer and the sales agent, observe a signal Ψ of the market condition and update their belief about the market condition Θ. Then, the manufacturer offers a menu of linear contracts to the sales agent (and a wholesale price contract to the retailer under endogenous wholesale pricing in Section 5). The sales agent subsequently chooses a contract from the menu or rejects participating. Next, the sales agent makes an effort to increase consumer demand. After observing the sales agent's effort, the retailer submits his order quantity to the manufacturer.[4] The manufacturer then compensates the sales agent and delivers the order quantity to the retailer, and lastly demand is realized.

4 Exogenous Wholesale Price

In this section, we focus on the case in which it is very costly for the manufacturer to change the terms of her contract with the retailer; that is, we assume that the wholesale price is fixed at $\overline{w}$. We relax this assumption in Section 5 and explore the case in which the wholesale price is set by the manufacturer.

[4] In Section 6, we discuss the case in which the sales agent makes an effort after the retailer places his order to the manufacturer.

4.1 Equilibrium Analysis

We use backward induction to obtain the equilibrium. First, we analyze the retailer's problem of his optimal order quantity. Note that since the retailer and the sales agent observe a signal of the market condition, their belief about the market condition is updated according to Bayes rule. Specifically, from the retailer's and the sales agent's perspective, given their observed signal ψ, the market condition is $\Theta|(\Psi = \psi) \sim N(\frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}, \frac{1}{h+\widetilde{h}})$.

The retailer observes the effort of the sales agent e. Therefore, the retailer's expected profit can be written as:

$$\mathbb{E}[\Pi_R(q)] = (p-\overline{w})q - p\int_{-\infty}^{q} \Phi\left(\frac{y - e - \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}}{\frac{1}{\sqrt{h+\widetilde{h}}}}\right) dy\,,$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. The retailer's objective function is concave in q; thus, from the first order condition, we obtain the retailer's optimal order quantity as $q^*(\psi, e) = q_0 + e + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}$, where q_0 satisfies $\Phi\left(q_0\sqrt{h+\widetilde{h}}\,\right) = \frac{p-\overline{w}}{p}$.

Next, given the retailer's optimal order quantity and the compensation scheme (α, β), we then derive the sales agent's optimal effort level e^*. The sales agent's net payoff when exerting effort e is

$$\pi_s(e, \alpha, \beta|\Psi = \psi) = \alpha\Big(q_0 + e + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}\Big) + \beta - \frac{1}{2k}e^2\,. \tag{1}$$

This is a concave function of e; consequently, the sales agent's optimal effort is $e^*(\alpha) = k\,\alpha$.

Finally, we study the manufacturer's problem of designing an optimal menu of linear contracts to offer to the sales agent. Let $(\alpha_\psi, \beta_\psi)$ be the designated contract from the menu for the sales agent who observes $\Psi = \psi$ as a signal of the market condition. Then, we can write the manufacturer's problem as follows:

$$\max_{\alpha_\psi\geq 0,\,\beta_\psi} \quad \mathbb{E}_\psi\Big[\big(\overline{w} - c - \alpha_\psi\big)\big(q_0 + k\alpha_\psi + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}\big) - \beta_\psi\Big]$$

$$s.t. \quad \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi|\Psi = \psi) \geq \pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'}|\Psi = \psi)\,, \quad \forall\psi, \forall\psi'\,, \tag{IC}$$

$$\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi|\Psi = \psi) \geq 0\,, \quad \forall\psi\,. \tag{IR}$$

Constraint (IC) is the incentive-compatibility constraint to ensure that the sales agent with signal ψ chooses the designated contract, i.e., contract $(\alpha_\psi, \beta_\psi)$ is preferred over any other contract $(\alpha_{\psi'}, \beta_{\psi'})$ by the sales agent when observing signal ψ. Constraint (IR) is the individual-rationality constraint that requires the sales agent to gain at least as much as its reservation value. We now present the optimal menu of linear contracts to offer to the sales agent:

PROPOSITION 1. *Denote $\underline{\psi}$ as the unique solution to $k(\overline{w}-c)H(\underline{\psi}) = A$, where $A \equiv \sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}$. Then, the optimal menu of linear contracts offered to the sales agent is $\{P_\psi(q) = \alpha_\psi q + \beta_\psi : \forall\psi\}$, where*

$$\alpha_\psi = \frac{A}{k}\left(\frac{1}{H(\underline{\psi})} - \frac{1}{H\left(\frac{\psi-\theta}{\sqrt{\sigma^2+\widetilde{\sigma}^2}}\right)}\right)^+, \text{ and} \tag{2}$$

$$\beta_\psi = \frac{\widetilde{h}}{h+\widetilde{h}}\int_{-\infty}^{\psi} a_y \,\mathrm{d}y - \alpha_\psi\left(q_0 + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}} + \frac{k}{2}a_\psi\right). \tag{3}$$

$H(\cdot) \equiv \frac{\phi(\cdot)}{1-\Phi(\cdot)}$ is the hazard rate of the standard normal distribution, and $\phi(\cdot)$ is the PDF of the standard normal distribution.

Given the market signal ψ, the corresponding order quantity of the retailer q_ψ, the effort level e_ψ, the expected effort of the sales agent $\mathbb{E}_\psi[e_\psi]$, and the expected profits of the retailer $\mathbb{E}[\Pi_R]$, and the manufacturer $\mathbb{E}[\Pi_M]$, in equilibrium, are as follows:

$$q_\psi = q_0 + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}} + k\alpha_\psi, \qquad e_\psi = k\,\alpha_\psi, \tag{4}$$

$$\mathbb{E}[e_\psi] = k(\overline{w}-c)\left(1-\Phi(\underline{\psi})\right)\left(1 - H(\underline{\psi})^2 + \underline{\psi}H(\underline{\psi})\right), \tag{5}$$

$$\mathbb{E}[\Pi_R] = (p-\overline{w})\left(q_0+\theta+\mathbb{E}[e_\psi]\right) - p\int_{-\infty}^{q_0}\Phi\left(\sqrt{h+\widetilde{h}}\,y\right)\mathrm{d}y, \tag{6}$$

$$\mathbb{E}[\Pi_M] = (\overline{w}-c)(q_0+\theta) + \frac{1}{2k}\mathbb{E}[e_\psi^2]. \tag{7}$$

As shown in (2), the commission rate α_ψ of the sales agent depends on both how well it knows consumer demand, which is measured by $h+\widetilde{h}$, and how much better it knows demand than the manufacturer, which is measured by $\frac{\widetilde{h}}{h}$. The commission rate α_ψ is increasing in the absolute measure of the sales agent's information (i.e., $h+\widetilde{h}$), whereas it is decreasing in the relative measure of the sales agent's information to the manufacturer (i.e., $\frac{\widetilde{h}}{h}$). Furthermore, the more efficient the sales agent is (the larger k), the higher the commission rate of the sales agent becomes.

When there is information asymmetry between the manufacturer and the sales agent, the manufacturer does not induce the sales agent who observes a signal of a very poor market condition to exert any effort. This is because the manufacturer cannot verify whether the sales agent is truthful in selecting a contract that is designed for a poor market condition signal. It could be that the sales agent who observes a prosperous market condition signal selects a contract designed for a poor market condition signal so that it can collect a larger payoff after the retailer's order quantity is realized. Therefore, there is a threshold such that the commission rate of the sales agent is zero if it observes a market condition signal lower than that threshold. From Proposition 1, this threshold on ψ is given by $\theta + \underline{\psi}\sqrt{\sigma^2 + \widetilde{\sigma}^2}$.

Note that from (4), the retailer orders what he would have ordered if there had been no sales agent, $q_0 + \frac{h\theta + \widetilde{h}\psi}{h + \widetilde{h}}$, plus the extra demand that the sales agent has promoted, $k\alpha_\psi$. The effort of the sales agent is proportional to the commission rate, and is increasing as it becomes more efficient (larger k). In the following sections, we discuss the expected effort of the sales agent and the expected profits of the manufacturer and the retailer in detail.

4.2 Impact of Downstream Signal Accuracy on Supply Chain

In this section, we analyze how the decisions and expected profits of supply chain members depend on the accuracy of the downstream parties' signal of demand. More specifically, in the presence of a manufacturer-hired sales agent, we study how the expected profits of the manufacturer and retailer depend on the availability of market demand information at the downstream level.

4.2.1 Impact on the Effort of the Sales Agent

First, we examine how the sales agent's effort changes if the downstream parties observe a more accurate demand forecast.

Proposition 2. *The expected effort of the sales agent decreases in the accuracy of the downstream signal of demand* $\widetilde{h}$.

As the accuracy of downstream signal improves, the retailer relies more on the demand signal when he decides his order quantity; specifically, from the retailer's perspective, the expected demand is written as $\frac{h}{h+\widetilde{h}}\,\theta + \frac{\widetilde{h}}{h+\widetilde{h}}\,\psi$. As the accuracy $\widetilde{h}$ increases, the weight $\frac{\widetilde{h}}{h+\widetilde{h}}$ that the retailer puts on his signal ψ in calculating the expected demand increases, whereas the weight $\frac{h}{h+\widetilde{h}}$ that he puts on the prior expected demand θ decreases. As a result, the order quantity of the retailer is influenced more by his forecast. In other words, because the manufacturer does not know the downstream forecast, she sees greater variability in the retailer's order quantity. Given that the payment of the sales agent depends on the order quantity of the retailer, when the manufacturer decides on the menu of contracts to offer to the sales agent, it becomes more difficult for the manufacturer to determine how the sales agent values a certain contract and whether that contract satisfies the agent's reservation utility. Consequently, the manufacturer has to pay a larger information rent to incentivize the sales agent to participate. Therefore, the marginal cost of inducing the sales agent to put a certain amount of effort increases for the manufacturer; thus, in equilibrium, the manufacturer offers a contract that induces less effort.

4.2.2 Impact on the Retailer's Expected Profit

If we consider the relationship between the manufacturer and the retailer under the fixed wholesale price contract, ignoring the presence of a manufacturer-hired sales agent, a retailer should benefit from having more accurate demand information because he can better match his order quantity with demand. In this section, we show that if we take the manufacturer-hired sales agent into consideration, the retailer's expected profit can decrease in the accuracy of his demand forecast. Proposition 3 characterizes how the retailer's expected profit changes as a function of his forecast accuracy $\widetilde{h}$, and provides conditions under which the retailer's expected profit decreases as more information about demand becomes available at the downstream level.

Proposition 3. *There exists a threshold $\widetilde{h}_1$ such that the retailer's expected profit is decreasing in $\widetilde{h}$, if and only if $\widetilde{h} < \widetilde{h}_1$. Furthermore, $\widetilde{h}_1$ is the unique solution to*

$$\sqrt{\frac{h}{\widetilde{h}_1}} \int_{v(\widetilde{h}_1)}^{\infty} \Big(1 - \Phi(x)\Big)\,\mathrm{d}x \;=\; \frac{p}{p-\overline{w}}\phi\Big(\Phi^{-1}(\frac{p-\overline{w}}{p})\Big)\,,$$

and $v(y) = H^{-1}\Big(\frac{1}{k(\overline{w}-c)} \cdot \sqrt{\frac{y}{h(h+y)}}\Big)$.

If the supply chain is integrated, the supply chain surplus would be higher as more information becomes available at the downstream level. However, this proposition implies that when supply chain is disintegrated, it may not be beneficial for the retailer to know more about market demand, especially if the downstream parties do not know much more about demand than the manufacturer. This insight is particularly helpful when the wholesale price is fixed, for example, due to competition among manufacturers.

The drivers of this result are (i) the lower expected effort of a sales agent with a more accurate demand forecast and (ii) better prediction of demand. The retailer's expected profit depends on both the effort of the sales agent and his ability to predict demand. When more information about demand is available at the downstream level, the sales agent's effort becomes lower, as shown in Proposition 2. Therefore, the retailer's expected profit can decrease because of the reduced sales agent's effort. On the other hand, a retailer with a more accurate demand forecast can better predict demand and, thus, better match his order quantity with demand; that is, the retailer's expected profit can be higher if he knows more about demand. When the downstream signal of demand is less accurate and below a threshold ($\widetilde{h} < \widetilde{h}_1$), the first driver (lower effort of the sales agent) dominates the trade-off; in other cases ($\widetilde{h} \geq \widetilde{h}_1$), the second driver (a better demand prediction) dominates.

Furthermore, the reason the first driver (the effect of sales effort) dominates the second driver (the effect of a better demand predictor) under less forecast accuracy, i.e., $\widetilde{h} < \widetilde{h}_1$, can be explained as follows. When $\widetilde{h}$ is very small, there is very little information asymmetry between the manufacturer and the sales agent; thus, the information rent paid to the sales agent is small. By increasing $\widetilde{h}$, this information rent increases considerably. Consequently, as $\widetilde{h}$ increases, the manufacturer reduces the commission rate paid to the sales agent substantially to avoid paying large information rent. This strategy results in a significant decrease in the sales agent's effort. The ability of the retailer to predict demand, however, does not change much if $\widetilde{h}$ increases in this range. Hence, when $\widetilde{h}$ is small, the impact of lower effort dominates the impact of better prediction.

Taylor and Xiao (2010) show that the retailer's expected profit can be decreasing in the accuracy of retailer's demand forecast. Their reason is that as downstream parties learn more about demand, the manufacturer increases the wholesale price, which hurts the retailer. We complement this work by providing another reason why the retailer may not be willing to invest in improving his demand forecast. In our model, although the wholesale price is fixed, the retailer's expected profit can decrease as his forecasting accuracy increases due to contracting considerations involving the sales agent's effort.

4.2.3 Impact on the Manufacturer's Expected Profit

Does the manufacturer benefit from the retailer's more accurate demand forecast? In a supply chain without a sales agent, one can show that when the wholesale price is fixed, the manufacturer's expected profit is monotone (increasing or decreasing) in the accuracy of the retailer's demand forecast. In this section, we show that in the presence of a manufacturer-hired sales agent, there exists a threshold above which the manufacturer's expected profit increases in the accuracy of the downstream parties' signal of demand, and below which it decreases.

PROPOSITION 4. *The manufacturer's expected profit is quasi-convex in the accuracy of the downstream parties' demand signal. Technically, there exists a threshold $\widetilde{h}_2$, such that the manufacturer's expected profit decreases in $\widetilde{h}$, if and only if $\widetilde{h} < \widetilde{h}_2$. Furthermore, for $\overline{w} > p/2$, $\widetilde{h}_2 < \infty$ is the unique solution of*

$$\int_{v(\widetilde{h}_2)}^{\infty} \sqrt{\frac{h}{\widetilde{h}_2}} \Big(1 - \Phi(x)\Big) \left(k(\overline{w} - c) - \sqrt{\frac{\widetilde{h}_2}{h(h + \widetilde{h}_2)}} \frac{1}{H(x)}\right) \mathrm{d}x \;=\; -k(\overline{w} - c)\Phi^{-1}(\frac{p - \overline{w}}{p}),$$

and for $\overline{w} < p/2$, the manufacturer's expected profit monotonically decreases in the accuracy of the downstream signal (i.e., $\widetilde{h}_2 = \infty$).

When the wholesale price is small (precisely, when $\overline{w} < p/2$), the expected profit of the manufacturer decreases in the accuracy of the downstream signal for the following two reasons. First, the safety stock of the retailer, i.e., the quantity ordered above the expected demand, decreases. Specifically, when the retailer's demand forecast is normally distributed with mean μ and standard deviation σ, the order quantity of the retailer is $\mu + z \times \sigma$, where z is a fractile solution that depends on the underage and overage costs. The value of z is positive when $\overline{w} < p/2$, and does not change as the accuracy of the demand forecast improves. Therefore, as the standard deviation of the demand forecast decreases (i.e., demand forecast improves), $z \times \sigma$ decreases; thus, the retailer orders less safety stock. Second, the manufacturer has to pay a larger information rent to the sales agent, which leads to a smaller demand-promoting effort on the part of the sales agent as the information accuracy of the downstream parties' signal increases, as discussed in Section 4.2.1. In summary, when the wholesale price is low, the manufacturer's expected profit decreases both due to the decrease in the retailer's safety stock and the decrease in the expected demand as a result of the decrease in the resulting effort of the sales agent.

On the other hand, when the wholesale price is high (i.e., $\overline{w} > p/2$), the retailer orders less than the expected demand (i.e., $z < 0$). Consequently, as the downstream parties learn more about the demand, (i) the retailer orders more, given the effort level, and (ii) the expected demand decreases due to the smaller induced effort level as explained previously. The former effect benefits the manufacturer, whereas the latter hurts her. When the information accuracy is large ($\widetilde{h} > \widetilde{h}_2$), the first driver dominates; thus, the manufacturer's expected profit increases as the downstream signal becomes more accurate. When the downstream signal is inaccurate ($\widetilde{h} < \widetilde{h}_2$), the second driver dominates the first one, and a more accurate downstream signal hurts the manufacturer.

4.3 Impact of Sales Agent Efficiency on the Supply Chain

When hiring a sales agent, most companies would seek to hire a more efficient one. As a result, one would expect that the manufacturer's expected profit increases if a more efficient sales agent is hired. But, would the retailer also benefit from a more efficient sales agent?

PROPOSITION 5. *The expected profits of the retailer and the manufacturer and the demand-promoting efforts of the sales agent are increasing in the efficiency of the sales agent, k, under a fixed wholesale price contract.*

As the agent becomes more efficient, it becomes easier for the sales agent to boost demand. With a specific commission contract, a more efficient sales agent consequently makes more effort to increase her net payoff. Knowing this behavior of the sales agent, the manufacturer induces the more efficient sales agent to increase effort, while it pays the agent less for each unit of effort. Therefore, demand increases at a lower cost, and both the retailer and the manufacturer benefit. However, under an endogenous wholesale price model, the manufacturer's optimal wholesale price may differ for sales agents with different efficiencies. In Section 5, we study the impact of an endogenous wholesale price on the supply chain.

Remark. Based on Propositions 3 and 4, we find that when $\widetilde{h} < \min\{\widetilde{h}_1, \widetilde{h}_2\}$, both the retailer's and the manufacturer's expected profits decrease in $\widetilde{h}$.

When the contract between the manufacturer and the retailer is fixed due to long term contracts or competitive factors, the above remark implies that if the downstream forecast information lacks accuracy substantially, the manufacturer should not provide her retailer or sales agent with technology/training to improve the forecast, because it can hurt both the manufacturer and the retailer. Rather, as Proposition 5 demonstrates, the manufacturer should focus more on hiring a cost efficient sales agent in this case.

5 Endogenous Wholesale Price

If the manufacturer hires a more efficient sales agent, she may charge a higher wholesale price to the retailer because the sales agent may increase the retailer's demand more. In order to incorporate this decision of the manufacturer, in this section, we allow the manufacturer to set the wholesale price optimally.

PROPOSITION 6. *The equilibrium wholesale price w^* is the solution to the following equation:*

$$q_0(w^*) + \theta + \mathbb{E}_\psi[e_\psi(w^*)] + \frac{\partial q_0(w^*)}{\partial w}(w^* - c) \;=\; 0\,, \tag{8}$$

where

$$\mathbb{E}_\psi[e_\psi(w)] = k(w-c)\Big(1 - \Phi\big(\underline{\psi}(w)\big)\Big)\Big(1 - H(\underline{\psi}(w))^2 + \underline{\psi}(w)H(\underline{\psi}(w))\Big)\,, \tag{9}$$

$$\underline{\psi}(w) = H^{-1}\left(\frac{A}{k(w-c)}\right)\,, \quad \textit{and} \quad q_0(w) \;=\; \frac{1}{\sqrt{h+\widetilde{h}}}\Phi^{-1}\left(\frac{p-w}{p}\right)\,. \tag{10}$$

The expression $q_0(w^*) + \theta + \mathbb{E}_\psi[e_\psi(w^*)]$ in (8) is the expected order quantity of the retailer from the manufacturer's perspective, and $\frac{\partial q_0(w^*)}{\partial w}$ is the marginal change in the order quantity of the retailer if the wholesale price increases, which is negative. The manufacturer sets the wholesale price in such a way that the marginal expected profit gain from charging a higher wholesale price on the order quantity of the retailer, i.e., $q_0(w^*) + \theta + \mathbb{E}_\psi[e_\psi(w^*)]$, offsets the marginal expected profit loss from the decrease in the retailer's order quantity. Equation (9) shows the expected effort of the sales agent given the wholesale price w, and equation (10) illustrates the safety stock of the retailer given the wholesale price w.

5.1 Impact of Downstream Signal's Accuracy on the Supply Chain

When the wholesale price is fixed, we demonstrated that the retailer's expected profit is quasi-convex in the accuracy of his forecast. However, in this section, we illustrate that the opposite holds if the manufacturer sets the wholesale price endogenously. We further investigate how the wholesale price, the demand-promoting effort of the sales agent, and the manufacturer's expected profit change as the downstream parties learn more about demand.

5.1.1 Impact on the Wholesale Price and Expected Effort

The effort of the sales agent and the wholesale price of the manufacturer affect the expected profits of all supply chain members. For example, a larger demand-promoting effort on the part of the sales agent may result in larger profits for the manufacturer and the retailer. In order to gain insights into how the accuracy of the downstream signal impacts the retailer and the manufacturer, we first investigate how these decisions (the manufacturer's wholesale price, and the sales agent's effort) change as the accuracy of the downstream signal improves.

Proposition 7.

(i) There exists a threshold $\underline{T}_0$ *such that if* $\widetilde{h} < \underline{T}_0$*, the optimal wholesale price and the expected effort of the sales agent decrease as the accuracy of the downstream parties' demand forecast improves (i.e., as* $\widetilde{h}$ *increases).*

(ii) There exists another threshold $\overline{T}_0$ *such that if* $\widetilde{h} > \overline{T}_0$*, the optimal wholesale price and the expected effort of the sales agent increases as the accuracy of the downstream parties' demand forecast improves.*

Two different factors determine the optimal wholesale price, as $\widetilde{h}$ increases: (i) the order quantity of a retailer with a more accurate demand forecast is less sensitive to the wholesale price; therefore the manufacturer would be willing to charge a higher wholesale price to a retailer with a more accurate demand forecast; (ii) the information rent paid to a sales agent with a more accurate forecast is substantially larger when $\widetilde{h}$ is small which creates an incentive for the manufacturer to charge a lower wholesale price for small values of $\widetilde{h}$.

With respect to the second factor, the optimal contract to be offered to the sales agent depends on both the profit margin of the manufacturer and on the information rent. A manufacturer with a larger profit margin has a greater incentive to pay a higher commission rate to the sales agent. In this case, the sales agent exerts more effort; consequently, the demand increases more. This incentive of the manufacturer does not directly depend on the accuracy of the downstream parties' demand forecast. However, with a higher commission rate offered to the sales agent, the manufacturer also pays a larger information rent; this information rent depends on $\widetilde{h}$. When downstream parties do not know much more about demand than the manufacturer (i.e., for small $\widetilde{h}$), the sales agent cannot collect large information rents. However, in this case, a small change in $\widetilde{h}$ affects the information rent considerably. Hence, a decrease in the wholesale price reduces the commission rate, which, in turn, substantially decreases the information rent that needs to be provided. Therefore, when $\widetilde{h}$ is small, the manufacturer has a large incentive to decrease the wholesale price.

These two factors also explain why the expected effort of the sales agent decreases in $\widetilde{h}$ for small values of $\widetilde{h}$, and increases for large values of $\widetilde{h}$. For small values of $\widetilde{h}$, the substantial increase in the information rent by an increase in $\widetilde{h}$ motivates the manufacturer to decrease the commission rate; consequently, the expected effort of the sales agent decreases. For large values of $\widetilde{h}$, the first factor, the order quantity effect, dominates; hence, the wholesale price increases. As a result, the expected effort of the sales agent increases in $\widetilde{h}$.

One of the interesting implications is that the sales agent's effort can be increasing in the accuracy of the downstream demand forecast; in other words, larger information asymmetry increases the agent's effort. This result seems counter-intuitive based on the result of a typical principal-agent model. However, our model includes the retailer in addition to the principal (the manufacturer) and the agent (the sales agent). Furthermore, the relationship between the manufacturer and the retailer through an endogenous wholesale price also affects the dynamics between the principal and the agent to reverse the conventional result.

5.1.2 Impact on the Retailer's Expected Profit

In the fixed wholesale price model, we showed that the expected profit of the retailer is quasi-convex in the accuracy of his demand signal ($\widetilde{h}$); this implies that if the retailer knows only a little more than the manufacturer about demand (under small $\widetilde{h}$), he should not invest in improving his forecast accuracy. The following proposition, however, demonstrates that this result dramatically changes if the wholesale price is endogenously set by the manufacturer.

PROPOSITION 8. *Under the endogenous wholesale price model,*

(i) there exists a threshold $\overline{T}_r$ such that if $\widetilde{h} > \overline{T}_r$, the retailer's expected profit decreases, as the accuracy of the downstream demand forecast $\widetilde{h}$ increases;

(ii) there exists another threshold $\underline{T}_r$ such that if $\widetilde{h} < \underline{T}_r$, the retailer's expected profit is monotone (increasing or decreasing), as the accuracy of the downstream demand forecast $\widetilde{h}$ increases.

Unlike the exogenous wholesale price model, for broad parameter ranges, the retailer's expected profit is quasi-concave in $\widetilde{h}$; this implies that a retailer with less accurate demand information is better off compared to a retailer with more accurate demand information, if he already knows much more than the manufacturer about demand (under a large $\widetilde{h}$). In Proposition 8, we demonstrate this result analytically under a small $\widetilde{h}$ and a large $\widetilde{h}$. However, for intermediate $\widetilde{h}$ regions, the retailer's expected profit remains quasi-concave in $\widetilde{h}$, as suggested by our extensive numerical studies. Similar arguments also apply to the manufacturer's expected profit being quasi-convex in Section 5.1.3.

When the downstream demand forecast is accurate (i.e., when $\widetilde{h}$ is large), the manufacturer charges a higher wholesale price, as discussed previously. This increase in the wholesale price has a significant, adverse effect on the retailer's expected profit; hence, the retailer's expected profit decreases as the forecast accuracy increases.

When the downstream demand forecast is less accurate (i.e., under a small $\widetilde{h}$), with an increase in $\widetilde{h}$, the retailer can better match his order quantity with demand. Moreover, as seen in Proposition 7, the optimal wholesale price and the expected effort of the sales agent decrease in $\widetilde{h}$, due to the agency issue between the manufacturer and the sales agent. The effects of the order quantity and the lower wholesale price favor the retailer, whereas the lower sales agent's effort hurts the retailer. Therefore, when $\widetilde{h}$ is small, the expected profit of the retailer can either increase or decrease as the accuracy of the downstream parties' forecast improves, depending on which effect dominates. When the forecast accuracy of the retailer is similar to that of the manufacturer, the wholesale price decreases as the retailer's forecast accuracy improves due to a more severe agency issue between the sales agent and the manufacturer; this decrease in the wholesale price can in turn increase the retailer's profit. Consequently, the retailer can actually benefit from the agency issue between the manufacturer and the sales agent.

This result provides managerial implications on the impact of the retailer's forecast accuracy. The direct benefit of more accurate forecasting for the retailer is that he can better match his order quantity with demand. However, the retailer should be also aware that (*i*) the sales agent working with him will also have more accurate demand forecast, which implies reduced effort by the sales agent, and hence the benefit of the sales agent to the retailer is smaller; and (*ii*) with a more accurate demand forecast, he faces a higher wholesale price. These two intricate implications of a more accurate demand forecast having a negative impact on the retailer convey that a net loss can result.

5.1.3 Impact on the Manufacturer's Expected Profit

We showed in Proposition 4 that the manufacturer's expected profit is quasi-convex in the accuracy of the downstream parties' forecast if the wholesale price is fixed. Would an endogenous wholesale price change the impact of the accuracy of the downstream parties' forecast on the manufacturer's expected profit? The following proposition answers this question:

Proposition 9. *Under the endogenous wholesale price model,*

(i) there exists a threshold $\underline{T}_m$ such that if $\widetilde{h} < \underline{T}_m$, the manufacturer's expected profit decreases, as the accuracy of the downstream demand forecast improves;

(ii) there exists another threshold $\overline{T}_m$ such that if $\widetilde{h} > \overline{T}_m$, the manufacturer's expected profit increases, as the accuracy of the downstream demand forecast improves.

From Proposition 7 and our extensive numerical study, we demonstrated that under the endogenous wholesale price model, the wholesale price and the sales agent's effort are quasi-convex with respect to the accuracy of downstream parties' demand forecast. Since the manufacturer's expected profit is increasing with respect to the wholesale price and the sales agent's effort, we must have that the manufacturer's expected profit is also quasi-convex with respect to the accuracy of the downstream demand forecast. In other words, when the forecast of the downstream parties is less accurate ($\widetilde{h}$ is small), the manufacturer's expected profit is decreasing in the accuracy of the downstream parties' forecast. In addition, when the forecast of the downstream parties is more accurate ($\widetilde{h}$ is large), the manufacturer's expected profit is increasing in the accuracy of the downstream parties' forecast.

This result addresses the question of how the manufacturer should perceive an improvement in the retailer's forecast, or equivalently, when should the manufacturer provide her retailers with technology/training that improve their forecast accuracy? The forecast accuracy of the retailer directly impacts the forecast accuracy of the sales agent who is assigned to him by the manufacturer. Consequently, the improvement in the forecast accuracy of the retailer can impact the manufacturer directly through the retailer as well as indirectly through the sales agent. Specifically, the improvement in the accuracy of the downstream demand forecast has the following impact on the manufacturer:

(i) If the wholesale price is high, the order quantity of the retailer increases, and if the wholesale price is low, the order quantity of the retailer decreases;

(ii) The retailer's order quantity depends more on his forecast, and consequently, less on the wholesale price. Thus, the manufacturer can charge a higher wholesale price;

(iii) Information asymmetry between the manufacturer and the sales agent increases, and hence it becomes more expensive for the manufacturer to encourage the sales agent to exert effort.

The last effect is exclusively relevant when the manufacturer employs sales agents who work at the retail location. For low retail margin products, i.e., for the products whose profit margin of the manufacturer is higher than the profit margin of the retailer, the first two effects suggest that the manufacturer should be willing to provide the retailer with the training and technology support in order to improve the retailer's forecasts. However, in doing so, the manufacturer should not ignore the indirect consequence of the third effect. Proposition 9 suggests that if the improvement in the forecast is substantial enough, then the benefits of improving the retailer's forecast outweigh the indirect negative impacts stemming from sales agent contracting.

5.2 Impact of Sales Agent Efficiency on the Supply Chain

When the sales agent is more efficient, each unit of her effort is less costly, and, thus, the manufacturer can easily motivate the sales agent to exert more effort. Under the fixed wholesale price contract, we consequently demonstrated that a more efficient sales agent exerts more effort, which benefits both the manufacturer and the retailer. The following proposition demonstrates that when the manufacturer sets the wholesale price, the retailer does not necessarily benefit from a more efficient sales agent.

PROPOSITION 10. *As the sales agent becomes more efficient, i.e., as k increases, the optimal wholesale price, the expected effort of the sales agent, and the manufacturer's expected profit increase. However, the retailer's expected profit increases only when the sales agent is less efficient than a threshold level, and then it decreases afterwards.*

As the sales agent becomes more efficient, the manufacturer finds it cheaper to provide the sales agent with an incentive to boost demand than to motivate the retailer to order more by reducing the wholesale price. The manufacturer, therefore, increases the wholesale price to afford offering a higher commission rate to the sales agent.

The efficiency of the sales agent has two effects on the retailer's expected profit. The first effect is the change in the wholesale price. As discussed above, the wholesale price is higher when the sales agent is more efficient. The higher wholesale price leads to a lower retailer's expected profit. Thus, the more efficient sales agent can hurt the retailer due to this higher wholesale price. The second effect is the change in the effort of the sales agent. The more efficient sales agent exerts more effort for two reasons: (i) with the given commission rate, it costs less for the agent to exert effort; thus the more efficient the sales agent, the greater the effort it makes; (ii) since the manufacturer sets a higher wholesale price under a more efficient sales agent, it can afford to offer a higher commission rate to the sales agent, which increases the effort of the sales agent even further. Hence, a more efficient sales agent can benefit the retailer because of its increased effort.

Considering both effects, whether the retailer prefers a more efficient sales agent or a less efficient one depends on which of the two effects, i.e., the increase in the wholesale price or the increase in the effort of the sales agent, is dominant. When the sales agent is not efficient (i.e., k is less than a threshold), the resulting effort level is small. Consequently, the manufacturer's wholesale price does not change much as a function of the efficiency of the sales agent. Hence, the first effect is negligible, and the second effect becomes dominant, which increases the retailer's expected profit. However, if the sales agent is relatively efficient (i.e., k is greater than the threshold), the wholesale price depends critically on the efficiency of the sales agent. Moreover, this change in the wholesale price impacts the retailer through all his order quantities. Therefore, the first effect dominates, and it decreases the retailer's expected profit as the sales agent becomes more efficient.

In Section 4.3, we confirmed that the profits of the manufacturer and the retailer increase as the sales agent becomes more efficient, when the wholesale price is fixed. However, if the wholesale price depends on the efficiency of the sales agent, e.g., if the contract is short-term and changing over time, the benefit that comes with an efficient sales agent is not always greater than the loss due to the higher wholesale price. Our result does *not* suggest that the retailer should choose a less efficient sales agent if he has the option to choose a sales agent *after* he contracts with the manufacturer and/or if the wholesale price is fixed via a long-term contract. The retailer should nonetheless understand the consequences of the contract depending on the efficiency of the manufacturer-hired sales agent.

6 Discussion and Extensions

We have studied the impact of a manufacturer-hired sales agent on the supply chain, focusing on (*i*) the accuracy of the downstream parties' demand forecast and (*ii*) the efficiency of the sales agent. We have considered two cases: the exogenous wholesale price model in which the wholesale price is fixed, and the endogenous wholesale price model in which the manufacturer sets the wholesale price. We illustrated that the retailer's expected profit is quasi-convex (quasi-concave) with respect to the accuracy of the downstream parties' forecast under the exogenous wholesale price model (the endogenous wholesale price model). Furthermore, we demonstrated that the retailer's expected profit increases in the efficiency of the sales agent under the fixed wholesale price model whereas it is quasi-concave under the endogenous wholesale price model. Table 1 summarizes our results.

Our work can be extended in several dimensions. In Section 6.1, we discuss a different sequence of events. We also compare the impact of a manufacturer-hired sales agent to that of a retailer-hired agent on the supply chain in Section 6.2. Finally, in Section 6.3, we study the case in which the sales agent has more accurate demand information than the retailer.

		Expected effort	Wholesale price	Retailer's expected profit	Manufacturer's expected profit
Exogenous wholesale price	Accuracy of downstream forecast increases	↓	–	⤻	⤻↓
	Sales agent's efficiency increases	↑	–	↑	↑
Endogenous wholesale price	Accuracy of downstream forecast increases	⤻	⤻	↷↓	⤻
	Sales agent's efficiency increases	↑	↑	↷	↑

Table 1: Summary of the comparative statics results. Note that ⤻ or ↷ indicates that the corresponding expression is quasi-convex or quasi-concave, respectively.

6.1 Different Sequence of Events

We assumed that the sales agent makes its effort before the retailer places his order to the manufacturer. However, there could be other examples in which the sales agent makes an effort after the retailer orders. To investigate how robust our results are with respect to this assumption, we analytically study the case in which the effort of the sales agent is made after the retailer determines his order quantity.

In such a setting, if the sales agent is paid based on the retailer's order quantity, it does not have any incentive to make an effort to boost demand. The reason is that the retailer has already submitted his order before the agent makes an effort; thus the sales agent's effort incurs only the cost without any additional payoff. This finding implies that with this new sequence of events, in order to have any impact on the supply chain, the sales agent's compensation should be based on a criterion other than the order quantity of the retailer. For example, consider the case in which the sales agent is paid based on the *realized consumer demand* and it makes its effort *after* the retailer orders from the manufacturer. In this case, we find that there is no difference in equilibrium–except the fixed salary of the sales agent, which does not play a major role in our analysis–between our original sequence of events and this new sequence of events, and all of our results remain valid (please see Proposition A.1 and the corresponding proof in Section A.2 of the Online Supplement).

Intuitively, the reason that the above two different timings result in the same equilibrium is due to two facts: (i) the level of information asymmetry between the manufacturer and the sales agent is the same under the two scenarios; and (*ii*) the manufacturer can adjust the constant salary of the sales agent (without changing the commission rate and thus incentive of the sales agent) to make any payment scheme based on demand ex-ante equivalent to another payment scheme based on the order quantity of the retailer. Consequently, the two scenarios result in the same outcome.

6.2 Manufacturer-hired Sales Agent vs. Retailer-hired Sales Agent

One question related to our paper is how and whether the impact of a sales agent on the supply chain is different if it is hired by the retailer (see, e.g., Chen and Xiao 2012) compared to the case in which it is hired by the manufacturer. For comparison, we focus on the case in which the primary task of the sales agent, regardless of who hires it, is to boost the market demand. We analytically study a problem (please see Section A.2 of the Online Supplement) similar to what we discussed previously, with the difference being that the retailer hires the sales agent and that the compensation of the retailer-hired sales agent is contingent on the retailer's sales quantity, i.e., the actual unit sold by the retailer. After solving the problem, we numerically compare the supply chain under the retailer-hired sales agent with that under the manufacturer-hired sales agent.

We observe the following: (*i*) the manufacturer is better off with a manufacturer-hired sales agent than with a retailer-hired agent as illustrated in panel (b) of Figure 2; (*ii*) the manufacturer's expected profit is quasi-convex in the efficiency of the retailer-hired sales agent; and (*iii*) the retailer is better off under a manufacturer-hired sales agent than a retailer-hired agent, when the sales agent is not efficient enough (i.e., in region A of panel (a) in Figure 2).

Interestingly, as also illustrated in panel (a), the retailer's expected profit can decrease as the retailer-hired sales agent becomes more efficient (i.e., as k increases). In this case, the optimal wholesale price set by the manufacturer is quasi-concave in k; as the retailer-hired sales agent becomes more efficient, the sale agent exerts more effort, which increases the marginal benefit of increasing the wholesale price. Consequently, the manufacturer sets a higher wholesale price as k increases. However, as k increases further, the manufacturer decreases the wholesale price to incentivize the retailer to induce more sales agent's effort. When the sales agent's efficiency is low (under a small k, i.e., between 0 and 6 in panel (a) of Figure 2), the wholesale price increases in k, which hurts the retailer. This finding interestingly implies that in such a case, the retailer is better off if he does not have the option of hiring a sales agent; that is, the retailer's expected profit is

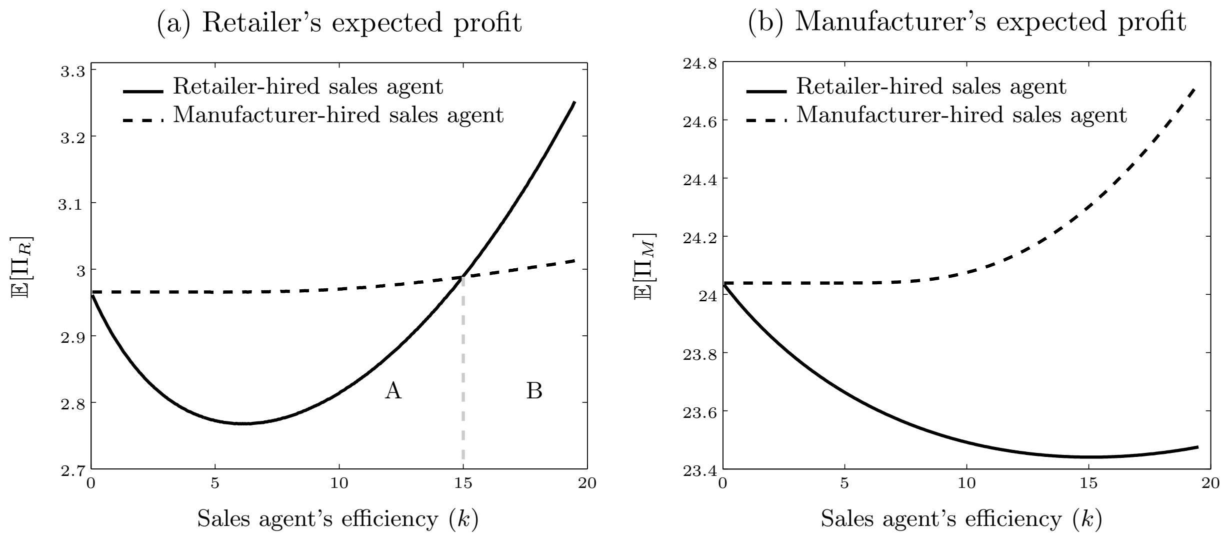

Figure 2: The expected profits of the retailer (panel (a)) and the manufacturer (panel (b)) as a function of the sales agent's efficiency (k) under the retailer-hired sales agent (solid line) and the manufacturer-hired sales agent (dashed line). Parameters values are: $p = 10$, $\theta = 50$, $c = 9$, $h = 0.0036$, and $\widetilde{h} = 0.0063$.

higher under the manufacturer-hired sales agent than under the retailer-hired agent in this region.

6.3 Additional Information of the Sales Agent

We assumed that the retailer and the sales agent share the same consumer demand information. Due to its job characteristics of gathering market demand information, the sales agent may have a more accurate demand forecast than the retailer (e.g., from the demand of the national market). Even in that case, all of our results remain valid (please see Section A.2 of the Online Supplement for the exact statement and the proof).

To understand this result, note that any information that does not impact the sales agent's payoff cannot impact the sales agent's effort, and consequently equilibrium of the game. Given that the sales agent's payoff is based on the order quantity of the retailer, if this extra information about demand helps the sales agent to boost the retailer's order quantity, then the equilibrium outcome may change due to this additional information. In order to impact the retailer's order quantity, the sales agent should credibly communicate its information with the retailer. Unless the retailer trusts the sales agent potentially due to behavioral reasons or repeated interactions, the sales agent cannot credibly communicate its information with the retailer to convince him to order more. The retailer has no reason to believe the sales agent's information about demand because he knows that the sales agent has an incentive to inflate its forecasts to induce the retailer to order more. As a result, the sales agent's additional information does not change the retailer's order quantity, and hence, the equilibrium outcome remains the same.

7 Concluding Remarks

Our paper helps to understand whether a retailer with a more accurate demand forecast is better off compared to a retailer with a less accurate demand forecast, specifically in a supply chain where sales agents are employed by the manufacturer. In this regard, we investigate the question of what is the impact of improvement in the retailer's forecast accuracy on his expected profit? The direct benefit for the retailer to have more accurate demand forecasting is that he can better match his order quantity with demand. However, more accurate demand forecasting by the retailer impacts him in other less obvious ways: if more information about demand is available at the downstream level, (i) the agency issue between the manufacturer and the sales agent is more severe and thus the manufacturer would not offer a high commission rate to the sales agent which means the sales agent's effort would be lower; (ii) the retailer would be less sensitive to the wholesale price and consequently, the manufacturer is more likely to charge a higher wholesale price. These two implications of improved forecasting at the retail level have a negative impact on the retailer, and may result in a net loss. Consequently, we suggest that if the manufacturer wants to encourage the retailer to improve his forecast accuracy, she may want to consider committing to a long-term wholesale price contract. In this fashion, the second negative impact of improved forecast accuracy to the retailer can be mitigated.

This paper addresses the question of how a manufacturer should perceive an improvement in the retailer's forecast, or equivalently, when the manufacturer should provide her retailer with technology/training that improve his forecast. The forecast accuracy of a retailer directly impacts the forecast accuracy of the sales agent who is assigned to him by the manufacturer. Consequently, an improvement in the forecast accuracy of the retailer impacts the manufacturer both through the retailer and through the sales agent. Specifically, an improvement in the accuracy of the retailer's forecast has the following impact on the manufacturer: (i) for low retail margin products, the order quantity of the retailer increases, and vice versa for high retail margin products; (ii) the retailer becomes less sensitive to the wholesale price, and thus the manufacturer charges a higher wholesale price; and (*iii*) information asymmetry between the manufacturer and the sales agent increases, and as a result, the manufacturer induces less effort from the sales agent.

This paper discusses the impact of an improvement in the efficiency of a manufacturer-hired sales agent on the retailer and the manufacturer. We confirm that the manufacturer always benefits from a more efficient sales agent. However, we show that the retailer can be hurt by the presence of such a sales agent in the supply chain. With a more efficient sales agent, the manufacturer also charges a higher wholesale price to the retailer, which can lead to a net loss for the retailer. The implication of this result is that if the efficiency of the sales agent can be improved by collaborating with the retailer, commitment to a long-term contract, i.e., to a fixed wholesale price, by the manufacturer can help encourage this collaboration and lead to better outcomes for the entire supply chain.

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Online Supplement for
The Impact of the Manufacturer-hired Sales Agent on a Supply Chain with Information Asymmetry

A.1 Proof of Propositions

Proof of Proposition 1. Consider any $\psi > \psi'$. We know from (IC), that

$$\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) \geq \pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'} | \Psi = \psi)$$
$$\pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'} | \Psi = \psi') \geq \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi')\,.$$

Therefore, using (1), we find that $\alpha_\psi > \alpha_{\psi'}$. Using this fact and (1), we can rearrange (IC), to get

$$\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) - \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi') \geq \frac{\widetilde{h}}{h+\widetilde{h}}(\psi - \psi')\alpha_{\psi'} \qquad \forall \psi, \forall \psi'\,. \tag{A.1}$$

Consider any ψ and ψ' such that $\psi > \psi'$. From (A.1) and similar inequality in which the role of ψ and ψ' is reversed, we obtain

$$\frac{\widetilde{h}}{h+\widetilde{h}}(\psi - \psi')\alpha_\psi \geq \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) - \pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'} | \Psi = \psi') \geq \frac{\widetilde{h}}{h+\widetilde{h}}(\psi - \psi')\alpha_{\psi'}\,.$$

Dividing these inequalities by $(\psi - \psi')$ and converging ψ close to ψ', we obtain

$$\frac{\partial \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi)}{\partial \psi} = \frac{\widetilde{h}}{h+\widetilde{h}}\alpha_\psi\,. \tag{A.2}$$

After we integrate both sides, it follows

$$\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Theta_r = \psi) = L + \frac{\widetilde{h}}{h+\widetilde{h}}\int_{-\infty}^{\psi} \alpha_y\, dy\,. \tag{A.3}$$

where L is a constant that the manufacturer can decide. However, IR (Individual Rationality) constraint should be satisfied for all ψ, which restricts the value of L. Using (1), we can substitute (A.3) in the objective function of the manufacturer, and after simplification, the manufacturer's problem can be written as

$$\max_{\alpha_\psi \geq 0, L} \Bigg(-L + (\overline{w} - c)(q_0 + \theta) + \int_{-\infty}^{\infty} \left(\left(k(\overline{w}-c)\alpha_\psi - \frac{k}{2}\alpha_\psi^2 \right) \sqrt{\frac{h\widetilde{h}}{h+\widetilde{h}}}\, \phi\left(\frac{\psi-\theta}{\sqrt{\sigma^2+\widetilde{\sigma}^2}} \right) - \frac{\widetilde{h}}{h+\widetilde{h}} \left(1 - \Phi\left(\frac{\psi-\theta}{\sqrt{\sigma^2+\widetilde{\sigma}^2}} \right) \right) \alpha_\psi \right) \mathrm{d}\psi \Bigg),$$

$$s.t. \quad \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Theta_r = \psi) \geq 0, \text{ for all } \psi\,.$$

Therefore, from the IR constraint and $\alpha_\psi \geq 0$, we obtain $L = 0$. Furthermore, from the first order condition, it follows that

$$\alpha_\psi^* = \left((\overline{w} - c) - \frac{A}{kH\left(\frac{\psi-\theta}{\sqrt{\sigma^2+\widetilde{\sigma}^2}} \right)} \right)^+ ,$$

where $A = \sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}$. Let $\underline{\psi}$ be the unique solution to $k(\overline{w}-c)H(\underline{\psi}) = A$, i.e., $\alpha_\psi^* = 0$ at $\psi = \underline{\psi}$. Then we have the results. ■

Proof of Proposition 2. Note that the hazard rate function $H(x)$ for the standard normal distribution is monotone increasing, and therefore, it is invertible. From Proposition 1, it follows that

$$\mathbb{E}[e(\psi)] = k(\overline{w}-c)\left(1 - \Phi(\underline{\psi})\right)\left(1 - H(\underline{\psi})^2 + \underline{\psi}H(\underline{\psi})\right) ,$$

where $\underline{\psi}$ is the unique solution of $k(\overline{w}-c)H(\underline{\psi}) = A$. Taking the first derivative of $\mathbb{E}[e(\psi)]$ with respect to $\widetilde{h}$ and simplifying, we have $\frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} = -\frac{h}{2\widetilde{h}(h+\widetilde{h})} k(\overline{w}-c)\phi(\underline{\psi})\left(H(\underline{\psi}) - \underline{\psi}\right) < 0$; that is, expected effort is decreasing in $\widetilde{h}$. ■

Proof of Proposition 3. From Proposition 1, the expected profit of the retailer is $\mathbb{E}[\Pi_R] = (p-\overline{w})(q_0+\theta+\mathbb{E}[e(\psi)]) - p\int_{-\infty}^{q_0} \Phi(u\sqrt{h+\widetilde{h}})\,\mathrm{d}u$. Therefore, we can derive the first order derivative of the $\mathbb{E}[\Pi_R]$ with respect to $\widetilde{h}$ and after simplification we have

$$\frac{\partial \mathbb{E}[\Pi_R]}{\partial \widetilde{h}} = \frac{1}{2(h+\widetilde{h})^{3/2}} \left(p\phi(q_0\sqrt{h+\widetilde{h}}) - (p-\overline{w})\sqrt{\frac{h}{\widetilde{h}}} \int_{\underline{\psi}}^{\infty} \left(1 - \Phi(x)\right) \mathrm{d}x \right).$$

This implies that $\frac{\partial \Pi_R}{\partial \widetilde{h}} < 0$, if and only if $\sqrt{\frac{h}{\widetilde{h}}} \int_{\underline{\psi}}^{\infty} \left(1 - \Phi(x)\right) \mathrm{d}x > \frac{p}{p-\overline{w}}\phi(\Phi^{-1}(\frac{p-\overline{w}}{p}))$. Note that the right hand side of the inequality is constant and given. Furthermore, the left hand side of the inequality is decreasing in $\widetilde{h}$ and $\lim_{\widetilde{h}\to 0} \sqrt{\frac{h}{\widetilde{h}}} \int_{\underline{\psi}}^{\infty} \left(1 - \Phi(x)\right) \mathrm{d}x = \infty$ and $\lim_{\widetilde{h}\to\infty} \sqrt{\frac{h}{\widetilde{h}}} \int_{\underline{\psi}}^{\infty} \Big(1 -$

$\Phi(x)\Big)\,\mathrm{d}x = 0$. Therefore, there exists $\widetilde{h}_1$ such that for $\widetilde{h} = \widetilde{h}_1$ the inequality holds as equality and for $\widetilde{h} < \widetilde{h}_1$, the inequality is satisfied and the expected profit of the retailer is decreasing in $\widetilde{h}$. ■

Proof of Proposition 4. From proposition 1, the expected profit of the manufacturer is $\mathbb{E}[\Pi_M] = (\overline{w}-c)(q_0+\theta) + \frac{k}{2}\int_{\underline{\psi}}^{\infty}\big(\overline{w}-c-\frac{A}{kH(x)}\big)^2\phi(x)\,\mathrm{d}x$. Therefore, it follows

$$\frac{\partial \mathbb{E}[\Pi_M]}{\partial \widetilde{h}} = -\frac{1}{2k(h+\widetilde{h})^{3/2}}\Big(k(\overline{w}-c)\Phi^{-1}(\frac{p-\overline{w}}{p}) + \int_{\underline{\psi}}^{\infty}\sqrt{\frac{h}{\widetilde{h}}}(1-\Phi(x))\Big(k(\overline{w}-c)-\frac{A}{H(x)}\Big)\,\mathrm{d}x\Big).$$

Note that for $\overline{w} < p/2$, $\Phi^{-1}(\frac{p-\overline{w}}{p})$ is positive. Also for $x > \underline{\psi}$, $k(\overline{w}-c) - \frac{A}{H(x)} > 0$ (recall that $k(\overline{w}-c)H(x) = A$). Therefore, when $\overline{w} < p/2$, $\frac{\partial \mathbb{E}[\Pi_M]}{\partial \widetilde{h}} < 0$ which implies that the expected profit of the manufacturer is decreasing and quasi-convex.

On the other hand, when $\overline{w} > p/2$, the expected profit of the manufacturer is decreasing in $\widetilde{h}$, if and only if

$$\int_{\underline{\psi}}^{\infty}\sqrt{\frac{h}{\widetilde{h}}}(1-\Phi(x))\Big(k(\overline{w}-c)-\frac{A}{H(x)}\Big)\,\mathrm{d}x > -k(\overline{w}-c)\Phi^{-1}(\frac{p-\overline{w}}{p}).$$

The right hand side of the above inequality is constant and given. The left hand side of the inequality is decreasing in $\widetilde{h}$ and positive when $\overline{w} > p/2$. Furthermore, $\lim_{\widetilde{h}\to 0}\int_{\underline{\psi}}^{\infty}\sqrt{\frac{h}{\widetilde{h}}}(1-\Phi(x))\Big(k(\overline{w}-c)-\frac{A}{H(x)}\Big)\,\mathrm{d}x = \infty$, and $\lim_{\widetilde{h}\to\infty}\int_{\underline{\psi}}^{\infty}\sqrt{\frac{h}{\widetilde{h}}}(1-\Phi(x))\Big(k(\overline{w}-c)-\frac{A}{H(x)}\Big)\,\mathrm{d}x = 0$. Therefore, there is a unique solution $\widetilde{h}_2$ such that for $\widetilde{h} = \widetilde{h}_2$, (i) the inequality holds as equality, (ii) for $\widetilde{h} < \widetilde{h}_2$, the above inequality is satisfied and thus the expected profit of the manufacturer is decreasing in $\widetilde{h}$, and (iii) for $\widetilde{h} > \widetilde{h}_2$, the expected profit of the manufacturer is increasing in $\widetilde{h}$. Thus, we conclude that the manufacturer's expected profit is quasi-convex. ■

Proof of Proposition 5. The effort at equilibrium is $\int_{\underline{\psi}}^{\infty}(k(\overline{w}-c)-\frac{A}{H(x)})\phi(x)\,\mathrm{d}x$. The first derivative of the effort with respect to k is $(\overline{w}-c)(1-\Phi(\underline{\psi})) > 0$. That is, effort is increasing in k. The expected profit of the manufacturer is $(\overline{w}-c)(q_0+\mu) + \frac{k}{2}\int_{\underline{\psi}}^{\infty}\Big((\overline{w}-c)-\frac{A}{kH(x)}\Big)^2\phi(x)\,\mathrm{d}x$. The first derivative of the manufacturer's expected profit with respect to k is

$$\frac{1}{2}\int_{\underline{\psi}}^{\infty}\Big((\overline{w}-c)-\frac{A}{kH(x)}\Big)^2\phi(x)\,\mathrm{d}x + \int_{\underline{\psi}}^{\infty}\frac{A}{kH(x)}\Big((\overline{w}-c)-\frac{A}{kH(x)}\Big)\phi(x)\,\mathrm{d}x > 0.$$

That is, the manufacturer's expected profit is increasing in k.

The retailer's expected profit is $(p-\overline{w})(q_0+\mu+\mathbb{E}[e]) - p\int_{-\infty}^{q_0}\Phi(x)\,\mathrm{d}x$. Since $\mathbb{E}[e]$ is increasing in k, and both q_0 and $\overline{w}$ are independent of k, the retailer's expected profit is increasing in k. ■

Proof of Proposition 6. First note that the expected profit function of the manufacturer is continuous and differentiable in w. Therefore, the optimal wholesale price should either satisfy the first order condition which is $q_0 + \theta + \mathbb{E}_\psi[e(\psi)] + \frac{\partial q_0}{\partial w}(w - c) = 0$ or it should be in the boundaries, i.e., either at p or 0. Note that for $w = 0$, the expected profit function of the manufacturer is zero. Furthermore, for $w \geq p$, the order quantity of the retailer is zero, and thus the manufacturer's expected profit is also zero. Therefore, the optimal wholesale price should satisfy $q_0+\theta+\mathbb{E}_\psi[e(\psi)]+\frac{\partial q_0}{\partial w}(w - c) = 0$. ■

Proof of Propositions 7, 8, and 9, when $\widetilde{h}$ is small.
From now on, to simplify the notation, we represent $q_0(w^*)$ by q_0 and $\underline{\psi}(w^*)$ by $\underline{\psi}$. The proof is consisted of several major steps, presented as claims. Claims (2) and (4) together establish Proposition 7, for small $\widetilde{h}$. Claims (5) and (6) establish Propositions 8, and 9, for small $\widetilde{h}$, respectively.

Claim 1. $\lim_{\widetilde{h}\to 0} \underline{\psi} = -\infty$, $\lim_{\widetilde{h}\to 0} w^*$ exists, $c < \lim_{\widetilde{h}\to 0} w^* < p$, $\lim_{\widetilde{h}\to 0} \mathbb{E}[e] = \lim_{\widetilde{h}\to 0} k(w^* - c)$, and $\lim_{\widetilde{h}\to 0} q_0$ and $\lim_{\widetilde{h}\to 0} \frac{\partial q_0}{\partial w}$ are finite.

To show this claim, let $g(w) = (w - c)\left(\frac{1}{\sqrt{h+\widetilde{h}}}\Phi^{-1}\left(\frac{p-w}{p}\right) + \theta\right)$. Then one can show that $g(w)$ is concave in w and thus $g(\cdot)$ has a unique maximizer. Let $w_0(\widetilde{h})$ be the maximizer of $g(\cdot)$ when accuracy of the downstream parties' signal is $\widetilde{h}$. Then by Berge's maximum theorem, $w_0(\widetilde{h})$ is continuous and has limit when $\widetilde{h} \to 0$. Note that $\lim_{\widetilde{h}\to\infty} w_0(\widetilde{h}) > c$, because otherwise $\lim_{\widetilde{h}\to 0} g'(w_0(\widetilde{h})) > 0$ (assuming θ is large enough so that a retailer, when there is no sales agent or information asymmetry in the supply chain, would order a positive amount from the manufacturer). Since $w_0(\widetilde{h})$ is increasing in $\widetilde{h}$, $w_0(\widetilde{h}) > c$ for all $\widetilde{h}$. Also, let $\widetilde{w}(\widetilde{h})$, represent any w that satisfies the first order condition of the manufacturer's problem of finding optimal wholesale price, when the accuracy of the downstream parties' signal is $\widetilde{h}$. Note that $\widetilde{w}(\widetilde{h}) > w_0(\widetilde{h})$, because otherwise by concavity of the $g(\cdot)$, the first order condition of the manufacturer's problem is not satisfied at $\widetilde{w}(\widetilde{h})$. Therefore, we have $\widetilde{w}(\widetilde{h}) > w_0(\widetilde{h}) > c$. Let $\underline{\psi}(\widetilde{w}(\widetilde{h})) = H^{-1}(\frac{A}{k(\widetilde{W}(\widetilde{h})-c)})$. Since $\lim_{\widetilde{h}\to 0} H^{-1}(\frac{A}{k(w^*(\widetilde{h})-c)}) < \lim_{\widetilde{h}\to 0} H^{-1}(\frac{A}{k(w_0(\widetilde{h})-c)}) = -\infty$, we have $\lim_{\widetilde{h}\to 0} \underline{\psi}(\widetilde{w}(\widetilde{h})) = -\infty$. Therefore, $\lim_{\widetilde{h}\to 0} k(1 - \Phi(\underline{\psi}(\widetilde{w}(\widetilde{h}))))(1 - H(\underline{\psi}(\widetilde{w}(\widetilde{h})))^2 + \underline{\psi}(\widetilde{w}(\widetilde{h}))H(\underline{\psi}(\widetilde{w}(\widetilde{h})))) - k(1 - \Phi(\underline{\psi}(\widetilde{w}(\widetilde{h})))) = 0$. One can show that, this implies that for any w such that the first order condition of the manufacturer's expected profit is satisfied, the first order condition is decreasing in w. Therefore, the manufacturer's expected profit function is quasi-concave when $\widetilde{h} \to 0$. Therefore, by Berge's maximum theorem $\lim_{\widetilde{h}\to 0} w^*$ exists. Suppose $\lim_{\widetilde{h}\to 0} w^* = p$, then $\lim_{\widetilde{h}\to 0} q_0 = \lim_{\widetilde{h}\to 0} \frac{1}{\sqrt{h+\widetilde{h}}}\Phi^{-1}\left(\frac{p-w}{p}\right) = -\infty$ and thus $\lim_{\widetilde{h}\to 0} q_0 + \mathbb{E}[e] + \theta = -\infty$. That is, the order quantity of the retailer is negative, which is a contradiction. Therefore, $\lim_{\widetilde{h}\to 0} w^* < p$. Note that Since $\lim_{\widetilde{h}\to 0} w^* < p$, $\lim_{\widetilde{h}\to 0} q_0$ and $\lim_{\widetilde{h}\to 0} \frac{\partial q_0}{\partial w}$ are finite.

Claim 2. $\lim_{\widetilde{h}\to 0} \frac{\partial w^*}{\partial \widetilde{h}} = -\infty$ and $\lim_{\widetilde{h}\to 0} \frac{\partial w^*}{\partial \widetilde{h}} / \frac{-\underline{\psi}}{2h\sqrt{\widetilde{h}}\frac{\partial^2 \mathbb{E}[\Pi_M]}{\partial w^2}} = 1$.

First, from first order condition of the manufacturer's optimization, one can show that

$$\frac{\partial w^*}{\partial \widetilde{h}} = \frac{1}{2(h+\widetilde{h})\frac{\partial^2 \Pi_M}{\partial w^2}}(-\mathbb{E}[e] - \theta + \frac{(1-\Phi(\underline{\psi}))(H(\underline{\psi}) - \underline{\psi})}{A(h+\widetilde{h})}).$$

Therefore,

$$\lim_{\widetilde{h}\to 0} \frac{\partial w^*}{\partial \widetilde{h}} / \frac{-\underline{\psi}}{2h\sqrt{\widetilde{h}}\frac{\partial^2 \mathbb{E}[\Pi_M]}{\partial w^2}} = 1$$

because

$$\lim_{\widetilde{h}\to 0} \frac{\partial^2 \mathbb{E}[\Pi_M]}{\partial w^2} = \lim_{\widetilde{h}\to 0} 2\frac{\partial q_0}{\partial w} + k + (\frac{\partial q_0}{\partial w})^2 (h+\widetilde{h}) q_0 (w^* - c)$$

is finite and negative.

Claim 3. $\lim_{\widetilde{h}\to 0} \frac{\partial q_0}{\partial \widetilde{h}} = +\infty$ and $\lim_{\widetilde{h}\to 0} \frac{\partial q_0}{\partial \widetilde{h}} / \frac{\partial w^*}{\partial \widetilde{h}} = \lim_{\widetilde{h}\to 0} \frac{\partial q_0}{\partial w}$.

$$\frac{\partial q_0}{\partial \widetilde{h}} = \frac{\partial q_0}{\partial w}\frac{\partial w^*}{\partial \widetilde{h}} - \frac{q_0}{2(h+\widetilde{h})}.$$

Since $\lim_{\widetilde{h}\to 0} q_0$ is finite and $\lim_{\widetilde{h}\to 0} \frac{\partial q_0}{\partial w}$ is finite and negative, $\lim_{\widetilde{h}\to 0} \frac{\partial q_0}{\partial \widetilde{h}} = \lim_{\widetilde{h}\to 0} \frac{\partial q_0}{\partial w}\frac{\partial w^*}{\partial \widetilde{h}} = +\infty$.

Claim 4. $\lim_{\widetilde{h}\to 0} \frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} = -\infty$ and $\lim_{\widetilde{h}\to 0} \frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} / \frac{\partial w^*}{\partial \widetilde{h}} = \lim_{\widetilde{h}\to 0} k - \frac{\partial^2 \mathbb{E}[\Pi_M]}{\partial w^2}$.

The proof of the claim is as follows: We know that

$$\frac{\partial E[e]}{\partial \widetilde{h}} = (1-\Phi(\underline{\psi}))(k\frac{\partial w^*}{\partial \widetilde{h}} - \frac{H(\underline{\psi}) - \underline{\psi}}{2A(h+\widetilde{h})}).$$

Therefore,

$$\lim_{\widetilde{h}\to 0} \frac{\partial E[e]}{\partial \widetilde{h}} = \lim_{\widetilde{h}\to 0} \frac{\partial w^*}{\partial \widetilde{h}}(k - \frac{\partial^2 \mathbb{E}[\Pi_M]}{\partial w^2}) = -\infty$$

Claim 5. $\lim_{\widetilde{h}\to 0} \frac{\partial \mathbb{E}[\Pi_R]}{\partial \widetilde{h}} / \frac{\partial w^*}{\partial \widetilde{h}} = \lim_{\widetilde{h}\to 0} -\frac{\partial q_0}{\partial w}\Big((p-w^*) - (w^*-c) + (p-w^*)(1 + \frac{\partial q_0}{\partial w}(h+\widetilde{h})q_0(w^*-c))\Big)$.

The proof of this claim is as follows: we know

$$\frac{\partial \mathbb{E}[\Pi_R]}{\partial \widetilde{h}} = \frac{\partial w^*}{\partial \widetilde{h}}\frac{\partial q_0}{\partial w}(w^*-c) + (p-w^*)\frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} - \frac{p}{2(h+\widetilde{h})}\int_{-\infty}^{q_0} x\phi\left(\sqrt{h+\widetilde{h}}\, x \,\mathrm{d}x\right)$$

Therefore,

$$\lim_{\widetilde{h}\to 0} \frac{\partial \mathbb{E}[\Pi_R]}{\partial \widetilde{h}} / \frac{\partial w^*}{\partial \widetilde{h}} = -\frac{\partial q_0}{\partial w}\left(((p-w^*) - (w^*-c)) + (p-w^*)(1 + \frac{\partial q_0}{\partial w}(h+\widetilde{h})q_0(w^*-c))\right).$$

Note that, if $p - w^* > w^* - c$, then $\lim \frac{\partial \mathbb{E}[\Pi_R]}{\partial \widetilde{h}} = -\infty$.

Claim 6. $\lim_{\widetilde{h}\to 0} \frac{\partial \mathbb{E}[\Pi_M]}{\partial \widetilde{h}} = -\infty$.

Taking the derivative of $\mathbb{E}[\Pi_M]$ in (7) with respect to $\widetilde{h}$ after substituting the equilibrium effort level e_ψ using α_ψ in Proposition 1 and simplifying, we obtain

$$\frac{\partial \mathbb{E}[\Pi_M]}{\partial \widetilde{h}} = -\frac{q_0}{2(h+\widetilde{h})}(w^* - c) - \frac{1}{2(h+\widetilde{h})} \int_{\underline{\psi}}^{\infty} \frac{\frac{1}{H(\underline{\psi})} - \frac{1}{H(x)}}{H(x)} \phi(x)\, \mathrm{d}x\,.$$

Define $g(y) \triangleq \int_y^\infty \frac{\frac{1}{H(y)} - \frac{1}{H(x)}}{H(x)} \phi(x)\, \mathrm{d}x$. It then follows that

$$g'(y) = -\int_y^\infty \frac{H(y) - y}{H(x)H(y)} \phi(x)\, \mathrm{d}x < 0\,,$$

which shows that $g(y)$ is strictly decreasing in y. Furthermore, we also obtain

$$g''(y) = \frac{H(y) - y}{H(y)^2}\phi(y) + \int_y^\infty \frac{1 - yH(y) + y^2}{H(x)H(y)} \phi(x)\, \mathrm{d}x > 0\,,$$

i.e., $g(y)$ is strictly convex in y. Using the fact that $g(\underline{\psi})$ is strictly convex and strictly decreasing in $\underline{\psi}$, and taking the limit, we then obtain that $\lim_{\underline{\psi}\to -\infty} g(\underline{\psi}) = \infty$. Therefore, by using this claim together with Claim 1 above, it follows that $\lim_{\widetilde{h}\to 0} \frac{\partial \mathbb{E}[\Pi_M]}{\partial \widetilde{h}} = -\infty$.

■

Proof of Propositions 7, 8, and 9, when $\widetilde{h}$ is large. The proof is consisted of several major steps, presented as claims. Claims (5) and (7) together establish Proposition 7, for large $\widetilde{h}$. Claims (8) and (9) establishes Propositions 8 and 9, for large $\widetilde{h}$, respectively.

Claim 1. $\lim_{\widetilde{h}\to\infty} w^* = p$.

To show this claim, let $g(w) = (w - c)\left(\frac{1}{\sqrt{h+\widetilde{h}}}\Phi^{-1}\left(\frac{p-w}{p}\right) + \theta\right)$. Then one can show that $g(w)$ is concave in w and thus $g(\cdot)$ has a unique maximizer. Let $w_0(x)$ be the maximizer of $g(\cdot)$ when $\widetilde{h} = x$. Then by Berge's maximum theorem, $w_0(x)$ is continuous and has limit when $\widetilde{h} \to +\infty$. Note that $\lim_{\widetilde{h}\to\infty} w_0(\widetilde{h}) = p$, because otherwise $\lim_{\widetilde{h}\to\infty} g'(w_0(\widetilde{h})) > 0$ which is a contradiction. Now let $w^*(\widetilde{h})$ be the maximizer of the manufacturer's expected profit function when the accuracy of the downstream parties' signal is $\widetilde{h}$. Note that by concavity of $g(w)$, we have, for $w < w_0(\widetilde{h})$, the first order condition of the manufacturer's problem to find the optimal wholesale price is positive. Thus $w^*(\widetilde{h}) > w_0(\widetilde{h})$. Since $w^*(\widetilde{h}) < p$, by sandwich theorem, we have that $\lim_{\widetilde{h}\to\infty} w^*(\widetilde{h}) = p$.

Claim 2. $\lim_{\widetilde{h}\to\infty} \underline{\psi}$ is finite. Also $\lim_{\widetilde{h}\to\infty} \mathbb{E}[e]$ is finite and positive.

Note that $\lim_{\widetilde{h}\to\infty} \underline{\psi} = \lim_{\widetilde{h}\to\infty} H^{-1}\left(\frac{\sqrt{\frac{\widetilde{h}}{h+\widetilde{h}}}}{k(w^*-c)}\right)$ which is finite, since $\lim_{\widetilde{h}\to\infty} \frac{k(w^*-c)}{\sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}} = k\sqrt{h}(p-c) > 0$ is finite and positive. Furthermore, $\mathbb{E}[e] = k(w^*-c)(1-\Phi(\underline{\psi}))(1-H(\underline{\psi})^2+\underline{\psi}H(\underline{\psi}))$. Since $\lim_{\widetilde{h}\to\infty} \underline{\psi}$ is finite, $\lim_{\widetilde{h}\to\infty} \mathbb{E}[e]$ is finite and positive.

Claim 3. $\lim \frac{\partial q_0}{\partial w}$ is finite and $\lim_{\widetilde{h}\to\infty} q_0 = 0$.

To show this, first note that since q_0 and $\frac{\partial q_0}{\partial w}$ are continuous functions of $\widetilde{h}$, their limit exist in the extended real line. Also note that

$$\lim_{x\to 0^+} \Phi^{-1}(x) + \sqrt{-\log\big(-2\pi x^2\log(2\pi x^2)\big)} = 0\,.$$

Therefore, $\lim_{\widetilde{h}\to\infty} q_0 = \lim_{\widetilde{h}\to\infty} \frac{1}{\sqrt{h+\widetilde{h}}}\Phi^{-1}\left(\frac{p-w^*}{p}\right) \le 0$. Furthermore, since $\lim_{\widetilde{h}\to\infty} \mathbb{E}[e]$ is finite and since $\lim_{\widetilde{h}\to\infty} q_0 + \mathbb{E}[e] + \theta$ should be positive (order quantity of retailer should be positive), we must have $\lim_{\widetilde{h}\to\infty} q_0 > -\infty$. Suppose, $-\infty < \lim_{\widetilde{h}\to\infty} q_0 < 0$, then

$$\lim_{\widetilde{h}\to\infty} \frac{\partial q_0}{\partial w} = \lim_{\widetilde{h}\to\infty} -\frac{1}{p\sqrt{h+hr}\phi(\sqrt{h+\widetilde{h}}q_0)} = -\infty\,,$$

which implies that $\lim_{\widetilde{h}\to\infty} q_0 + \mathbb{E}[e] + \theta + \frac{\partial q_0}{\partial w}(w^*-c) = -\infty$. That is, the first order condition of the manufacturer's optimization is negative, which is a contradiction. Therefore, we must have $\lim_{\widetilde{h}\to\infty} q_0 = 0$. In this case, by the first order condition of the manufacturer's optimization, we must have $\lim_{\widetilde{h}\to\infty} \frac{\partial q_0}{\partial w} = \frac{-\mathbb{E}(e)-\theta}{p-c}$ which is finite and negative.

Claim 4. $\lim_{\widetilde{h}\to\infty} \frac{\partial^2\mathbb{E}[\Pi_M]}{\partial w^2} = -\infty$ and $\lim_{\widetilde{h}\to\infty} \frac{\partial^2\mathbb{E}[\Pi_M]}{\partial w^2} / \left(\frac{\partial q_0}{\partial w}\right)^2 (h+\widetilde{h})q_0(w^*-c) = 1$.

The proof of this claim is as follows: One can show that $\frac{\partial^2\mathbb{E}[\Pi_M]}{\partial w^2} = 2\frac{\partial q_0}{\partial w} + k(w^*-c)(1-\Phi(\underline{\psi})) + (\frac{\partial q_0}{\partial w})^2(h+\widetilde{h})q_0(w^*-c)$. Since $\lim_{\widetilde{h}\to\infty} \frac{\partial q_0}{\partial w}$ and $\lim_{\widetilde{h}\to\infty} k(w^*-c)(1-\Phi(\underline{\psi}))$ is finite, and since $\lim_{\widetilde{h}\to\infty}(h+\widetilde{h})q_0 = \lim_{\widetilde{h}\to\infty} \sqrt{h+\widetilde{h}}\Phi^{-1}\left(\frac{p-w}{p}\right) = -\infty$, the claim follows.

Claim 5. $\lim_{\widetilde{h}\to\infty} \frac{\partial w^*}{\partial \widetilde{h}} = 0^+$ and $\lim_{\widetilde{h}\to\infty} \frac{\partial w^*}{\partial \widetilde{h}} / \frac{\mathbb{E}[e]+\theta}{-2\left(\frac{\partial q_0}{\partial w}\right)^2(h+\widetilde{h})^2 q_0(w^*-c)} = 1$.

Applying envelope theorem on the first order condition of the manufacturer's optimization problem, one can show that

$$\frac{\partial w^*}{\partial \widetilde{h}} = \frac{1}{2(h+\widetilde{h})\frac{\partial^2\mathbb{E}[\Pi_M]}{\partial w^2}}\left(-\mathbb{E}[e] - \theta + \frac{(1-\Phi(\psi))(H(\underline{\psi})-\underline{\psi})}{A(h+\widetilde{h})}\right). \tag{A.4}$$

Therefore using the previous claims, we have the result.

Claim 6. $\lim_{\widetilde{h}\to\infty} \frac{\partial q_0}{\partial \widetilde{h}} = 0^+$ and $\lim_{\widetilde{h}\to\infty} \frac{\partial q_0}{\partial \widetilde{h}} / \frac{-q_0}{2(h+\widetilde{h})} = 1$.

First, one can show that

$$\frac{\partial q_0}{\partial \widetilde{h}} = \frac{\partial q_0}{\partial w}\frac{\partial w^*}{\partial \widetilde{h}} - \frac{q_0}{2(h+\widetilde{h})}.$$

Therefore,

$$\lim_{\widetilde{h}\to\infty} \frac{\partial q_0}{\partial \widetilde{h}} / \frac{\mathbb{E}[e]+\theta}{-2\left(\frac{\partial q_0}{\partial w}\right)^2 (h+\widetilde{h})^2 q_0 (w^*-c)} = \lim_{\widetilde{h}\to\infty} \frac{\partial q_0}{\partial w} + \frac{\left(\frac{\partial q_0}{\partial w}\right)^2 (h+\widetilde{h}) q_0^2 (w^*-c)}{\mathbb{E}[e]+\theta}.$$

Since $\lim_{\widetilde{h}\to\infty}(h+\widetilde{h})q_0^2 = \lim_{\widetilde{h}\to\infty} \Phi^{-1}\left(\frac{p-w}{p}\right)^2 = -\infty$ and $\lim_{\widetilde{h}\to\infty} \frac{\partial q_0}{\partial w}$ is finite, the result follows.

Claim 7. $\lim_{\widetilde{h}\to\infty} \frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} = 0^+$ and $\lim_{\widetilde{h}\to\infty} \frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} / \left(k(1-\Phi(\underline{\psi}))\frac{\partial w^*}{\partial \widetilde{h}}\right) = 1$.

The following argument proves this claim. One can show that

$$\frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} = k\big(1-\Phi(\underline{\psi})\big)\left(\frac{\partial w^*}{\partial \widetilde{h}} - \frac{H(\underline{\psi})-\underline{\psi}}{2k\sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}(h+\widetilde{h})^2}\right). \tag{A.5}$$

Therefore,

$$\begin{aligned}
&\lim_{\widetilde{h}\to\infty} \frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} / \frac{\mathbb{E}[e]+\theta}{-2\left(\frac{\partial q_0}{\partial w}\right)^2 (h+\widetilde{h})^2 q_0 (w^*-c)} \\
&= \lim_{\widetilde{h}\to\infty} k\big(1-\Phi(\underline{\psi})\big)\left(1 + \frac{H(\underline{\psi})-\underline{\psi}}{k\sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}(\mathbb{E}[e]+\theta)}\left(\frac{\partial q_0}{\partial w}\right)^2 q_0 (w^*-c)\right) \\
&= \lim_{\widetilde{h}\to\infty} k\big(1-\Phi(\underline{\psi})\big).
\end{aligned}$$

Claim 8. $\lim_{\widetilde{h}\to\infty} \frac{\partial \mathbb{E}[\Pi_R]}{\partial \widetilde{h}} = 0^-$.

First note that

$$\frac{\partial \mathbb{E}[\Pi_R]}{\partial \widetilde{h}} = -\frac{\partial w^*}{\partial \widetilde{h}}(q_0+\theta+\mathbb{E}[e]) + (p-w^*)\frac{\partial \mathbb{E}[e]}{\partial \widetilde{h}} - \frac{1}{2(h+\widetilde{h})}\left((p-w^*)q_0 - p\int_{-\infty}^{q_0} \Phi\left(\sqrt{h+\widetilde{h}}\,x\right)\mathrm{d}x\right).$$

Therefore,

$$\lim_{\widetilde{h}\to\infty} \frac{\partial \mathbb{E}[\Pi_R]}{\partial \widetilde{h}} / \frac{\partial w^*}{\partial \widetilde{h}} = \lim_{\widetilde{h}\to\infty} k(1-\Phi(\underline{\psi}))(p-w^*) - (q_0+\theta+\mathbb{E}[e]) + \frac{(\frac{\partial q_0}{\partial w})^2(h+\widetilde{h})q_0(w^*-c)}{E[e]+\theta}\left((p-w^*)q_0 - p\int_{-\infty}^{q_0}\Phi\left(\sqrt{h+\widetilde{h}}\,x\right)\mathrm{d}x\right)$$
$$= \lim_{\widetilde{h}\to\infty}(\theta+\mathbb{E}[e]).$$

The last equality is because $\lim_{\widetilde{h}\to\infty} w^* = p$, $\lim_{\widetilde{h}\to\infty} q_0 = 0$ and $\lim_{\widetilde{h}\to\infty} \sqrt{h+\widetilde{h}}\,q_0 = 0$.

Claim 9. $\lim_{\widetilde{h}\to\infty} \frac{\partial \mathbb{E}[\Pi_M]}{\partial \widetilde{h}} = 0^+$.

Notice that

$$\frac{\partial \mathbb{E}[\Pi_M]}{\partial \widetilde{h}} = \frac{\partial w^*}{\partial \widetilde{h}}(q_0+\theta) + \frac{\partial q_0}{\partial \widetilde{h}}(w^*-c) + \int_{\underline{\psi}}^{\infty}\left(\frac{\partial w^*}{\partial \widetilde{h}} - \frac{1}{2(h+\widetilde{h})^2 k\sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}H(\underline{\psi})}\right)\left(k(w^*-c) - \frac{\sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}}{H(x)}\right)\phi(x)\,\mathrm{d}x\,.$$

Therefore,

$$\lim_{\widetilde{h}\to\infty} \frac{\partial \mathbb{E}[\Pi_M]}{\partial \widetilde{h}} / \frac{\mathbb{E}[e]+\theta}{-2\left(\frac{\partial q_0}{\partial w}\right)^2(h+\widetilde{h})^2 q_0(w^*-c)}$$
$$= \lim_{\widetilde{h}\to\infty}(q_0+\theta) - \frac{q_0(w^*-c)}{2(h+\widetilde{h})}$$
$$+ \lim_{\widetilde{h}\to\infty}\int_{\underline{\psi}}^{\infty}\left(1 + \frac{\left(\frac{\partial q_0}{\partial w}\right)^2 q_0(w^*-c)}{k\sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}H(\underline{\psi})(\mathbb{E}[e]+\theta)}\right)\left(k(w^*-c) - \frac{\sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}}{H(x)}\right)\phi(x)\,\mathrm{d}x$$
$$= \lim_{\widetilde{h}\to\infty}(q_0+\theta+\mathbb{E}[e]) - \frac{q_0(w^*-c)}{2(h+\widetilde{h})} = \lim_{\widetilde{h}\to\infty}\theta+\mathbb{E}[e]\,.$$

■

Proof of Proposition 10. Let $f(x) = \frac{1-\Phi(x)}{H(x)}\big(1-H(x)^2+xH(x)\big)$. By definition of q_0 and $\mathbb{E}(e)$, we know $\frac{\partial q_0}{\partial k} = \frac{\partial q_0}{\partial w}\frac{\partial w^*}{\partial k}$, $\frac{\partial^2 q_0}{\partial w \partial k} = \frac{\partial w^*}{\partial k}\frac{\partial^2 q_0}{\partial w^2}$, and $\frac{\partial \mathbb{E}[e]}{\partial k} = \frac{\partial w^*}{\partial k}\frac{\partial \mathbb{E}(e)}{\partial w} - \frac{Af'(\underline{\psi})}{k(H(\underline{\psi})-\underline{\psi})}$.

By the first order condition to find the optimal wholesale price (equation (8)), $\frac{\partial w^*}{\partial k} = \frac{Af'(\underline{\psi})}{k(H(\underline{\psi})-\underline{\psi})\frac{\partial^2\mathbb{E}[\Pi_M]}{\partial w^2}} > 0$. That is, the wholesale price is increasing in efficiency of the sales agent.

This implies that $\frac{\partial q_0}{\partial k} = \frac{\partial q_0}{\partial w}\frac{\partial w^*}{\partial k} < 0$. Furthermore, $\frac{\partial \mathbb{E}[e]}{\partial k} = -\frac{1}{\frac{\partial^2 \mathbb{E}[\Pi_M]}{\partial w^2}}(1-\Phi(\underline{\psi}))(w^*-c)(k(w^*-c) - \frac{\partial^2 \mathbb{E}[\Pi_M]}{\partial w^2}) > 0$. In other words, the expected effort of the sales agent is increasing in the efficiency of the sales agent.

Next, we show that the retailer's expected profit function is quasi-concave in the efficiency of the sales agent. One can show that $\frac{\partial \mathbb{E}[\Pi_R]}{\partial k} = \frac{\partial w^*}{\partial k}\frac{\partial q_0}{\partial w}(p-w^*)\frac{w^*-c}{p}\left(\frac{1}{\Phi(\sqrt{h+\tilde{h}q_0})} + \frac{\sqrt{h+\tilde{h}q_0}}{\phi(\sqrt{h+\tilde{h}q_0})} - 2\frac{p}{w^*-c}\right)$. Since $\frac{\partial w^*}{\partial k}\frac{\partial q_0}{\partial w}(p-w^*)\frac{w^*-c}{p}$ is negative, to show that the retailer's expected profit function is quasi-concave in k, it is enough to show $\left(\frac{1}{\Phi(\sqrt{h+\tilde{h}q_0})} + \frac{\sqrt{h+\tilde{h}q_0}}{\phi(\sqrt{h+\tilde{h}q_0})} - 2\frac{p}{w^*-c}\right)$ is increasing in k. Furthermore, since $\frac{\partial(-2\frac{p}{w^*-c})}{\partial k} > \frac{\partial(-2\frac{p}{w^*})}{\partial k} > \frac{\partial(-\frac{p}{w^*})}{\partial k} > 0$, we only need to show $\left(\frac{1}{\Phi(\sqrt{h+\tilde{h}q_0})} + \frac{\sqrt{h+\tilde{h}q_0}}{\phi(\sqrt{h+\tilde{h}q_0})} - \frac{p}{w^*}\right)$ is increasing in k. In addition, since $\frac{\partial q_0}{\partial k} < 0$ and $w^* = p(1-\Phi(\sqrt{h+\tilde{h}q_0}))$, we only need to show $g_1(x) = \frac{1}{\Phi(x)} + \frac{x}{\phi(x)} - \frac{1}{1-\Phi(x)}$ is decreasing in x.
Note that $g_1'(x) = \phi(x)g_2(x)$ where $g_2(x) = -\frac{1}{\Phi(x)^2} + \frac{1+x^2}{\phi(x)^2} - \frac{1}{(1-\Phi(x))^2}$. Also $g_2'(x) = \frac{1}{\phi(x)^2}(2H(-x)^3 - 2H(x)^3 - x^2)$. Since $2H(-x)^3 - 2H(x)^3 - x^2$ is decreasing in x, $g_2(x)$ is quasi-concave and its maximum is obtained at $x = 0$, with the value $2\pi - 8$. Therefore, $g_2(x) < 0$ for all x and hence, $g_1(x)$ is decreasing in x. In conclusion, the retailer's expected profit is quasi-concave in k.

Next, we show that the manufacturer's expected profit is increasing in k. Let $k_1 < k_2$, and w_1 and w_2 be the optimizer of the manufacturer's expected profit when $k = k_1$ and $k = k_2$, respectively. Also let $\mathbb{E}[\Pi_M(k,w)]$ be the optimal expected profit of the manufacturer when the wholesale price is w and efficiency of the sales agent is k. Then we have

$$\begin{aligned}
\mathbb{E}[\Pi_M(k_1,w_1)] &= (w_1-c)(q_0(w_1)+\theta) + \frac{k_1}{2}\int_{\underline{\psi}(k_1,w_1)}^{\infty}((w_1-c) - \frac{A}{k_1 H(x)})^2\phi(x)\,\mathrm{d}x \\
&< (w_1-c)(q_0(w_1)+\theta) + \frac{k_2}{2}\int_{\underline{\psi}(k_2,w_1)}^{\infty}((w_1-c) - \frac{A}{k_2 H(x)})^2\phi(x)\,\mathrm{d}x \\
&\le (w_2-c)(q_0(w_2)+\theta) + \frac{k_2}{2}\int_{\underline{\psi}(k_2,w_2)}^{\infty}((w_2-c) - \frac{A}{k_2 H(x)})^2\phi(x)\,\mathrm{d}x \\
&= \mathbb{E}[\Pi_M(k_2,w_2)]\,.
\end{aligned}$$

That is, the manufacturer's expected profit function is increasing in efficiency of the sales agent. ■

A.2 Extensions of the Model Setup

In this section, we provide the technical statements and outline the proofs of the results discussed in the extensions in Section 6 of the paper. We first present the results related to the observability and the sequence of events for the sales agent's effort:

Proposition A.1.

(i) If the effort of the sales agent is not observable by the retailer, the agent's equilibrium effort level is equal to zero.

(ii) If the sales agent exerts an effort after the retailer makes the order quantity decision, the agent's effort level is zero in equilibrium.

(iii) If the sales agent's compensation scheme depends on the realized consumer demand, the equilibrium commission rate, effort, order quantity, payoff of the sales agent, and expected profits of the retailer and the manufacturer are the same as the one presented in Proposition 1. Furthermore, in this case, the sequence of events for the sales agent's effort does not affect this equilibrium outcome; that is, whether the sale agent exerts an effort before or after the retailer orders does not influence the equilibrium outcome.

Proof of Proposition A.1. First, for part (i), consider the case in which the effort of the sales agent is not observable by the retailer. Suppose that the equilibrium effort of the sales agent is $e_0(\psi)$, which could be a mixed strategy. In equilibrium, the retailer has a consistent belief about the sales agent's effort level, $e_0(\psi)$. Therefore, the retailer maximizes his expected profit given this consistent belief; that is, he solves the following optimization problem

$$\mathbb{E}[\Pi_R(q)] = (p - \overline{w})q - p \int_{-\infty}^{q} \mathbb{E}\left(\Phi\left(\frac{y - e_0(\psi) - \frac{h\theta + \widetilde{h}\psi}{h+\widetilde{h}}}{\frac{1}{h+\widetilde{h}}}\right)\right) \mathrm{d}y\,,$$

where the expectation in the right hand side of the equation is with respect to the strategy of the sales agent. Maximizing this expected profit, we obtain that the optimal order quantity q^* satisfies

$$\frac{p-\overline{w}}{p} = \mathbb{E}\left(\Phi\left(\frac{q^* - e_0(\psi) - \frac{h\theta + \widetilde{h}\psi}{h+\widetilde{h}}}{\frac{1}{h+\widetilde{h}}}\right)\right)\,.$$

Then the sales agent maximizes its expected payoff given as

$$\pi_s = \alpha(q^*) + \beta - \frac{1}{2k}e^2\,.$$

Note that the retailer's order quantity q^* does not depend on the actual effort of the sales agent (which cannot be observed by the retailer). It only depends on the belief of the retailer from the sales agent effort $e_0(\psi)$ which is consistent with the equilibrium effort of the sales agent. Consequently, the first order condition of this expected profit function becomes $-\frac{1}{k}e < 0$. In other words, the sales agent's expected payoff function is decreasing in its effort. Thus, it follows that the equilibrium sales agent's effort level is zero.

Second, for part (ii), suppose that the sales agent is paid based on the order quantity of the retailer but makes its effort after the retailer places his order. This implies that the sales agent is the last player to decide in this model. Therefore, by solving backwards, we consider the sales agent's problem first. The payoff function of the sales agent given that it has accepted a contract with constant salary β and commission rate α on order quantity of the retailer is:

$$\pi_s = \alpha q + \beta - \frac{1}{2k} e^2 .$$

This expected payoff function is again decreasing in e, and thus the optimal effort is $e^* = 0$.

Finally, for part (iii), consider the case that the sales agent is paid based on the realized consumer demand and makes its effort after the retailer places his order. The sales agent determines its effort at the last step of the game. Therefore, we solve the sales agent's problem first. Notice that from the sales agent's perspective $D|(e, \Psi = \psi) \sim N(e + \frac{h\theta + \widetilde{h}\psi}{h+\widetilde{h}}, \frac{1}{h+\widetilde{h}})$. The payoff function of the sales agent who has accepted a contract with constant salary β and commission rate α on the realized demand is:

$$\pi_s = \alpha D + \beta - \frac{1}{2k} e^2 .$$

Therefore,

$$\mathbb{E}[\pi_s(e, \alpha, \beta)] = \alpha(e + \frac{h\theta + \widetilde{h}\psi}{h + \widetilde{h}}) + \beta - \frac{1}{2k} e^2 .$$

It follows that the optimal strategy of the sales agent is $e^*(\alpha) = k\alpha$, which is the same as the one presented in Proposition 1.

Next, we focus on the retailer's expected profit to find his optimal order quantity. The retailer's expected profit function can be simplified to

$$\mathbb{E}[\Pi_R(q)] = (p - \overline{w})q - p \int_{-\infty}^{q} \Phi \left(\frac{y - k\alpha - \frac{h\theta + \widetilde{h}\psi}{h+\widetilde{h}}}{\frac{1}{h+\widetilde{h}}} \right) .$$

Then the optimal order quantity of the retailer becomes $q^*(\psi, \alpha) = q_0 + k\alpha + \frac{h\theta + \widetilde{h}\psi}{h+\widetilde{h}}$, where $q_0 = \frac{1}{\sqrt{h+\widetilde{h}}} \Phi^{-1} \left(\frac{p - \overline{w}}{p} \right)$.

The manufacturer's expected profit function is then:

$$\mathbb{E}[\Pi_M] = \mathbb{E}_\psi \left[(\overline{w} - c - \alpha_\psi)(q_0 + k\alpha_\psi + \frac{h\theta + \widetilde{h}\psi}{h + \widetilde{h}} + \alpha_\psi q_0 - \beta_\psi) \right] .$$

From the revelation principle, it follows that the manufacturer maximizes her expected profit given that she should satisfy the following to constraints:

$$\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) \geq \pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'} | \Psi = \psi), \quad \forall \psi, \forall \psi',$$
$$\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) \geq 0, \quad \forall \psi.$$

Next, similar to the proof of Proposition 1, we show that $\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) \geq \pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'} | \Psi = \psi)$ is equivalent to $\frac{\partial \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi)}{\partial \psi} = \frac{\widetilde{h}}{h+\widetilde{h}} \alpha_\psi$. Fix ψ and consider any $\psi' < \psi$. We must have

$$\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) \geq \pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'} | \Psi = \psi)$$
$$\pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'} | \Psi = \psi') \geq \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi').$$

Equivalently,

$$\alpha_\psi \left(\frac{k}{2} \alpha_\psi + \frac{h\theta + \widetilde{h}\psi}{h + \widetilde{h}} \right) + \beta_\psi \geq \alpha_{\psi'} \left(\frac{k}{2} \alpha_{\psi'} + \frac{h\theta + \widetilde{h}\psi}{h + \widetilde{h}} \right) + \beta_{\psi'},$$
$$\alpha_{\psi'} \left(\frac{k}{2} \alpha_{\psi'} + \frac{h\theta + \widetilde{h}\psi'}{h + \widetilde{h}} \right) + \beta_{\psi'} \geq \alpha_\psi \left(\frac{k}{2} \alpha_\psi + \frac{h\theta + \widetilde{h}\psi'}{h + \widetilde{h}} \right) + \beta_\psi,$$

which implies that $\alpha_\psi > \alpha_{\psi'}$ for $\psi > \psi'$. Therefore, we have:

$$\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) - \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi') \geq \frac{\widetilde{h}}{h + \widetilde{h}} (\psi - \psi') \alpha_{\psi'} \qquad \forall \psi, \forall \psi'. \tag{A.6}$$

By following similar lines of proofs as in Proposition 1, it follows that $\frac{\partial \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi)}{\partial \psi} = \frac{\widetilde{h}}{h+\widetilde{h}} \alpha_\psi$. Note that any α_ψ and β_ψ that satisfy $\frac{\partial \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi)}{\partial \psi} = \frac{\widetilde{h}}{h+\widetilde{h}} \alpha_\psi$ also satisfy $\pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) \geq \pi_s(k\alpha_{\psi'}, \alpha_{\psi'}, \beta_{\psi'} | \Psi = \psi)$. Therefore, the manufacturer's problem becomes

$$\max_{\alpha_\psi \geq 0, L} \Big(-L + (\overline{w} - c)(q_0 + \theta)$$
$$+ \int_{-\infty}^{\infty} \left(\left(k(\overline{w} - c)\alpha_\psi - \frac{k}{2} \alpha_\psi^2 \right) \sqrt{\frac{h\widetilde{h}}{h + \widetilde{h}}} \phi \left(\frac{\psi - \theta}{\sqrt{\sigma^2 + \widetilde{\sigma}^2}} \right) - \frac{\widetilde{h}}{h + \widetilde{h}} \left(1 - \Phi \left(\frac{\psi - \theta}{\sqrt{\sigma^2 + \widetilde{\sigma}^2}} \right) \right) \alpha_\psi \right) \mathrm{d}\psi \Big)$$
$$s.t. \quad \pi_s(k\alpha_\psi, \alpha_\psi, \beta_\psi | \Psi = \psi) \geq 0.$$

It then follows that

$$\alpha_\psi^* = \left((\overline{w} - c) - \frac{A}{kH\left(\frac{\psi-\theta}{\sqrt{\sigma^2+\widetilde{\sigma}^2}}\right)} \right)^+ ,$$

$$\beta_\psi^* = \frac{\widetilde{h}}{h+\widetilde{h}} \int_{-\infty}^{\psi} \alpha_y \,\mathrm{d}y - \alpha_\psi \left(\frac{h\theta + \widetilde{h}\psi}{h+\widetilde{h}} + \frac{k}{2}\alpha_\psi^* \right) ,$$

where $A = \sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}$. Note that the commission rate of the sales agent remains the same as in Proposition 1 and the analysis following Proposition 1 carry out. As a result, the equilibrium outcome is equivalent to that presented in Proposition 1. Furthermore, for the case in which the sales agent makes its effort before the retailer orders, the analysis remains the same as long as the compensation scheme of the sales agent depends on the realized demand. This completes the proof.

■

Next, we present the equilibrium outcome of the case in which the sales agent has more accurate demand information than the retailer. Specifically, as in the original model, both the sales agent and the retailer observe a noisy signal that is normally distributed with a mean equal to the market condition, i.e., $\Psi|(\Theta = \hat{\theta}) \sim N(\hat{\theta}, \widetilde{\sigma}^2)$. Note that the accuracy of this signal is denoted by $\widetilde{h} \equiv \frac{1}{\widetilde{\sigma}^2}$. In addition, only the sales agent observes another additional independent noisy signal that is also normally distributed with a mean equal to the market condition, i.e., $\Psi_s|(\Theta = \hat{\theta}) \sim N(\hat{\theta}, \widetilde{\sigma}_s^2)$. We denote the accuracy of this signal by $\widetilde{h}_s \equiv \frac{1}{\widetilde{\sigma}_s^2}$. All other structure and the sequence of events are the same as the one in the original model.

PROPOSITION A.2. *If the sales agent observes additional independent market condition signal Ψ_s, the equilibrium outcome remains the same as the one presented in Proposition 1 in which the sales agent does not observe Ψ_s.*

Proof of Proposition A.2. Suppose the sales agent observes an additional signal $\Psi_s \sim N(\Theta, \frac{1}{\widetilde{h}_s})$. Consider the following system of beliefs for the retailer:

$$\Psi_s \sim N(\Theta, \frac{1}{\widetilde{h}_s}) .$$

Together with this belief system, we show that the following strategies, which are the same as the one presented in Proposition 1, are a Perfect Bayesian equilibrium of the game:

$$q(e, \psi) = q_0 + e + \frac{h\theta + \widetilde{h}\psi}{h+\widetilde{h}} ,$$

$$
\begin{aligned}
&e(\alpha,\beta,\psi,\psi_s) = k\alpha\,,\\
&\alpha(\psi,\psi_s) = \frac{A}{k}\left(\frac{1}{H(\underline{\psi})} - \frac{1}{H\left(\frac{\psi-\theta}{\sqrt{\sigma^2+\widetilde{\sigma}^2}}\right)}\right)^+\,,\\
&\beta(\psi,\psi_s) = \frac{\widetilde{h}}{h+\widetilde{h}}\int_{-\infty}^{\psi} a_y\,\mathrm{d}y - \alpha(\psi,\psi_s)\left(q_0 + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}} + \frac{k}{2}a_\psi\right)\,.
\end{aligned}
$$

First, note that the specified system of beliefs is consistent with the equilibrium strategies and prior belief of the retailer. We use the backward induction to show that these strategies are sequentially rational. Suppose that the retailer observes $e = A\left(\frac{1}{H(\underline{\psi})} - \frac{1}{H\left(\frac{\psi-\theta}{\sqrt{\sigma^2+\widetilde{\sigma}^2}}\right)}\right)^+$. With the specified system of beliefs $D|(\Psi=\psi,e) \sim N(e + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}, \frac{1}{h+\widetilde{h}})$. Therefore, the retailer's expected profit can be written as:

$$
\mathbb{E}[\Pi_R(q)] \;=\; (p-\overline{w})q - p\int_{-\infty}^{q}\Phi\left(\frac{y-e-\frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}}{\frac{1}{\sqrt{h+\widetilde{h}}}}\right)dy\,,
$$

Note that $q(e,\psi) = q_0 + e + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}} = \arg\max_q \mathbb{E}[\Pi_R(q)]$. Therefore, the retailer would not deviate from this strategy.

Next, assume that the sales agent, who has observed ψ and ψ_s, has chosen a contract with commission rate α and fixed salary β. In such a case, the sales agent's payoff is

$$
\pi_s(e,\alpha,\beta|\Psi\;=\;\psi) \;=\; \alpha\Big(q_0 + e + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}\Big) + \beta - \frac{1}{2k}e^2\,. \tag{A.7}
$$

Notice that $k\,\alpha = \arg\max_e\, \pi_s(e,\alpha,\beta|\Psi\;=\;\psi)$. Therefore, the sales agent would not deviate from this strategy, either.

Lastly, the manufacturer's expected profit function can be written as

$$
\mathbb{E}_{\psi,\psi_s}\Big[\big(\overline{w}-c-\alpha_\psi\big)\big(q_0 + k\alpha_\psi + \frac{h\theta+\widetilde{h}\psi}{h+\widetilde{h}}\big) - \beta_\psi\Big]\,.
$$

By the revelation principle, there exists a payoff-equivalent revelation mechanism that has an equilibrium where the players truthfully report their types. Therefore, in order to find a commission rate and a constant salary that are sequentially rational for the manufacturer, we solve the following constrained optimization problem:

$$\max_{\alpha_\psi \geq 0, \beta_\psi} \quad \mathbb{E}_{\psi,\psi_s}\Big[\big(\overline{w} - c - \alpha_{\psi,\psi_s}\big)\big(q_0 + k\alpha_{\psi,\psi_s} + \frac{h\theta + \widetilde{h}\psi}{h + \widetilde{h}}\big) - \beta_{\psi,\psi_s}\Big]$$

$$s.t. \quad \pi_s(k\alpha_{\psi,\psi_s}, \alpha_{\psi,\psi_s}, \beta_{\psi,\psi_s} | \Psi = \psi, \Psi_s = \psi_s) \geq \pi_s(k\alpha_{\psi',\psi'_s}, \alpha_{\psi',\psi'_s}, \beta_{\psi',\psi'_s} | \Psi = \psi, \Psi_s = \psi_s), \quad \forall\psi, \forall\psi', \forall\psi_s, \forall\psi'_s, \qquad (IC')$$

$$\pi_s(k\alpha_{\psi,\psi_s}, \alpha_{\psi,\psi_s}, \beta_{\psi,\psi_s} | \Psi = \psi, \Psi_s = \psi_s) \geq 0, \quad \forall\psi, \forall\psi_s. \qquad (IR')$$

Now fix ψ and ψ_s. Suppose α_{ψ,ψ_s} and β_{ψ,ψ_s} satisfy (IC') against α_{ψ',ψ_s} and β_{ψ',ψ_s} as follows:

$$\pi_s(k\alpha_{\psi,\psi_s}, \alpha_{\psi,\psi_s}, \beta_{\psi,\psi_s} | \Psi = \psi, \Psi_s = \psi_s) \geq \pi_s(k\alpha_{\psi',\psi_s}, \alpha_{\psi',\psi_s}, \beta_{\psi',\psi_s} | \Psi = \psi, \Psi_s = \psi_s), \quad \forall\psi' < \psi.$$

Note that α_{ψ',ψ_s} and β_{ψ',ψ_s} should also satisfy (IC'). Therefore, we obtain:

$$\pi_s(k\alpha_{\psi',\psi_s}, \alpha_{\psi',\psi_s}, \beta_{\psi',\psi_s} | \Psi = \psi', \Psi_s = \psi_s) \geq \pi_s(k\alpha_{\psi,\psi_s}, \alpha_{\psi,\psi_s}, \beta_{\psi,\psi_s} | \Psi = \psi', \Psi_s = \psi_s), \quad \forall\psi' < \psi.$$

Using (A.7), we find that $\alpha_{\psi,\psi_s} > \alpha_{\psi',\psi_s}$ for all $\psi > \psi_s$. Using this fact and (A.7), we then obtain:

$$\pi_s(k\alpha_{\psi,\psi_s}, \alpha_{\psi,\psi_s}, \beta_{\psi,\psi_s} | \Psi = \psi, \Psi_s = \psi_s) - \pi_s(k\alpha_{\psi',\psi_s}, \alpha_{\psi',\psi_s}, \beta_{\psi',\psi_s} | \Psi = \psi', \Psi_s = \psi_s) \geq \frac{\widetilde{h}}{h + \widetilde{h}}(\psi - \psi')\alpha_{\psi',\psi_s} \qquad \forall\psi, \forall\psi'. \quad \text{(A.8)}$$

From (A.8) and similar inequality in which the role of ψ and ψ' is reversed, it follows that

$$\frac{\widetilde{h}}{h + \widetilde{h}}(\psi - \psi')\alpha_{\psi,\psi_s} \geq \pi_s(k\alpha_{\psi,\psi_s}, \alpha_{\psi,\psi_s}, \beta_{\psi,\psi_s} | \Psi = \psi, \Psi_s = \psi_s) - \pi_s(k\alpha_{\psi',\psi_s}, \alpha_{\psi',\psi_s}, \beta_{\psi',\psi_s} | \Psi = \psi', \Psi_s = \psi_s) \geq \frac{\widetilde{h}}{h + \widetilde{h}}(\psi - \psi')\alpha_{\psi',\psi_s}.$$

Dividing these inequalities by $(\psi - \psi')$ and converging ψ close to ψ', we obtain

$$\frac{\partial \pi_s(k\alpha_{\psi,\psi_s}, \alpha_{\psi,\psi_s}, \beta_{\psi,\psi_s} | \Psi = \psi, \Psi_s = \psi_s)}{\partial \psi} = \frac{\widetilde{h}}{h + \widetilde{h}}\alpha_{\psi,\psi_s}. \qquad \text{(A.9)}$$

After we integrate both sides, it follows

$$\pi_s(k\alpha_{\psi,\psi_s}, \alpha_{\psi,\psi_s}, \beta_{\psi,\psi_s} | \Psi = \psi, \Psi_s = \psi_s) = L + \frac{\widetilde{h}}{h + \widetilde{h}} \int_{-\infty}^{\psi} \alpha_{y,\psi}\, dy, \qquad \text{(A.10)}$$

where L is a constant that the manufacturer can decide. This implies that any commission rate

α_{ψ,ψ_s} and constant salary β_{ψ,ψ_s} that satisfy (IC') should satisfy (A.10). One can also verify that any commission rate α_{ψ,ψ_s} and constant salary β_{ψ,ψ_s} that satisfy (A.10) would satisfy (IC'). Therefore, (IC') constraint is equivalent to (A.10). We then replace this into objective function of the manufacturer and solve for optimal commission rate α_{ψ,ψ_s} and constant salary β_{ψ,ψ_s}. We find that the optimal solutions are:

$$\alpha^*_{\psi,\psi_s} = \left((\overline{w} - c) - \frac{A}{kH\left(\frac{\psi-\theta}{\sqrt{\sigma^2+\widetilde{\sigma^2}}}\right)} \right)^+ ,$$

where $A = \sqrt{\frac{\widetilde{h}}{h(h+\widetilde{h})}}$. Let $\underline{\psi}$ be the unique solution to $k(\overline{w} - c)H(\underline{\psi}) = A$. Consequently, the manufacturer does not want to deviate either, and hence these strategies are an equilibrium of this game. This completes the proof. One might wonder whether a separating equilibrium in which the sales agent can signal its type to the retailer exists. We next provide sketch of a proof that shows no separating equilibrium in which the sales agent signals its effort exists.

Suppose to the contrary that from the sales agent's effort, the retailer can infer ψ_s by observing effort e. In such case, there must be a function f such that $f(e, \psi) = \psi_s$. Equivalently, there must be a function g such that $g(e, \psi) = \frac{h\theta+\widetilde{h}\psi+\widetilde{h}_s\psi_s}{h+\widetilde{h}+\widetilde{h}_s}$. By Berge maximum theorem, one can show that g is continuous. Also since the effort is informative, g must be bijection and surjective.

Then, the retailer's optimal strategy is

$$q(\psi, e) = \arg\max_{q>0} (p - \overline{w})q - p \int_{-\infty}^{q} \Phi\left(\frac{y - e - g(e, \psi)}{\frac{1}{\sqrt{h+\widetilde{h}+\widetilde{h}_s}}} \right) = q_0 + e + g(e, \psi)\,.$$

Therefore, the sales agent optimal strategy is

$$e(\psi, \psi_s) = \arg\max_{e} \alpha(\psi, \psi_s)(q_0 + e + g(e, \psi)) + \beta(\psi, \psi_s) - \frac{1}{2k}e^2\,.$$

The optimal strategy of the sales agent should satisfy the first order condition. The first order condition of this optimization is

$$\alpha(\psi, \psi_s) = \frac{1}{k(1 + g_1(e(\psi, \psi_s), \psi))} e(\psi, \psi_s)\,,$$

where g_1 is the derivative of g with respect to its first argument. The optimal payoff of the sales agent is then

$$\pi_s(e(\psi,\psi_s),\psi,\psi_s) = \frac{1}{2k}e^2(\psi,\psi_s)(\frac{1-g_1(e(\psi,\psi_s),\psi)}{1+g_1(e(\psi,\psi_s))}) + \frac{1}{k}\frac{e}{1+g_1(e(\psi,\psi_s),\psi)}(q_0+g(e(\psi,\psi_s),\psi))\,. \tag{A.11}$$

The manufacturer's optimization is then

$$\begin{aligned} \max_{\alpha_{\psi,\psi_s}\geq 0,\beta_{\psi,\psi_s}} \quad & (\overline{w}-c)(q_0+e+g(e,\psi)) - \pi_s(e,\psi,\psi_s) - \frac{1}{2k}e^2 \\ s.t. \quad & \pi_s(e_{\psi,\psi_s},\psi,\psi_s) \geq \pi_s(e_{\psi',\psi_s'},\psi,\psi_s) \qquad \forall \psi,\psi_s,\psi',\psi_s' \\ & \pi_s(e_{\psi,\psi_s},\psi,\psi_s) \geq 0 \\ & \alpha(\psi,\psi_s) = \frac{1}{k(1+g_1(e(\psi,\psi_s),\psi))}e(\psi,\psi_s)\,. \end{aligned} \tag{A.12}$$

Fix ψ and choose ψ_s and ψ_s'. Suppose $\pi_s(e_{\psi,\psi_s},\psi,\psi_s) > \pi_s(e_{\psi,\psi_s'},\psi,\psi_s)$. By (A.12), we must have $\pi_s(e_{\psi,\psi_s'},\psi,\psi_s') \geq \pi_s(e_{\psi,\psi_s},\psi,\psi_s')$. Summing these two inequalities and using (A.11), we must have $0 > 0$, which is a contradiction. Therefore, we must have $\pi_s(e_{\psi,\psi_s},\psi,\psi_s) \leq \pi_s(e_{\psi,\psi_s'},\psi,\psi_s)$. By switching the role of ψ_s, and ψ_s' in this argument, we must also have $\pi_s(e_{\psi,\psi_s},\psi,\psi_s) \geq \pi_s(e_{\psi,\psi_s'},\psi,\psi_s)$. Therefore, $\pi_s(e_{\psi,\psi_s},\psi,\psi_s) = \pi_s(e_{\psi,\psi_s'},\psi,\psi_s)$ for all ψ.

This implies $e_{\psi,\psi_s} = e_{\psi,\psi_s'}$. That is, the effort of the sales agent is independent of ψ_s. Hence the effort is not informative. ∎

A.3 Reference to the Toyota Area Sales Managers Job Description

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Job Description

Area Sales Manager - Middletown DSSO-TFS000LD

Description

Toyota Financial Services
Our people are the driving force behind our success and we're moving forward!
Join a dynamic company known for rapid growth and solid success.

As an **Area Sales Manager**, you will create impact by:

- Planning and conducting regularly scheduled dealership visits to maintain or improve existing dealer relationships, sign and activate new TFS dealers and facilitate increased sales
- Clearly communicating finance and insurance product/service offerings and developing promotional plans and programs to meet or exceed sales objectives, including contract volume and market share
- Providing day-to-day and ongoing support and training to assigned dealerships to promote product understanding, improve sales and provide "best in class" customer service
- Providing effective communication and training to credit associates
- Monitoring competitor rates and programs and preparing analyses for DSSO Manager to ensure DSSO maintains a competitive position in all market areas
- Obtaining proprietary business financial information from dealership owners and providing consultation in order to obtain compliance with TFS standards

Qualifications

TFS is looking for individuals with strong business sense and practical expertise. Successful candidates must have:

- Minimum 4 years experience in a captive finance and/or insurance environment
- B.A./B.S. Degree or equivalent finance/business experience
- Dealer contact and successful record of credit, collection and wholesale preferred
- Excellent verbal and written communication skills and the ability to interface with all levels of dealership personnel
- Strong verbal and written communication skills
- Strong organizational skills and an attention to details
- Working knowledge of Microsoft Office applications (Word, Excel, PowerPoint, etc.) and Lotus Notes strongly preferred

Turn toward great benefits:

- Work/Life benefits (flextime, 9/80 work schedules offered where applicable, tuition

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reimbursement)
- Vehicle lease and purchase (Associates are eligible date of hire, access to favorable rates and more incentives!)
- Medical, dental and vision insurance (Associates are eligible date of hire & premiums are paid by Toyota)
- Matching 401(k) and fully funded Pension Plan
- Paid time off (vacation, sick, personal, holidays)

About Toyota Financial Services
Headquartered in Torrance, Calif., Toyota Financial Services (TFS) is the finance and insurance brand for Toyota in the United States, offering retail auto financing and leasing through Toyota Motor Credit Corporation (TMCC) and extended service contracts through Toyota Motor Insurance Services (TMIS). Lexus Financial Services is the brand for financial products for Lexus dealers and customers. TFS currently employs over 3,000 associates nationwide, and has managed assets totaling more than $79 billion. It is part of a worldwide network of comprehensive financial services offered by Toyota Financial Services Corporation, a wholly-owned subsidiary of Toyota Motor Corporation.
EOE. M/F/D/V.

Job Field Sales
Primary LocationUS-NY-New York-New York
Organization TFS - Toyota Financial Services
Travel Yes, 75% of the time

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