Private Information, Diversification, and Asset Pricing *†

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Private Information, Diversification, and Risk Premium

Abstract

We investigate the effects of private information and diversification on risk premiums in a noisy rational expectations model in which risky asset payoffs have a factor structure. Information in our model is composed of private signals that are informative about both systematic factors and idiosyncratic shocks. Taking the large economy limit, we show that the APT pricing relation holds with asymmetric information. Private information about systematic factors affects risk premiums only through its effects on factor risk premiums; private information only about idiosyncratic shocks has no effects on risk premiums. Consistent with our intuition, factor risk premiums are decreasing in the fraction of informed to uninformed investors, sensitivity of private signals to systematic factors, and precision of private signals. More subtly, although idiosyncratic risks are not priced, factor risk premiums are increasing in the volatility of idiosyncratic shocks.

1 Introduction

Two major insights in modern finance are diversification, the vanishing of idiosyncratic risk premiums in large economies, and price discovery, the revelation of private information in equilibrium prices. While the implications of diversification on risk premiums are well known from arbitrage pricing theory (APT) under homogeneous beliefs, less is understood about how private signals impact on risk premiums in large economies. To the best of our knowledge, there is no previous study that addresses the interaction of diversification and price discovery in the large economy limit.

In this paper, we consider the effects of private signals on risk premiums in a noisy rational expectations model in which risky asset payoffs obey a factor structure. Private signals for each asset include components related to systematic factors as well as idiosyncratic shocks underlying those payoffs. Informed investors receive these signals and uninformed investors draw inferences about the information contained in these signals from prices. Although we begin with finite economies, our principal interest lies in characterizing risk premiums for large economies in which the number of risky assets and related private signals go to infinity.

We show that for large economies, the APT pricing relation holds under heterogeneous beliefs, private information about idiosyncratic shocks only matters as a source of noise in drawing inferences about systematic factors from private signals or prices, and private information about systematic factors affects risk premiums only through factor risk premiums. Comparative statics are that factor risk premiums are increasing in the fraction of informed to uninformed investors, sensitivity of private signals to systematic factors, and precision of private signals and decreasing in the volatility of idiosyncratic shocks.

These results are intuitive. As expected, information about systematic factors does affect risk premiums by resolving uncertainty about systematic factors representing risks that are priced in large as well as finite economies. Private information about idiosyncratic shocks changes expectations of future risky asset payoffs, but in large economies only affects risk premiums through its impact on resolving uncertainty about systematic factors. While private information on idiosyncratic shocks also resolves uncertainty about those shocks, thereby diminishing idiosyncratic risk, this aspect does not matter if the risk is not priced. Since private signals are more informative than prices about systematic factors than increasing the fraction of informed investors results in a greater resolution of uncertainty about those factors. Greater sensitivity of private signals to systematic factors, higher precision, and lower volatility of idiosyncratic shocks imply more is learned about those factors, thereby reducing risk premiums.

The issues addressed in our study are important in light of the recent interest of empiricists in linking characteristics of a firm's information environment to its expected return (e.g., Aboody, Hughes, and Liu, 2004; Bhattacharya and Daouk, 2002; Botosan, 1997; Botosan and Plumlee, 2002; Botosan, Plumlee, and Xie, 2004; Healy, Hutton, and Palepu, 1999; Francis, LaFond, Olsson, and Schipper, 2002; and Easley, Hvidkjaer, and O'Hara, 2002). In particular, the notion that risk premiums may vary with asymmetries in information pertaining to idiosyncratic shocks as in Easley and O'Hara (2004) bears scrutiny since they do not consider the full implications of diversification in large economies.

While our principal results pertain to the case of imperfect private information about systematic factors, other cases yield intuitively complementary results. At one extreme, if private signals are simply risky asset payoffs plus noise (e.g., Admati, 1985), then in the limit as the economy expands factor realizations become perfectly revealed to all investors, which implies risk premiums equal to zero. At the other extreme, if private signals only pertain to idiosyncratic shocks and not as well to systematic factors (e.g., Easley and O'Hara, 2004), then in large economies information leads to no resolution of uncertainty about priced risks, which implies risk premiums are unaffected.

A key element of the information structure that we impose is that private signals are informative about systematic factors as well as idiosyncratic shocks. This information structure is consistent with evidence from Seyhun (1992) and Lakonishok and Lee (2001) that corporate insiders are able to time the market. The further notion that private signals at the firm (asset) level may contain a systematic component is the observation that data supplied by financial reports for which some investors may have advance knowledge typically includes fundamentals such as revenues, earnings, and cash flows that are plausibly affected by systematic factors as well as idiosyncratic shocks; evidence dates back to Ball and Brown (1968). These data also include descriptions of business risk factors and management's discussion and analysis of prospective performance. While it is likely that private signals at the firm-level are far more informative of idiosyncratic shocks, as we will show, even an infinitesimally small amount of information on systematic factors extracted from private signals for each firm in large economies, when aggregated, can have a finite effect on factor risk premiums.

We note that our asymmetric information structure allows us to solve for equilibrium prices and risk premiums explicitly; which is infeasible under Admati's (1982) diverse information structure in a factor model setting. Having an explicit pricing solution is especially useful because it allows us to examine how changes in model parameters such as the fraction of informed to uninformed investors affect factor risk premiums. Furthermore, having a closed form solution for finite economies enables us to examine the convergence properties of risk premiums as the number of assets is varied.

Supported by the empirical findings of Chordia, Roll, and Subrahmanyam (2000) and Huberman and Hulka (2001), we introduce a systematic component to the random supply of risky asset shares that commonly serves as the source of noise in rational expectations models. Without this systematic component or some alternative structure, prices in large economies will eliminate asymmetry in information by fully revealing private signals. In other words, when the number of assets and related signals is large, only systematic noise can prevent information, idiosyncratic or systematic, contained in private signals of informed investors from being perfectly inferred from prices by uninformed investors.

Like us, Admati (1985) considers the interplay between private information and equilibrium prices in a noisy rational expectations framework.[1] Rather than a factor structure, Admati's principal analysis assumes asset payoffs are distributed normally and satisfy a general variance-covariance matrix. Admati (1982) recognizes the advantages of a factor structure in characterizing economy-wide information, but, as previously noted, an explicit solution in the case of diverse information poses difficulties due to mathematical complexities.

Our study is an extension of Easley and O'Hara (2004). They also examine the effects of private information on risk premiums in a noisy rational expectations framework with multiple assets very similar to ours. Their model differs from ours in two principal respects: they assume risky asset payoffs and related signals are independent and identically distributed (all risk is idiosyncratic) and they only consider the case in which the number of assets is finite.[2] We offer two

[1]Brennan and Cao (1997) employ a similar structure.

[2]Easley and O'Hara (2004) also assume that there are a fixed set of signals, some of which are public and the rest private. Although it would be simple to incorporate public signals, in our

perspectives on their characterization of risk premiums. First, our characterization of risk premiums in the finite case reduces to their characterization when factor loadings (betas) in our model are set equal to zero. Taking the limit as the number of assets goes to infinity in this case results in no risk premium. Alternatively, one might interpret the assets in Easley and O'Hara (2004) as analogous to the systematic factors in our model. Removing all idiosyncratic risks and assuming factor independence in our model would then result in equivalent (factor) risk premiums.

The rest of the paper is organized as follows: Section 2 describes the setup for our model and studies an economy with a finite number of risky assets; Section 3 studies the limit of a large economy as the number of risky assets goes to infinity; and Section 4 concludes the paper.

2 Finite Economy

In this section, we consider an economy with a finite number of risky assets. We present a noisy rational expectation model in which the asset payoffs and the random supply of the assets have factor structures. We solve the equilibrium in closed form and consider special cases that serve as a benchmarks when information is symmetric.

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model all signals are private to informed investors. Accordingly, unlike Easley and O'Hara (2004), our comparative statics do not encompass the case in finite economies where the proportions of signals that are public or private are allowed to change.

2.1 The Setup

We assume that payoffs of N risky assets are generated by a factor structure of the form

$$\nu = \bar{\nu} + \beta F + \Sigma^{1/2}\epsilon. \tag{1}$$

The mean of asset payoffs $\bar{\nu}$ is an $N \times 1$ constant vector, the factor F is a $K \times 1$ vector of mean normal random variables with covariance matrix Σ_F, the factor loading β is an $N \times K$ constant matrix, the idiosyncratic risk ϵ is a vector of standard normal random variables, and Σ is an $N \times N$ diagonal matrix.

The supply of risky assets, x, is a vector of $N \times 1$ random variables with mean vector $\bar{x}$ and covariance matrix Σ_x and η_x is a standard normal random variable:

$$x = \bar{x} + \beta_x F_x + \Sigma_x^{1/2}\eta_x. \tag{2}$$

The noisiness of the supply is necessary in our setting to prevent prices from fully revealing the informed investors' private signal (defined below) and can be interpreted as caused by trading for liquidity reasons. The presence of a systematic component is based on the reasonable view that liquidity trading is influenced by market-wide forces that may or may not correspond to factors influencing risky asset payoffs. If we interpret the random supply as due to a liquidity effect, then our assumption of systematic components in random supply is supported by empirical studies that find there are systematic components of liquidity; for example, Chordia, Roll, and Subrahmanyam (2000) and Huberman and Hulka (2001). IPO waves are also suggestive of systematic components. Without a systematic component in the random supply, then in the limiting case, as the number of risky assets becomes large (implying an infinite number of independent asset specific signals), prices would still be fully revealing of the informed investors' private signals. In other words, noisy supply is necessary but not sufficient to ensure that asymmetric information is not a moot issue in large economies; there also needs to be a systematic component. We further assume for simplicity that F_x is independent of the factors generating asset payoffs.[3]

We assume that there are two classes of investors, informed and uninformed, with each class containing an infinite number of identical agents. The informed investors all receive private signal s on asset payoffs and the uninformed can only (imperfectly) infer the signal from market prices. This specification is used by Grossman and Stiglitz (1980) and Easley and O'Hara (2004). In Admati (1985), there are infinitely many agents each of whom receives independent signals. It can be argued that our assumption and Admati's are two special cases of a general information structure where investors have both diverse and asymmetric information: while we emphasize asymmetry, Admati emphasizes diversity. Technically speaking, the correlation between the private signals across informed investors is perfect in our model and zero in Admati's model. While in our analysis price will be a function of informed investors' private information, price is a function of the realized asset payoffs in Admati's case when the number of assets is infinite due to the elimination of signal noise through aggregation of signals across assets.

We assume all investors have the following utility

$$U = -\mathrm{E}[\exp(-AW_1)], \tag{3}$$

where A is the investor's absolute risk aversion coefficient and W_1 is the investor's

[3]Noisy rational expectation equilibrium models with many assets having a factor structure in asset payoffs, but not in the random supply of risky assets, have been considered in Caballe and Krishnan (1994); Daniel, Hirshleifer, and Subrahmanyam (2001); Kodres and Pritsker (2002); and Pasquariello (2004).

terminal wealth. The budget constraint is:

$$W_1 = W_0 R_f + D'(\nu - R_f p), \tag{4}$$

where W_0 is the investor's initial wealth and D is a vector containing the numbers of shares invested in risky assets.

Under the normality, the utility maximization problem becomes a mean-variance problem

$$\begin{aligned} \max_D \quad & \mathrm{E}[W_1|J] - \frac{A}{2}\mathrm{var}[W_1|J], \\ s.t. \quad & W_1 = W_0 R_f + D'(\nu - R_f p), \end{aligned}$$

where J represents the investor's information set. The first-order condition implies optimal demand takes the following form:

$$D_J^* = \frac{1}{A}\Sigma_{\nu|J}^{-1}\mathrm{E}[\nu - R_f p|J]. \tag{5}$$

When asset payoffs do not depend on systematic factors, $\beta = 0$, it is easy to show investors' demands for securities are increasing in expected asset payoffs and the precision of information about asset payoffs, and decreasing in risk aversion. In the more general case where asset payoffs do depend on systematic factors, $\beta \neq 0$, the demand for asset i depends not only on investors' posterior precision of beliefs on payoffs for asset i, but also on their posterior beliefs on payoffs for other assets. The informed and the uninformed have different demands because they condition on different information sets J.

2.2 Informed Investors

The informed investors receive private signal s which takes the form

$$s = \nu - \bar{\nu} - \beta F + bF + \Sigma_s^{1/2}\eta. \tag{6}$$

The $N \times 1$ constant vector b reflects the relative information content of the signal with respect to the systematic factors and η is an $N \times 1$ standard normal random variable. To conform with the interpretation of factor models, we will assume that F, ϵ, η, and η_x are are jointly normal and independent and the matrices Σ_s and Σ_x are diagonal.

Our specification of asset payoffs is distinct from an alternative specification where asset payoffs do not follow a factor structure, but satisfy a general variance-covariance matrix (e.g., Admati, (1985)). Though a factor structure such as (1) implies a specific variance-covariance matrix, a general variance-covariance matrix does not imply a corresponding factor structure. Admati (1985) entertains such constructions and concludes that a factor model is the natural context in which to consider private signals on economy-wide phenomena. Under her information structure, investors receive private signals about both systematic factors and idiosyncratic shocks. However, because of mathematical complexities an explicit solution was not obtained.

The signal s for each risky asset specified in the above equation is a linear combination of information about the systematic components of the asset's payoff, information about the idiosyncratic component of that payoff, and noise. The signal s can also be interpreted as a combination of two signals: a signal about the idiosyncratic component of asset payoffs, $s_1 = \Sigma^{1/2}\epsilon + (bF + \Sigma_s^{1/2}\eta)$, where $(bF + \Sigma_s^{1/2}\eta)$ is interpreted as noise; and a signal about the systematic component, $s_2 = bF + (\Sigma^{1/2}\epsilon + \Sigma_s^{1/2}\eta)$, where $(\Sigma^{1/2}\epsilon + \Sigma_s^{1/2}\eta)$ is interpreted as noise. The assumption that informed investors receive private information not only about the idiosyncratic component, but also about the systematic components of risky asset payoffs, although uncommon in the theoretical literature, is intuitive. Informed investors such as corporate insiders are likely to know more than the general public about the firm's fundamentals such as revenues, earnings, and cash flows. To the extent that the fundamentals are generated by a factor structure, private information is likely to contain both components. Consistent with this assumption, Seyhun (1992) and Lakonishok and Lee (2001) show that aggregated trading by corporate insiders is predictive of future market returns.

Our specification of signals differs in two respects from that of Admati (1982) in the context of her factor model: the signals in our model are perfectly correlated across informed investors while in her model investors receive diverse signals, and the "two signals" constructively received by informed investors in our model are correlated with covariance matrix Σ_s conditional on ν and F, whereas the two signals for a given investor in Admati (1982) are uncorrelated. Assuming that the signals about the idiosyncratic component of an asset's payoff and the systematic component are uncorrelated as in Admati's signal specification, changes the expressions in the limiting case as N goes to infinity, but does not affect either the structure of the explicit solution or the qualitative results that follow from that solution.

To calculate the conditional expectations and covariance matrixes, we need to derive the joint density function of ν and F conditional on information s.

Remark 1 *The moments of the joint distribution of ν and F conditional on signal s are*

$$
\begin{aligned}
\mathrm{E}[\nu|s,F] &= \overline{\nu} + \beta F + \Sigma_{\nu|s,F}\Sigma_s^{-1}(s - bF), \\
\mathrm{E}[F|s] &= \Sigma_{F|s} b'(\Sigma + \Sigma_s)^{-1} s, \\
\Sigma_{\nu|s,F}^{-1} &= \Sigma^{-1} + \Sigma_s^{-1} \\
\Sigma_{F|s}^{-1} &= \Sigma_F^{-1} + b'(\Sigma + \Sigma_s)^{-1} b \\
\hat{\Sigma}_s &= \Sigma + b\Sigma_F b' + \Sigma_s.
\end{aligned}
$$

The proof is given in the Appendix. From these moments, it follows that, conditional on signal s, the payoff is of the form

$$\nu = \overline{\nu} + \Sigma_{\nu|s,F}\Sigma_s^{-1}s + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)F + \Sigma_{\nu|s,F}^{1/2}\epsilon_{\nu|s,F}, \tag{7}$$

where, conditional on s and F, $\epsilon_{\nu|s,F}$ is an standard normal random variable. We note that from the perspective of an informed investor loadings on the systematic factors (conditional betas) are $\beta_s = \beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b$. The precision matrix of the factors has increased from Σ_F^{-1} to $\Sigma_{F|s}^{-1} = \Sigma_F^{-1} + b'(\Sigma + \Sigma_s)^{-1}b$.

From equation (7), the expectation of ν conditional on s is

$$\mathrm{E}[\nu|s] \quad = \quad \overline{\nu} + \Sigma_{\nu|s,F}\Sigma_s^{-1}s + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}b'(\Sigma + \Sigma_s)^{-1}s \tag{8}$$

and the variance of ν conditional on s is

$$\Sigma_{\nu|s} = \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}(\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)'. \tag{9}$$

Equations (8) and (9) can be substituted into the demand function to calculate the investors' demand D_J^* for risky assets:

$$D_s^* = \frac{1}{A}\Sigma_{\nu|s}^{-1}(\bar{\nu} + \Phi_s s - R_f p), \tag{10}$$

where

$$\Phi_s = \Sigma_{\nu|s,F}\Sigma_s^{-1} + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}b'(\Sigma + \Sigma_s)^{-1}.$$

2.3 Uninformed Investors

The uninformed investors do not observe the signal s, but can imperfectly infer s from the equilibrium price.

We conjecture that the equilibrium prices have the following form:

$$p = C + B(s - \lambda(x - \bar{x})),$$

where C is an $N \times 1$ vector and B and λ are $N \times N$ matrices. We will assume that B is invertible. Therefore, observing the price p is equivalent to observing θ which is defined as

$$\theta = B^{-1}(p - C) = s - \lambda(x - \bar{x}).$$

Substituting equations (2) and (6), we can write

$$\theta = \nu - \bar{\nu} - \beta F + bF + \Sigma_s^{1/2}\eta + \lambda\beta_x F_x + \lambda\Sigma_x^{1/2}\eta_x. \tag{11}$$

Therefore, we can interpret θ as another signal which has sensitivity b to the factor F and idiosyncratic shocks with covariance matrix Σ_θ, where

$$\Sigma_\theta = \Sigma_s + \lambda(\beta_x \Sigma_{Fx} \beta_x' + \Sigma_x)\lambda'.$$

Note that signal θ is less informative than signal s, i.e., its conditional variance-covariance matrix is larger than that of s, i.e., $\Sigma_\theta = \Sigma_s + \lambda\Sigma_x\lambda' \geq \Sigma_s$. We should remark that λ is in general non-diagonal; the idiosyncratic shocks $\Sigma_s^{1/2}\eta + \lambda\Sigma_x^{1/2}\eta_x$, although independent of F, are not independent of each other.

When systematic factors in the random supply are uncorrelated with systematic factors in asset payoffs, as we assumed, the signal s is a sufficient statistic for (s, θ). However, it is plausible that the two systematic factors are correlated. In this case, the signal s is no longer a sufficient statistic for (s, θ). While the uninformed will continue to condition on only θ, the informed will now condition on both s and θ, a departure from the above analysis in which the informed only conditioned on s. We assume independence for tractability. Nonetheless, we are confident that our analysis can be extended to accommodate the case of correlated factors and that our results are robust with respect to the relaxation of the independence assumption. The crucial aspect for risk premiums to be affected by asymmetric information is whether the informed investors learn more about systematic factors that influence asset payoffs than uninformed investors in equilibrium; this can be modeled with or without the correlation between the two classes of systematic factors.

To calculate the conditional expectations and covariance matrixes, we need to derive the moments of the joint density function of ν and F conditional on information θ.

Remark 2 *The moments of the joint distribution of ν and F conditional on the signal θ are*

$$\begin{aligned}
\mathrm{E}[\nu|\theta, F] &= \overline{\nu} + \beta F + \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}(\theta - bF), \\
\mathrm{E}[F|\theta] &= \Sigma_{F|\theta} b'(\Sigma + \Sigma_{\theta})^{-1}\theta, \\
\Sigma_{\nu|\theta,F}^{-1} &= \Sigma^{-1} + \Sigma_{\theta}^{-1}, \\
\Sigma_{F|\theta}^{-1} &= \Sigma_{F}^{-1} + b'(\Sigma + \Sigma_{\theta})^{-1} b, \\
\hat{\Sigma}_{\theta} &= \Sigma + b\Sigma_F b' + \Sigma_{\theta}.
\end{aligned}$$

The proof is given in the Appendix. From these moments, it follows that, conditional on signal θ, the payoff is of the form

$$\nu = \overline{\nu} + \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}\theta + (\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)F + \Sigma_{\nu|\theta,F}^{1/2}\epsilon_{\nu|\theta,F}, \tag{12}$$

where $\epsilon_{\nu|\theta,F}$ is a standard normal random variable. We note that, from the perspective of an uninformed investor, loadings on the systematic factors (conditional betas) are $\beta_\theta = \beta + \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b$. The precision matrix of the factors has increased from Σ_F^{-1} to $\Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + b'(\Sigma + \Sigma_\theta)^{-1}b$.

From equation (12), the expectation of ν conditional on θ is

$$\mathrm{E}[\nu|\theta] = \overline{\nu} + \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}\theta + (\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)\Sigma_{F|\theta}b'(\Sigma + \Sigma_{\theta})^{-1}\theta \tag{13}$$

and the variance of ν conditional on θ is

$$\Sigma_{\nu|\theta} = \Sigma_{\nu|\theta,F} + (\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)\Sigma_{F|\theta}(\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)'. \tag{14}$$

Equations (13) and (14) can be substituted into the demand function to calculate the uninformed investors' demand D_J^* for risky assets:

$$D_{\theta}^* = \frac{1}{A}\Sigma_{\nu|\theta}^{-1}(\bar{\nu} + \Phi_{\theta}\theta - R_f p), \tag{15}$$

where

$$\Phi_{\theta} = \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1} + (\beta - \Sigma_{\nu|\theta,F}\Sigma_{\theta}^{-1}b)\Sigma_{F|\theta}b'(\Sigma + \Sigma_{\theta})^{-1}.$$

2.4 Equilibrium

Imposing the market clearing condition that the total demand from the informed and the uninformed investors equals the supply, we obtain the following equation:

$$x = \frac{\mu}{A}\Sigma_{\nu|s}^{-1}(\bar{\nu} + \Phi_s s - R_f p) + \frac{1-\mu}{A}\Sigma_{\nu|\theta}^{-1}(\bar{\nu} + \Phi_{\theta}\theta - R_f p),$$

where μ is the proportion of informed investors. Defining $\bar{\Sigma}_{\nu} = \left(\mu\Sigma_{\nu|s} + (1-\mu)\Sigma_{\nu|\theta}^{-1}\right)^{-1}$, we derive the following expression for the prices of risky assets:

$$\begin{aligned} p &= \frac{1}{R_f}\left(\bar{\nu} + \bar{\Sigma}_{\nu}\left(\mu\Sigma_{\nu|s}^{-1}\Phi_s s + (1-\mu)\Sigma_{\nu|\theta}^{-1}\Phi_{\theta}\theta - Ax\right)\right) \\ &= \frac{1}{R_f}\left(\bar{\nu} - \bar{\Sigma}_{\nu}A\bar{x}\right) + \frac{1}{R_f}\bar{\Sigma}_{\nu}\mu\Sigma_{\nu|s}^{-1}\Phi_s\left(s - \left(\mu\Sigma_{\nu|s}^{-1}\Phi_s\right)^{-1}A(x-\bar{x})\right) \\ &+ \frac{1}{R_f}\bar{\Sigma}_{\nu}(1-\mu)\Sigma_{\nu|\theta}^{-1}\Phi_{\theta}(s - \lambda(x-\bar{x})). \end{aligned} \tag{16}$$

Comparing the above expression to the conjectured form of the price p, it must be true that

$$\lambda = (\mu\Sigma_{\nu|s}^{-1}\Phi_s)^{-1}A. \tag{17}$$

Note that λ is solved in terms of the parameters of the model. The matrices $\Sigma_{\nu|\theta}$, Φ_θ, and $\bar{\Sigma}_\nu$ are expressed in terms of λ as well as the parameters of the model; they are solved once λ is solved.

Theorem 1 *Given that informed investors receive a private signal, s, that is informative about both idiosyncratic and systematic components of asset payoffs, a partially revealing noisy rational expectations equilibrium exists, and prices of risky assets satisfy*

$$p = \frac{1}{R_f}\bar{\nu} - \frac{1}{R_f}\bar{\Sigma}_\nu A\bar{x} + \frac{1}{R_f}\bar{\Sigma}_\nu \left(\mu\Sigma_{\nu|s}^{-1}\Phi_s + (1-\mu)\Sigma_{\nu|\theta}^{-1}\Phi_\theta\right)(s - \lambda(x - \bar{x})) \quad (18)$$

This equation confirms the conjectured form of the price

$$p = C + B(s - \lambda(x - \bar{x})),$$

where $C = \frac{1}{R_f}\left(\bar{\nu} - \bar{\Sigma}_\nu A\bar{x}\right)$ *and* $B = \frac{1}{R_f}\bar{\Sigma}_\nu\left(\mu\Sigma_{\nu|s}^{-1}\Phi_s + (1-\mu)\Sigma_{\nu|\theta}^{-1}\Phi_\theta\right)$. *The risk premium of assets satisfies*

$$\mathrm{E}[\nu - R_f p] = A\bar{\Sigma}_\nu\bar{x} = A\left(\mu\Sigma_{\nu|s}^{-1} + (1-\mu)\Sigma_{\nu|\theta}^{-1}\right)^{-1}\bar{x}. \quad (19)$$

Proof: The price p and the expressions for B and C are derived by combining the equations (16) and (17). The equation for the risk premium follows immediately. Note that the posterior precisions $\Sigma_{\nu|s}^{-1}$ and $\Sigma_{\nu|\theta}^{-1}$ do not depend on realizations of signals s and θ, respectively.

The first term in the price p is the expected payoff without information discounted by the risk-free return. This is the price if investors are risk-neutral ($A = 0$) and there are no signals in the economy. The second term is the discount in price associated with risk, thus the risk premium. The third term is the correction to the expected payoff associated with signals and noisy supply.

The risk premium is determined by the geometric average of the covariance matrices of asset payoffs conditional on s and θ, $\Sigma_{\nu|s}$ and $\Sigma_{\nu|\theta}$. That is, the risk premium compensates the average of the risks conditional on s and θ. Two properties of the risk premium follow. First, from equation (9), $\Sigma_{\nu|s} = \Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}(\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)'$ and similarly for $\Sigma_{\nu|\theta}$, the average risk includes idiosyncratic risk $\Sigma_{\nu|s,F}$ and $\Sigma_{\nu|\theta,F}$. Therefore, idiosyncratic risks are priced. Second, the average covariance matrix, $\bar{\Sigma}_\nu$ depends on β nonlinearly, thus the risk premium depends on β nonlinearly.

2.5 Symmetric Information

When all investors are informed, $\mu = 1$, Theorem 1 implies that the risk premium is

$$\mathrm{E}[\nu - R_f p] = \Sigma_{\nu|s} A\bar{x} = \Big(\Sigma_{\nu|s,F} + (\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)\Sigma_{F|s}(\beta - \Sigma_{\nu|s,F}\Sigma_s^{-1}b)'\Big)A\bar{x}.$$

In such an economy, an econometrician who observes the return but not the signal will conclude that the risk premium depends on β as well as some firm-specific characteristics, $\Sigma_{\nu|s,F}\Sigma_s^{-1}b$. Thus, firms with the same β but different $\Sigma_{\nu|s,F}\Sigma_s^{-1}b$ may have different expected returns. This economy seems potentially to provide a theory for the empirical findings of Daniel and Titman (1998).

At the other extreme, when all investors are uninformed, $\mu = 0$, $\lambda \to \infty$; i.e., the inferred signal θ is infinitely more noisy than s and thus is not informative at all. It follows immediately that the covariance matrix conditional on θ, $\Sigma_{\nu|\theta}$, is the same as Σ and the factor covariance matrix conditional on θ, $\Sigma_{F|\theta}$, is the same as Σ_F. Furthermore, factor loadings conditional on θ are the same as unconditional factor loadings, i.e.,$\beta_\theta = \beta$. From Theorem 1, the risk premium is

$$\mathrm{E}[\nu - R_f p] = \Sigma_{\nu|\theta} A\bar{x} = \Big(\Sigma + \beta\Sigma_F\beta'\Big)A\bar{x}. \tag{20}$$

The above can also be described as the risk premium in a finite economy with homogeneous beliefs. In this case, there is no updating of beliefs, idiosyncratic risk is priced, and β appears linearly in the risk premium.

We conclude this section by observing that in an economy with a finite number of assets idiosyncratic as well as systematic risk is priced, information on idiosyncratic shocks reduces idiosyncratic risk and hence the risk premium, information can increase or decrease factor loadings, the risk premium depends on beta nonlinearly, and information on the systematic factor reduces systematic risk and hence the factor risk premium. As we will demonstrate in Section 3, only the last property survives in the limit as the number of risky assets goes to infinity.

3 Large Economy Limit

In this section, we study the effects of private signals on risk premiums when the economy is large in the sense that $N \to \infty$. We begin by revisiting the limiting procedure employed by Ross (1976) in his derivation of the APT pricing relation for the case where there are no signals in the economy. We then apply the same limiting procedure to derive the risk premium under the information structure assumed in the previous section.

3.1 APT Limiting Procedures

We seek a limiting procedure that produces the APT pricing relation in the standard case of homogeneous beliefs. In his formal derivation of that relation in an economy without private signals, Ross (1976) imposes the restriction that relative risk aversion be uniformly bounded as the number of assets and, hence, wealth increases.[4] We implement this restriction in our model by assuming that absolute risk aversion decreases in proportion to the number of assets; i.e.,$A = \frac{\gamma}{N}$, where γ is a constant. This assumption ensures both that idiosyncratic risks are not priced[5] and the factor risk premium does not become infinite and therefore undefined.

To be precise, the risk premium from the previous section when there are no private signals is

$$\mathrm{E}[\nu - R_f p] = A(\beta\Sigma_F\beta' + \Sigma)\bar{x} = A(\beta\Sigma_F\beta'\bar{x} + \Sigma\bar{x}). \tag{21}$$

The above risk premium for asset i has two components; $A\beta_i\Sigma_F\beta'\bar{x}$, where $A\Sigma_F\beta'\bar{x}$, the risk premium associated with the factor, is independent of the asset and of order N^1 and $A\Sigma_{ii}\bar{x}_i$ (assuming that Σ is diagonal), where Σ_{ii} is the idiosyncratic variance, and is of order N^0. There are two problems when taking the limit as $N \to \infty$ if A is a constant. First, idiosyncratic risk is priced because the second component is non-zero. Second, the risk premium is undefined because the first component goes to infinity.

Both problems are resolved by our assumption that $A = \frac{\gamma}{N}$ where γ is a constant. The risk premium as $N \to \infty$ in this case is finite and proportional to beta:

$$\mathrm{E}[\nu - R_f p] = \beta\frac{\gamma\Sigma_F\beta'\bar{x}}{N} \tag{22}$$

[4]Rather than hold the number of investors constant as the economy expands, we could assume that the number of investors increases proportionately to with the number of assets(e.g. Ou-Yang (2004)). Under this assumption, per capita wealth remains constant, thereby satisfying the bound on relative risk aversion.

[5]This assumption is stronger than the usual assumption of no asymptotic arbitrage under which idiosyncratic risks are not priced for an infinite number of assets, but may be priced for a finite number of assets. In other words, the APT pricing relation need not hold for all assets in the limit as the number of assets goes to infinity when asymptotic arbitrage is ruled out.

We now return to the setting with private signals. As we will show, the APT pricing relation is robust with respect to the effects of those signals (and hence heterogeneous beliefs) on risk premiums.

3.2 Effects of Private Signals on Risk Premiums

We begin our analysis of the effects of private signals on risk premiums in the large economy limit with two special cases that have appeared in the literature; private information only on idiosyncratic shocks and private information on total asset payoffs. We then consider the case where private information on systematic factors has finite aggregate precision.

3.2.1 Private Information Only on Idiosyncratic Components of Asset Payoffs

Suppose informed investors receive private signals on just the idiosyncratic components of risky asset payoffs. In this case, $b = 0$ and the signals can be written as

$$s = \nu - \bar{\nu} - \beta F + \Sigma_s^{1/2} \eta. \tag{23}$$

Note that when $\beta \neq 0$, the asset payoffs are correlated. In the special case where all asset payoffs are uncorrelated, i.e., $\beta = 0$, this structure reduces to the setting considered by Easley and O'Hara (2004). It is easy to see that, for finite N, information solely about idiosyncratic shocks reduces uncertainty about priced risks; $\Sigma_{\nu|J,F}$. However, as we show below, in the limit as $N \to \infty$, elimination of idiosyncratic risks through diversification implies that private signals containing only idiosyncratic components have no effects on risk premiums:

Proposition 1 *Given that informed investors receive private signals only about the idiosyncratic components of asset payoffs, in the limit as $N \to \infty$, the risk premium satisfies*

$$\mathrm{E}[\nu - R_f p] = \beta\gamma\Sigma_F\beta'\bar{x}/N. \quad (24)$$

The proof is given in the Appendix. Note that $\beta'\bar{x}$ is of order N and hence $\beta'\bar{x}/N$ is of order 1 when $N \to \infty$. Thus, we have a finite risk premium. Notably, the risk premium in this case is the same as the risk premium without information, $\mu = 0$, $\beta\gamma\Sigma_F\beta'\bar{x}/N$, implying that this is the risk premium for all μ. In other words, there is no resolution of uncertainty about systematic factors from private signals that do not contain a systematic component; investors remain with their prior (homogeneous) beliefs.[6] It is also clear that, in the setting studied by Easley and O'Hara (2004), where $\beta = 0$, the risk premium is reduced to zero, i.e., $\mathrm{E}[\nu - R_f p] = 0$.

More generally, we expect that the same results will hold as long as $b'(\Sigma + \Sigma_s)^{-1}b \to 0$ and $b'(\Sigma + \Sigma_\theta)^{-1}b \to 0$ when $N \to \infty$. Intuitively, diversification works at the power of $1/N$, implying that if the systematic component of the signal has a power less than $1/N$, then it will be eliminated by diversification.

Note that although private information on idiosyncratic shocks does not affect risk premiums in this case, it does affect asset prices and portfolio holdings of informed and uninformed investors and, hence, their expected utilities.

[6] Note further, that the proposition holds notwithstanding systematic components in the random supply, $\beta_x\sigma_{fx} \neq 0$. This result is quite intuitive: Even if all the agents are informed, $\mu = 1$, there is no resolution of uncertainty about the factor that affects asset payoffs, implying that the random supply of assets is irrelevant for asset pricing.

3.2.2 Private Information on Total Risky Asset Payoffs

Suppose now that informed investors receive private signals about total asset payoffs. In this case, $b = -\beta$ and the signals can be written as

$$s = \nu - \bar{\nu} + \Sigma_s^{1/2}\eta. \tag{25}$$

This is a special case of Admati (1985) where the covariance matrix of the assets has the form of a factor structure and the signals for different assets are uncorrelated.

In this case, $\Sigma_{F|s}^{-1} = \Sigma_F^{-1} + \beta'(\Sigma + \Sigma_s)^{-1}\beta$, which goes to infinity as $N \to \infty$. Therefore, we have

$$\Sigma_{F|s} = 0.$$

Similarly, $\Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + (\beta + \beta_x)'(\Sigma + \Sigma_\theta)(\beta + \beta_x)$, which also goes to infinity as long as $\beta + \beta_x$ goes to a constant as $N \to \infty$; thus we also have

$$\Sigma_{F|\theta} = 0.$$

It is easy to show that the above two equations imply that the risk premium is zero.

The intuition here is also clear. Infinitely many private signals about asset payoffs implies that informed investors learn the systematic factor perfectly and set their demands such that prices fully reveal the systematic factor F and, thus, eliminate the risk associated with that factor.

3.2.3 Private Information with Finite Aggregate Precision

We have considered the cases where $(b'(\Sigma+\Sigma_s)^{-1}b, b'(\Sigma+\Sigma_\theta)^{-1}b) \to 0$ and $(b'(\Sigma+\Sigma_s)^{-1}b, b'(\Sigma+\Sigma_\theta)^{-1}b) \to \infty$. The more interesting case is where the limit of $(b'(\Sigma+\Sigma_s)^{-1}b, b'(\Sigma+\Sigma_\theta)^{-1}b)$ is a non-zero finite constant; what we call finite aggregate precision. This happens, for instance, if the elements of $\sqrt{N}b$ go to a non-zero constant when $N \to \infty$. In effect, under this structure, as the economy expands the informativeness of the private signal for a given asset about the factor is decreasing. Thus, even though signal noise and idiosyncratic risk are becoming diversified away as the number of assets increases, informed investors' aggregate information about the factor in the limit is imperfect.[7]

The risk premium in this case is given by the following proposition.

Proposition 2 *Given that informed investors receive private signals informative about both idiosyncratic and systematic components of asset payoffs with finite aggregate precision, in the limit as $N \to \infty$, the risk premium is*

$$\mathrm{E}[\nu - R_f p] = \beta\gamma \left(\mu\Sigma_{F|s}^{-1} + (1-\mu)\Sigma_{F|\theta}^{-1}\right)^{-1} \frac{\beta'\bar{x}}{N}$$

and the factor risk premium is

$$\lambda = \gamma \left(\mu\Sigma_{F|s}^{-1} + (1-\mu)\Sigma_{F|\theta}^{-1}\right)^{-1} \frac{\beta'\bar{x}}{N}.$$

The proof is given in the Appendix.

We can arrange the risk premium as follows

$$\beta \left(\mu + (1-\mu)\Sigma_{F|s}\Sigma_{F|\theta}^{-1}\right)^{-1} (I_K + \Sigma_F k'(\Sigma + \Sigma_s)^{-1}k)^{-1} \left(\gamma\Sigma_F \frac{\beta'\bar{x}}{N}\right).$$

The term $\Sigma_F^{-1}\gamma\frac{\beta'\bar{x}}{N}$ determines the risk premium without private signals; it does not contain parameters that characterize information structure. The term $(I_K + \Sigma_F k'(\Sigma + \Sigma_s)^{-1}k)^{-1}$

[7] As mentioned earlier, an alternative information structure that would preclude learning the factor realization perfectly and preserve our qualitative results is to assume informed investors receive two uncorrelated signals; one about idiosyncratic shocks and the other about the systematic factor. This is similar to the information structure assumed by Admati (1982) in the context of her factor model, the difference being that in our model all informed investors receive the same signals while in Admati they receive diverse signals.

determines the effects of private signals in reducing the factor risk premium; it does not contain parameters that characterize the asymmetric structure of the information. The term $\left(\mu + (1-\mu)\Sigma_{F|s}\Sigma_{F|\theta}^{-1}\right)^{-1}$ determines the effects of information asymmetry on the risk premium. The effect on the equilibrium risk premium attributable to the informed investors is different from the effect attributable to the uninformed investors, implying that the fraction of the informed investors in the economy plays an important role in the determination of that risk premium. In particular, the greater the proportion of uninformed to informed, the larger the risk premium since with less average information there is less resolution of uncertainty about systematic factors.

As the above proposition depicts, the risk premium is entirely determined by beta. In fact, it is proportional to beta; i.e., the APT pricing relation (Ross, 1976) holds. Information about the systematic factors in asset payoffs and the systematic component in the random supply affect the covariances of the factors. The risk premium is proportional to the geometric average of the factor covariance matrices conditional on s and θ, $\Sigma_{F|s}$ and $\Sigma_{F|\theta}$. Since $\Sigma_{F|s} < \Sigma_{F|\theta}$, the risk premium decreases with the fraction of the informed; in particular, the risk premium with private information is always smaller than the risk premium without private information. As made evident within the proof of Proposition 2 contained in the Appendix, without a systematic component in the random supply, the conditional factor covariance matrices would be equal, consistent with our earlier claim that prices would then fully reveal the informed investors' private signal. Hence, the systematic component in the random supply plays a crucial role in our extension of APT to a case with heterogeneous beliefs. We further observe that factor loadings are not affected by private signals in the large N limit.

For comparative statics, we can gain a useful expression of the risk premium in terms of model primitives by assuming identically distributed risky asset payoffs and related signals: i.e., the covariance matrices of the payoffs, signals, and the random supply are all proportional to the identity matrix; the betas of all risky asset payoffs are equal; and the sensitivities of the signals to (for convenience) a single factor are equal.

The risk premium for in the large N limit for this case (maintaining the assumption of finite aggregate precision) is[8]

$$\gamma\sigma_f^2\beta^2\bar{x}\left(1+\frac{\sigma_f^2k^2}{\sigma^2+\sigma_s^2}\left(\mu+\frac{1-\mu}{1+(\sigma^2+\sigma_s^2)\left(\frac{\gamma\beta\sigma_{fx}\beta_x}{\mu k}\right)^2}\right)\right)^{-1}1_{N\times 1}$$

$$= \gamma\sigma_f^2\beta^2\bar{x}\left(1+\frac{\sigma_f^2k^2}{\sigma^2+\sigma_s^2}\right)^{-1}\left(\mu+(1-\mu)\frac{1+\frac{\sigma_f^2k^2}{(\sigma^2+\sigma_s^2)+(\sigma^2+\sigma_s^2)^2\left(\frac{\gamma\beta\sigma_{fx}\beta_x}{\mu k}\right)^2}}{1+\frac{\sigma_f^2k^2}{\sigma^2+\sigma_s^2}}\right)^{-1}1_{N\times 1}.$$

The first term in the above expression, $\gamma\sigma_f^2\beta^2\bar{x}$, is the risk premium without information. It depends on the risk aversion, the beta, and the factor variance.

The risk premium when all investors are informed is given by $\gamma\sigma_f^2\beta^2\bar{x}\left(1+\frac{\sigma_f^2k^2}{\sigma^2+\sigma_s^2}\right)^{-1}$; it decreases with the systematic sensitivity k of the signal to the factor and increases with the variance of the payoff σ^2 and variance of the signal σ_s^2.

The term $k'(\Sigma+\Sigma_s)^{-1}k$ is the aggregate information on the factor F. The term $\mu+\frac{1-\mu}{1+(\sigma^2+\sigma_s^2)\left(\frac{\gamma\beta\sigma_{fx}\beta_x}{\mu k}\right)^2}$ determines the effect of asymmetric information, and μ and $\beta_x\sigma_{fx}$ only affect this term. Therefore, when either μ or $\beta_x\sigma_{fx}$ changes while keeping other parameters fixed, only the asymmetries of the information structure are affected while the risk premiums for the cases where all investors are uninformed ($\mu=0$) and all investors are informed ($\mu=1$) do not change.

When $\beta_x\sigma_{fx}=0$, the risk premium reduces to that of the case with $\mu=1$. This

[8] Details on the derivation in this special case are available from the authors upon request.

is due to the fact that the price fully reveals the signal, s, in the large N limit if there are no systematic components in the random supply. Note also that σ_x does not appear in the formula, because the idiosyncratic component of the random supply is diversified away. When $k = 0$, the information is only idiosyncratic and the risk premium reduces to that of the case with no information, $\gamma\sigma_f^2\beta^2\bar{x}$, even if $\beta_x\sigma_{fx} \neq 0$.

The risk premium decreases with the fraction of the informed investors, μ, the precision of the private signals, $1/\sigma_s^2$, and the sensitivity of those signals to the systematic component, k^2; it increases with factor loadings, β, the volatility of the idiosyncratic component of asset payoffs, σ, and risk aversion, γ.

3.2.4 Risk Premiums and the Size of Economy

Still assuming identically distributed risky asset payoffs and related signals, we now step back to the case with a finite number of risky assets and numerically examine the behavior of risk premiums as that number changes.

Under a set of plausible parameter values where $\gamma = 3$, $\sigma = 30\%$, $\beta = 1$, $\sigma_f = 20\%$, $\sigma_s = 25\%$, $\sigma_{fx} = 30\%$, $\beta_x = 1$, $\sigma_x = 30\%$, and $k = -1$, Figures 1 plots the risk premium against the fraction of the informed investors for various numbers of risky assets. The risk premium decreases with N as we would expect. In particular, we observe that there is substantial convergence to the risk premium in the limiting case as the number of assets reaches the hundreds. This suggests that the risk premium in the limit as the number of assets goes to infinity may be a reasonable approximation to the risk premium in a finite economy where the number of assets measures in the thousands.

4 Conclusion

In this paper we provide an explicit solution to a noisy rational expectations model that characterizes the effects of private signals on risk premiums. Risky asset payoffs in this model obey a factor structure. Private signals are informative of systematic factors as well as idiosyncratic risks. Our principal result is that, in large economies where the number of risky assets goes to infinity, the APT (Ross, 1976) pricing relation holds and private signals affect risk premiums only through factor risk premiums.

On the effects of information asymmetry as such, we show that a higher proportion of informed to uninformed investors leads to a greater resolution of uncertainty as manifested by a smaller aggregate posterior factor covariance matrix and, hence, lower factor risk premiums. This result depends on the presence of a systematic component in the random supply of risky assets. Eliminating the systematic component of the random supply would remove the asymmetry of information between informed and uninformed investors by causing prices to become fully revealing of private signals, a less interesting case.

It seems reasonable that as long as the precision of aggregate posterior beliefs about systematic factors is finite, the information supplied by the private signal for an individual risky asset about systematic factors when there are many assets must be small. In turn, the effect of such information on beliefs with respect to any aspect of an individual risky asset's payoff including systematic components must be small, approaching the null effect in the limit as the number of assets goes to infinity. We have confirmed this intuition when distributions are normal, utility functions are CARA with the absolute risk aversion coefficient decreasing in the number of assets, and investors can be ordered by statistical sufficiency of their information with respect to systematic factors. We conjecture that the absence of an effect of private signals on betas in large economies would hold up in more general cases where distributions and utility functions depart from these assumptions.

Last, we observe that our results in the case of asymmetric information establishes the validity of APT in a setting with heterogeneous expectations. If, as conjectured above, the effects of information supplied by private signals in large economies can be reduced to heterogeneous posterior beliefs on systematic factors, without affecting loadings on those factors, then we further conjecture as does Ross (1976) that the APT pricing relation holds in such economies with private signals, provided that limiting procedures for that relation to hold in the absence of private signals are met. These conjectures suggest that our results may apply to a far broader context than the setting for our model.

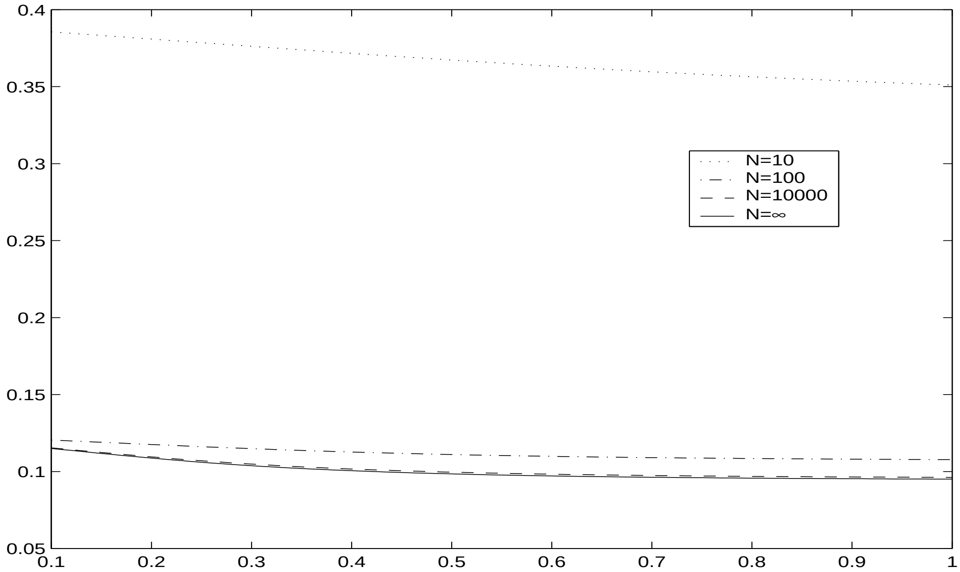

Figure 1. The Risk Premium. This graph plots the average risk premium in the ID case against the fraction of informed investors, for various numbers of risky assets, N. The parameters are: $\gamma = 3$, $\sigma = 30\%$, $\beta = 1$, $\sigma_f = 20\%$, $\sigma_s = 25\%$, $\sigma_{fx} = 30\%$, $\beta_x = 1$, $\sigma_x = 30\%$, and $k = -1$.

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Appendix

In the Appendix, we will use the following identity extensively:

$$(\Sigma + \beta\Omega\beta')^{-1} = \Sigma^{-1} - \Sigma^{-1}\beta(\Omega^{-1} + \beta'\Sigma^{-1}\beta)\beta'\Sigma^{-1}.$$

The Proof of Remark 1.

We solve for the filtering rule, given signal s. Our assumptions have specified the distribution functions $f(\nu|F,s)$, $f(\nu|F)$, and $f(F)$. Therefore,

$$f(v,F,s) = f(s|\nu,F)f(\nu|F)f(F).$$

We can rewrite the above as

$$f(v,F,s) = f(\nu|s,F)f(F|s)f(s).$$

Focusing on the exponential terms of the joint normal distribution densities, we obtain

$$\begin{aligned}
&-\ln f(\nu,F,s) = -\ln f(s|\nu,F) - \ln f(\nu|F) - \ln f(F)\\
&\frac{1}{2}(s-(\nu-\overline{\nu}-\beta F)-bF)'\Sigma_s^{-1}(s-(\nu-\overline{\nu}-\beta F)-bF)\\
&+\frac{1}{2}(\nu-\overline{\nu}-\beta F)'\Sigma^{-1}(\nu-\overline{\nu}-\beta F)+\frac{1}{2}F'\Sigma_F^{-1}F\\
&=\frac{1}{2}(\nu-\overline{\nu}-\beta F)'\left(\Sigma^{-1}+\Sigma_s^{-1}\right)(\nu-\overline{\nu}-\beta F)-(\nu-\overline{\nu}-\beta F)'\Sigma_s^{-1}(s-bF)\\
&+\frac{1}{2}(s-bF)'\Sigma_s^{-1}(s-bF)+\frac{1}{2}F'\Sigma_F^{-1}F\\
&=\frac{1}{2}(\nu-\mathrm{E}[\nu|s,F])\Sigma_{\nu|s,F}^{-1}(\nu-\mathrm{E}[\nu|s,F])\\
&+\frac{1}{2}(s-bF)'(\Sigma+\Sigma_s)^{-1}(s-bF)+\frac{1}{2}F'\Sigma_F^{-1}F\\
&=\frac{1}{2}(\nu-\mathrm{E}[\nu|s,F])\Sigma_{\nu|s,F}^{-1}(\nu-\mathrm{E}[\nu|s,F])+\frac{1}{2}s'(\Sigma+\Sigma_s)^{-1}s
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}(bF)'(\Sigma+\Sigma_s)^{-1}bF - s'(\Sigma+\Sigma_s)^{-1}bF + \frac{1}{2}F'\Sigma_F^{-1}F \\
&= \frac{1}{2}(\nu - \mathrm{E}[\nu|s,F])\Sigma_{\nu|s,F}^{-1}(\nu - \mathrm{E}[\nu|s,F]) \\
&+\frac{1}{2}(F - E[F|s])'\Sigma_{F|s}^{-1}(F - \mathrm{E}[F|s]) + \frac{1}{2}s'\hat{\Sigma}_s^{-1}s \\
&= -\ln f(\nu|s,F) - \ln f(F|s) - \ln f(s),
\end{aligned}
$$

The distribution functions $f(\nu|s,F)$, $f(F|s)$, and $f(s)$ can then identified from the above equation, with

$$
\begin{aligned}
\mathrm{E}[\nu|s,F] &= \overline{\nu} + \beta F + \Sigma_{\nu|s,F}\Sigma_s^{-1}(s - bF), \\
\mathrm{E}[F|s] &= \Sigma_{F|s}b'(\Sigma+\Sigma_s)^{-1}s, \\
\Sigma_{\nu|s,F}^{-1} &= \Sigma^{-1} + \Sigma_s^{-1}, \\
\Sigma_{F|s}^{-1} &= \Sigma_F^{-1} + b'(\Sigma+\Sigma_s)^{-1}b, \\
\hat{\Sigma}_s &= \Sigma + b'\Sigma_F b + \Sigma_s.
\end{aligned}
$$

The Proof of Remark 2.

The structure of the filtering rule, given signal θ, is the same as that for s. The proof proceeds in exactly the same fashion.

Proof of Proposition 1.

Because $b = 0$, we have

$$
\begin{aligned}
\Sigma_{F|s} &= \Sigma_F; \\
\Sigma_{\nu|s} &= \Sigma_{\nu|s,F} + \beta\Sigma_F\beta'; \\
\Sigma_{F|\theta} &= \Sigma_F; \\
\Sigma_{\nu|\theta} &= \Sigma_{\nu|\theta,F} + \beta\Sigma_F\beta'.
\end{aligned}
$$

Intuitively, the matrices $\Sigma_{\nu|s}$ and $\Sigma_{\nu|\theta}$ differ only in the idiosyncratic matrices $\Sigma_{\nu|s,F}$ and $\Sigma_{\nu|\theta,F}$ which do not matter for the risk premium and thus should produce the risk premium $\gamma\beta\Sigma_F\beta'\bar{x}$. The formal proof is as follows. From

$$\Sigma_\theta = \Sigma_s + \lambda(\beta_x\Sigma_{Fx}\beta_x' + \Sigma_x)\lambda' \geq \Sigma_s,$$

we know that

$$\Sigma \geq (\Sigma^{-1} + \Sigma_\theta^{-1})^{-1} = \Sigma_{\nu|\theta,F} \geq (\Sigma^{-1} + \Sigma_s^{-1})^{-1} = \Sigma_{\nu|s,F}.$$

It follows that

$$\begin{aligned}
&\Sigma + \beta\Sigma_F\beta' = (\mu(\Sigma + \beta\Sigma_F\beta') + (1-\mu)(\Sigma + \beta\Sigma_F\beta'))^{-1} \\
&\geq \left(\mu(\Sigma_{\nu|s,F} + \beta\Sigma_F\beta') + (1-\mu)(\Sigma_{\nu|\theta,F} + \beta\Sigma_F\beta')\right)^{-1} = \bar{\Sigma}_\nu \\
&\geq \left(\mu(\Sigma_{\nu|s,F} + \beta\Sigma_F\beta') + (1-\mu)(\Sigma_{s|\theta,F} + \beta\Sigma_F\beta')\right)^{-1} = \Sigma_{\nu|s,F} + \beta\Sigma_F\beta'.
\end{aligned}$$

Hence,

$$\lim_{N\to\infty}\frac{1}{N}(\Sigma + \beta\Sigma_F\beta') = \lim_{N\to\infty}\frac{1}{N}\beta\Sigma_F\beta' \geq \lim_{N\to\infty}\frac{1}{N}\bar{\Sigma}_\nu \geq \lim_{N\to\infty}\frac{1}{N}\Sigma_{\nu|s,F} + \beta\Sigma_F\beta' = \lim_{N\to\infty}\frac{1}{N}\beta\Sigma_F\beta.$$

Therefore, the average risk premium is

$$\mathrm{E}[\nu - R_f p] = \gamma\frac{1}{N}\bar{\Sigma}_\nu\bar{x} \to \gamma\frac{1}{N}\left(\Sigma_{\nu|s,F} + \beta\Sigma_F\beta'\right)\bar{x} \to \gamma\frac{1}{N}\beta\Sigma_F\beta'\bar{x}.$$

Proof of Proposition 2.

For the case of non-identically distributed risky asset payoffs, the leading order terms in the large N limit are

$$\begin{aligned}
\Sigma_{\nu|s,F} &= \left(\Sigma^{-1} + \Sigma_s^{-1}\right)^{-1}, \\
\Sigma_{F|s} &= \left(\Sigma_F^{-1} + \frac{1}{N}k'(\Sigma + \Sigma_s)^{-1}k\right)^{-1}.
\end{aligned}$$

The variance of ν conditional on s

$$\begin{aligned}\Sigma_{\nu|s} &= \Sigma_{\nu|s,F} + \beta\Sigma_{F|s}\beta' + O(N^{-1/2}),\\ \Phi_s &= \Sigma_{\nu|s,F}\Sigma_s^{-1} + \frac{1}{\sqrt{N}}\beta\Sigma_{F|s}k'(\Sigma+\Sigma_s)^{-1} + O(N^{-1}).\end{aligned}$$

Both first terms in the above equations are diagonal matrices. The second terms are due to factors. We use $O(N^\alpha)$ to denote matrices with all of their elements generally non-zero and of order N^α. In the case of identical assets, $O(N^\alpha) \propto N^\alpha 1_{N\times N}$. These terms will be negligible, in the large N limit, as far as the risk premium is concerned. The Φ_s^{-1} matrix is

$$\begin{aligned}\Phi_s^{-1} &= \Sigma_s\left(I_N + \frac{1}{\sqrt{N}}\Sigma_{\nu|s,F}^{-1}\beta\Sigma_{F|s}k'(\Sigma_s^{-1}\Sigma+I_N)^{-1}\right)^{-1}\Sigma_{\nu|s,F}^{-1}\\ &= \Sigma_s\left(I_N - \Sigma_{\nu|s,F}^{-1}\beta\left(\sqrt{N}\Sigma_{F|s}^{-1} + k'(\Sigma_s^{-1}\Sigma+I_N)^{-1}\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1}k'(\Sigma_s^{-1}\Sigma+I_N)^{-1}\right)\Sigma_{\nu|s,F}^{-1}\end{aligned}$$

and

$$\begin{aligned}\Phi_s^{-1}\Sigma_{\nu|s} &= \Sigma_s\left(I_N - \Sigma_{\nu|s,F}^{-1}\beta\Sigma_{F|s}\left(\sqrt{N}I_K + k'(\Sigma_s^{-1}\Sigma+I_N)^{-1}\Sigma_{\nu|s,F}^{-1}\beta\Sigma_{F|s}\right)^{-1}k'(\Sigma_s^{-1}\Sigma+I_N)^{-1}\right)\\ &\times \left(I_N + \Sigma_{\nu|s,F}^{-1}\beta\Sigma_{F|s}\beta'\right)\\ &= \Sigma_s\left(I_N - \Sigma_{\nu|s,F}^{-1}\beta\left(\sqrt{N}\Sigma_{F|s}^{-1} + k'(\Sigma_s^{-1}\Sigma+I_N)^{-1}\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1}k'(\Sigma_s^{-1}\Sigma+I_N)^{-1}\right.\\ &- \left.\Sigma_{\nu|s,F}^{-1}\beta\left(\sqrt{N}\Sigma_{F|s}^{-1} + k'(\Sigma_s^{-1}\Sigma+I_N)^{-1}\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1}\sqrt{N}\beta'\right)\\ &\to \Sigma_s\left(I_N + \frac{1}{\sqrt{N}}\Sigma_{\nu|s,F}^{-1}\beta\left(\frac{1}{N}k'\Sigma^{-1}\beta\right)^{-1}\beta'\right).\end{aligned}$$

Therefore,

$$\lambda = \frac{\gamma}{\mu N}\Phi_s^{-1}\Sigma_{\nu|s} = \frac{1}{\sqrt{N^3}}\gamma\mu^{-1}\Sigma_s\Sigma_{\nu|s,F}^{-1}\beta\left(\frac{1}{N}k'\Sigma^{-1}\beta\right)^{-1}\beta'.$$

The signal θ is now

$$\theta = s - \frac{1}{\sqrt{N}}\gamma\mu^{-1}\Sigma_s\Sigma_{\nu|s,F}^{-1}\beta\left(\frac{1}{N}k'\Sigma^{-1}\beta\right)^{-1}\frac{\beta'\beta_x}{N}F_x \;\equiv\; s - \frac{1}{\sqrt{N}}\Lambda F_x,$$

with $\Lambda = \gamma\mu^{-1}\Sigma_s\Sigma_{\nu|s,F}^{-1}\beta\left(\frac{1}{N}k'\Sigma^{-1}\beta\right)^{-1}\frac{\beta'\beta_x}{N}$. The idiosyncratic component of the random supply disappears; it is diversified away. The covariance matrix of the payoffs, conditional on θ, is

$$\Sigma_\theta = \Sigma_s + \frac{1}{N}\Lambda\beta_x\Sigma_{Fx}\beta_x'\Lambda'.$$

Note that Σ_s is a diagonal matrix while $\Lambda\beta_x\Sigma_{Fx}\beta_x'\Lambda'$ is a matrix with all of its matrix elements being of order 1. Therefore, when Σ_θ is multiplied by a vector of 1's from the right, the second term has the same order of magnitude as the first term. We can show that

$$\Sigma_{\nu|\theta,F} = \Sigma_{\nu|s,F} + O\left(N^{-1}\right).$$

As will be shown later, the contribution of such terms to the risk premium goes to zero in the limit as $N \to \infty$. The factor covariance matrix, conditional on θ, is

$$\Sigma_{F|\theta}^{-1} = \Sigma_F^{-1} + \frac{1}{N}k'\left(\Sigma + \Sigma_s + \frac{1}{N}\Lambda\beta_x\Sigma_{Fx}\beta_x'\Lambda'\right)^{-1}k.$$

Note that, when multiplied by vectors of 1's from left and from right, the term $\frac{1}{N}\Lambda\beta_x\Sigma_{Fx}\beta_x'\Lambda'$ produces a $K \times K$ matrix with elements of order N, the same as matrix $\Sigma + \Sigma_s$.

The variance of ν, conditional on θ,

$$\Sigma_{\nu|\theta} = \Sigma_{\nu|s,F} + \beta\Sigma_{F|\theta}\beta'.$$

The matrix $\Sigma_{\nu|s,F}$ is diagonal, while all the elements of the matrix $\beta\Sigma_{F|\theta}\beta'$ are of order 1. The terms neglected earlier produce matrices with all elements of order N^{-1}.

From the identity,

$$\begin{aligned}&\mu\Sigma_{\nu|s}^{-1} + (1-\mu)\Sigma_{\nu|\theta}^{-1}\\ =\ &\Sigma_{\nu|s,F}^{-1} - \Sigma_{\nu|s,F}^{-1}\beta\left(\mu\left(\Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1} + (1-\mu)\left(\Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|\theta,F}^{-1}\beta\right)^{-1}\right)\beta'\Sigma_{\nu|s,F}^{-1}\end{aligned}$$

we can write

$$\left(\mu\Sigma_{\nu|s}^{-1} + (1-\mu)\Sigma_{\nu|\theta}\right)^{-1} = \Sigma_{\nu|s,F} + \beta M^{-1}\beta',$$

where

$$\begin{aligned} M &= \left(\mu\left(\Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1} + (1-\mu)\left(\Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1}\right)^{-1} - \beta'\Sigma_{\nu|s,F}\beta \\ &= \left(\mu\left(\Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1} + (1-\mu)\left(\Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1}\right)^{-1} \\ &\times \left(\mu\left(\Sigma_{F|s}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1}\Sigma_{F|s}^{-1} + (1-\mu)\left(\Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1}\Sigma_{F|\theta}^{-1}\right). \end{aligned}$$

In the large N limit, $\beta'\Sigma_{\nu|s,F}^{-1}\beta$ is of order N, therefore, $\Sigma_{F|s}^{-1}+\beta'\Sigma_{\nu|s,F}^{-1}\beta \to \beta'\Sigma_{\nu|s,F}^{-1}\beta$. Similarly, $\Sigma_{F|\theta}^{-1} + \beta'\Sigma_{\nu|s,F}^{-1}\beta \to \beta'\Sigma_{\nu|s,F}^{-1}\beta$, so

$$\begin{aligned} M &\to \beta'\Sigma_{\nu|s,F}^{-1}\beta\left(\mu\left(\beta'\Sigma_{\nu|s,F}^{-1}\beta\right)^{-1}\Sigma_{F|s}^{-1} + (1-\mu)\left(\beta'\Sigma_{\nu|\theta,F}^{-1}\beta\right)^{-1}\Sigma_{F|\theta}^{-1}\right) \\ &= \mu\Sigma_{F|s}^{-1} + (1-\mu)\Sigma_{F|\theta}^{-1}. \end{aligned}$$

The risk premium is given by

$$\gamma\beta\left(\mu\Sigma_{F|s}^{-1} + (1-\mu)\Sigma_{F|\theta}^{-1}\right)^{-1}\frac{\beta'\bar{x}}{N}$$

and the factor risk premium is given by

$$\gamma\left(\mu\Sigma_{F|s}^{-1} + (1-\mu)\Sigma_{F|\theta}^{-1}\right)^{-1}\frac{\beta'\bar{x}}{N}.$$