Licensing and Competition for Services in Open Source Software

Terrence August*
Rady School of Management
University of California, San Diego

Hyoduk Shin†
Rady School of Management
University of California, San Diego

Tunay I. Tunca‡
Smith School of Business
University of Maryland

March, 2013

Abstract

Open source software is becoming increasingly prominent and the economic structure of open source development is changing. In recent years, firms motivated by revenues from software services markets have become the primary contributors to open source development. In this paper we study the role of services in open source software development and explore the choice between open source and proprietary software. Specifically, our economic model jointly analyzes the investment and pricing decisions of the originators of software and of subsequent open source contributors.

We find that if a contributor is efficient in software development, the originator should adopt an open-source strategy, allowing the contributor to offer higher total quality and capture the higher end of the market while the originator focuses on providing software services to lower end consumers. Conversely, if the contributor is not efficient in development, the originator should adopt a proprietary software development strategy, gaining revenue from software sales and squeezing the contributor out of the services market. In certain cases an increase in originator development efficiency can result in increased contributor profits. Finally, we find that, somewhat counter-intuitively, an increase in contributor development efficiency can reduce overall social welfare.

*Rady School of Management, University of California, San Diego, La Jolla, CA 92093-0553. e-mail: taugust@ucsd.edu
†Rady School of Management, University of California, San Diego, La Jolla, CA 92093-0553. e-mail: hdshin@ucsd.edu
‡Smith School of Business, University of Maryland, College Park, MD 20742. e-mail: ttunca@rhsmith.umd.edu

1 Introduction

As the employment of information technology in modern corporations broadens and evolves, software plays an increasing role in the efficiency of firms and their ultimate level of competitiveness. Consequently, implementation, integration and cost efficiency of software selection is crucial to a company's overall information technology strategy. Emerging as a response to this need, in a field that has been traditionally served by proprietary software solutions, open source software has become more prominent in the past decade, and its availability and quality have increased dramatically. For example, products and applications based on Linux and Apache software initiatives are widely used today, and many other open source products such as JBoss (Java middleware) and MySQL (DBMS) have emerged and generated increased usage in recent years (Kontzer 2005, Brunelli 2005, and Broersma 2005).

Open source software is arguably of comparable or, in some cases, even higher quality than its proprietary counterparts in terms of technical features. But high quality software and useful features alone have little value to a business unless it is able to utilize them through a cohesive integration of its systems and business processes and can maintain these integrated systems up and running in a reliable fashion. In fact, integration is precisely where the open source software industry has concentrated its efforts in the last decade.[1] Firms realize what matters to customers is the trade-off between the quality of the open source software solution and the price they pay. Due to customers' needs for value-added services, software companies now see potential in harnessing other developers' contributions to open software while still maintaining a lucrative source of revenues binded to services rather than product sales (Vance 2009). Today, while product license sales is the dominant revenue model for software firms who elect to keep their products proprietary, integration, support, and consulting services have become the most widespread and promising revenue model for firms that choose an open source path (Watson et al. 2008).

An important question that emerges is when would it be in a software firm's best interest to pursue an open source strategy. Strategic considerations play an important role in this decision. The highest

[1] *Integration* refers to getting software installed on a company's servers and integrated with business processes and pre-existing information systems as effectively as possible to increase the overall quality of the solution provided. *Support* refers to ongoing maintenance of these integrated systems, from troubleshooting bugs which arise to efficiently providing security vulnerability patches and more. Information technology *consulting* often refers to the provision of human resources to advise, implement, and deploy IT solutions which may include custom-built applications that run on top of open source software. Altogether, the integration, support, and consulting offerings are generally referred to as *services*.

quality services are expected to be offered by the firms who have invested the most effort into the open source project by contributing source code and managing the architecture of the software. Through their accumulated expertise, these firms have an advantage in the services market as they can provide the best integration with existing systems as well as most effectively provide support and consulting. However, with open source software, firms' development efforts to improve quality of integration and services can also benefit other service providers. In particular, because service providers can use publicly available software documentation, service modules and utilities written by firms who have invested in development, they can provide more effective integration and support, and thus offer a higher quality software solution (LaMonica 2005, Virijevich 2005). Considering the effects of such complementarities on competition in the market for integration and support, investments on software development and building expertise gain additional strategic importance.

With open source software, diverse market environments can emerge. In certain cases, the software originator assumes the role of primary developer while subsequent contributors undertake useful but relatively minor development roles. Red Hat, with its JBoss division, leads development and dominates the services market for JBoss Application Server, which is an example of this setting (Kerstetter 2004, Sager 2005, Adamson 2005). In other cases, a small firm or organization may originate a software product, but a large outside firm leads investment toward the development of the software product, thus making significant contributions even though they are subsequent to the originator's efforts. For example, IBM has over 700 developers working on open source software and self-reportedly plans to invest $1B into development and support of its projects in this domain (Capek et al. 2005). Similarly, HP has over 2,500 developers focused on open source projects (Hewlett-Packard 2007). These companies are strongly incentivized by the global services market. In 2007, IBM generated $55.3B from global services while only $20B from software (IBM 2007). The strategic interactions between these players determine the final market outcome and have an important role in shaping software development choice.

In this paper, we present a model that captures the economic incentives for firms to contribute to open source software development and then explore the strategic and policy decisions of a software originator and regulators, respectively. We explore joint development and pricing equilibria in software markets for both

open source and proprietary software strategies. If a firm chooses an open source strategy, she makes her source code publicly available which has several effects. First, the software receives a development boost as outside contributors help write and refine the product's source code, as well as produce related modules and utilities. Second, the firm must rely heavily on services as the primary revenue source since her source code is freely available to users. Third, the availability of source code makes it easier for competitors in the services market to build expertise and improve the quality of their own offerings, which means the originating firm faces tougher competition in this market. For example, a large contributor such as IBM can dedicate vast resources to the development of an open source project in order to increase his mastery of the software and improve his services packages. This investment can both benefit and harm an originator operating in an open source environment. Alternatively, the originating firm may choose a proprietary option, in which case the source code of her product remains closed. In this case, the firm incurs the entire product development cost alone. On the upside, in addition to potentially charging for software and integration services, the firm retains the ability to charge a price per each copy of the product regardless of whether she provides the services aspect.

Using this model, we study three main issues: First, under both open-source and proprietary licensing strategies, we thoroughly examine and identify the equilibrium outcomes in a market for software where an originator and contributor invest in development and firms compete for revenues from providing services. Second, building on this equilibrium analysis, we explore how software development efficiency for both the originator and subsequent contributors impacts the originator's choice of source code licensing strategy. Third, we study the implications of an open approach on social welfare. The U.S. government has recently begun to advocate open source development in certain circumstances and, more importantly, continues to play a significant role in setting policy to help increase the value generated by information goods in the economy (Herz et al. 2006). Accordingly, we also study social surplus, and provide insights into how governance and policy can utilize the economic incentives influenced by competition in the software services market to improve welfare.

In essence, what lies at the core of our paper is the management of two goods that are complements, namely the software and the integration and services provided for it. The main trade-off in this software

licensing decision with services is as follows: the originator can either develop a product alone and be able to charge for the software herself in return (proprietary strategy), or she can open up the source code to outside contributors to share the burden of investment in development but rely only on services and integration income for revenue, albeit with possibly higher quality of software and services thanks to the efforts of the contributor firm (open source strategy). There are a number of factors that play critical roles in determining the best licensing strategy such as the relative efficiencies of the originator and the contributor firms, and capabilities of the firms reaping benefits from each others' software developments.

Contributor efficiency is a double-edged sword for the originator. When the contributor is strong in development efficiency and utilizing developments to improve quality, it can pay off for the originator to open the source code since this can result in significantly increased total software and service quality at little cost to the originator. However, a strong contributor can also be a formidable competitor that can steal significant market share. The originator has to balance these two factors when making her licensing decision. We find that if the originator is sufficiently efficient in development, her licensing decision mainly depends on her ability to harness the contributor's development to improve quality: if the originator is adept at improving the quality of her software/service package by utilizing the contributor's development, then an open source strategy is optimal, otherwise the originator is better off keeping the software proprietary. We find that an increase in originator development costs can in fact decrease the originator's service price and increase the contributor's service price. This occurs because decreased originator development efficiency reduces originator effort in investment and increases differentiation between the originator and the contributor's offerings. Consequently, the contributor can be better off by increasing his efforts in development, quality and price. We also show that increased contributor efficiency can unexpectedly decrease welfare. This result can manifest when the contributor is highly efficient because if the originator opens up the source code, the originator can be squeezed out of the services market as the contributor uses his development efficiency and strategic pricing to open up a large gap between the overall attractiveness of his offering and that of the originator. In such circumstances, the originator may instead choose a proprietary strategy, which results in lower software and service quality and decreased welfare.

2 Literature Review

One of our key research goals in this paper is to explore the economic incentives of the developers and firms producing open source software. One primary question is why do developers contribute to a project without being paid for their effort, at least by traditional means. In the existing literature, researchers have empirically substantiated the existence of intrinsic motivations such as intellectual stimulation and community building as well as extrinsic motivations such as job market signaling and being paid to contribute (see, e.g., Lerner and Tirole 2002, Hars and Ou 2002, von Krogh and von Hippel 2006, Roberts et al. 2006, Iansiti and Richards 2006). However, to our knowledge, no study has investigated a theoretical model that explicitly structures the real monetary incentives and revenue sources of the firms that contribute to open source projects for profit under market competition. We focus on these incentives, which emerge through the market for services.

Recently, the market for services has been recognized as a powerful extrinsic motivation for developers' contributions to open source software (see, e.g., Raymond 1999, Lerner and Tirole 2002, Lerner and Tirole 2005a, Capek et al. 2005). The services market has the potential to generate significant revenues for contributors who invest in open source development. One reason is due to the existence of many indirect methods to capture a software program's "sale value" which refers to its value as a salable commodity (Raymond 1999). Lerner and Tirole (2005a) note that for-profit firms such as IBM which offer services that are complementary to open source software have largely benefited in their consulting businesses. Watson et al. (2008) suggest that the contemporary second-generation open source (OSSg2) companies that generate the bulk of their revenues from services have the most promising business model in the open source domain. Our paper contributes to the open source software literature by formally analyzing the impact of the services market on the incentives of originator and contributor firms as profit-seeking entities.

When one turns to the motivations of *firms* that profit from open source, revenues generated by providing implementation, integration and other related services turn out to be a significant factor. Iansiti and Richards (2006) empirically demonstrate that large IT vendors such as IBM and Oracle invest in open source software development projects that provide or complement revenues from applications and services

they provide. Watson et al. (2008) point to the fact that JBoss contributed 85%, 95%, and 60% of the total effort toward JBoss AS, Hibernate, and Tomcat, which are its three leading open source products, and the majority of JBoss's revenues come from its provision of services. Further, for most open source projects, although there may be many independent, small service providers, the major contributors to development (other than perhaps the originators) are a small number of firms prominent in the specialty (Mockus et al. 2000, Kogut and Metiu 2001, Koch and Schneider 2002, Kuk 2006). Based on these empirical observations, our paper focuses on the motivations of firms both in development and competition, accounting for profit-motivated contribution incentives via a market for services.

A major question we study in our setting is whether a software originator should choose to make her product proprietary or open source. Related to this question, Lerner and Tirole (2005b) predict proprietary projects that seem less likely to succeed may be turned into open source projects but will also face lower acceptance by the open community. Gaudeul (2004b) finds that proprietary strategies are favored when developer wages are closely balanced with their costs. Using a dynamic model of pricing and investment, Haruvy et al. (2008) study the problem in the context of an exogenous complementary product or service on the open source path.

Additionally, several papers look at direct competition between open source and proprietary firms (e.g., Gaudeul 2004a, Bessen 2006, Casadesus-Masanell and Ghemawat 2006, Lee and Mendelson 2008, Zhu and Zhou 2012). In this paper, unlike these existing studies, we focus on the competition in the services market for a given source code strategy. Our paper thus contributes to this literature by exploring the originator's strategy of whether to proceed with an open source or proprietary strategy in consideration of the market for services.

3 Model

A software *originator* determines whether to pursue a proprietary or open source software licensing strategy, which we denote with $\rho \in \{P, O\}$. Throughout the paper, we use superscripts P for proprietary and O for open source to indicate the licensing strategy. After the licensing decision, the originator invests in the

design and development of the software product. A secondary firm, the *contributor*, may choose to put forth effort to invest in development and increase the quality of the software solution.[2] We denote the originator with subscript o and the contributor with subscript c. In our model, we focus on a profit-motivated originator and a profit-motivated contributor who intend to generate revenues from offering software integration and other services to consumers. It is also important to note that there are typically other contributors to open source software that do not have commercial motivations. These contributors have non-strategic motivations such as hobbyism, altruism, and idealism. One can incorporate these non-strategic contributions from the open source software community as an exogenous, random additive component in the quality of the software and service for all providers, but this simple arithmetic shift would not alter the nature of the insights or the results from the model. In order to keep the model simpler, we will not incorporate this component in our model and analysis and, instead, maintain focus on contributors with strategic motivations.

After purchasing or deciding to use the software, a consumer needs to have the software installed and integrated, i.e., obtain service, for it to become usable and the consumer to derive value from it. The value derived from software usage for a consumer depends on many factors including product features, reliability, integration with business systems and processes, and support. In turn, the quality (and hence the value to the consumer) of features such as integration and support depends on the provider of the service. A service provider firm with more expertise, better utility and support tools, and preparation provides a higher quality service. Each firm that provides service and integration essentially offers its own level of service quality, and the overall value a consumer derives from using the software depends on the firm it contracts to provide integration and service (see Raymond 1999 and Farber 2004, among others). A firm may improve its expertise and total quality by investing effort in these areas. Therefore, although under both licensing strategies both firms share the same software product, because of the potential difference in service qualities, the *total quality* of the offering of a firm, which we define as the combined quality of the software used and the service provided by that firm, can differ between the two firms. We denote the

[2]Note, for simplicity, we include a single large contributor in the model. Further, in the market we study, contributions are extremely concentrated amongst very few contributors (see, e.g., Mockus et al. 2000, Koch and Schneider 2002, Kuk 2006), and it is often the case that a single large contributor emerges to compete against the originator and together dominate the market's strategic interactions, as in the case of Apache Geronimo.

total quality for the originator by Q_o and the total quality for the contributor by Q_c. The total quality of the contributor firm's offering benefits at a certain rate from the effort invested by the software originator toward development of the product. Furthermore, if the product is open source, the contributor's additional developments are available to the public, so in that case, the total quality of the originator's offering also benefits from the contributor firm's efforts. Therefore, a software and related services marketplace with the possibility of open source licensing has an interesting characteristic: The originator and the contributor are complementary to each other in improving the total quality of the software and services while they also compete against each other in the marketplace for providing these services. The software policy chosen by the originator (i.e., proprietary or open source) changes the incentives for these two firms and therefore influences their strategic choices of effort invested in the software.

There is a continuum of consumers who have heterogeneous preferences on the total quality of a complete software solution they receive. We model this consumer type characteristic as uniformly distributed, $\theta \in \Theta = [0, 1]$ where θ denotes a specific consumer's sensitivity to quality or "type." A consumer with type θ obtains value θQ_o, if she contracts with the originator and θQ_c if she contracts with the contributor.

The originator selects an effort in development $e_o \in \mathbb{R}_+$ and incurs a quadratic convex cost of effort $\beta_o e_o^2/2$. Similarly, the contributor invests effort $e_c \in \mathbb{R}_+$ at a cost $\beta_c e_c^2/2$ subsequent to the originator's investment decision. Effort in open source development amounts to labor and resources, and, particularly in the software industry, firms exhibit significant variation in size and worker abilities, as well as differences in the shadow prices of worker time and resource availability for any given project. Therefore, β_o and β_c are in general different and firms demonstrate heterogeneity in this dimension (see, e.g., Oi 1983). Note that, for the originator, e_o refers to the additional effort in development of the software *beyond* the base amount required for the release of the software in the minimal acceptable form.

As we mentioned above, depending on the software policy decision (O or P) a firm's offering will be positively impacted by other firms' investments in publicly available open source code, service support components, utility contributions, and information. Under an open source policy (i.e., for $\rho = O$), the total quality offered by the originator depends not only on her own effort e_o^O but also on the effort exerted by the contributor such that $Q_o^O = s_{oo}^O e_o^O + s_{oc}^O e_c^O$. Similarly, the total quality for the contributor depends

on the originator's effort and is given by $Q_c^O = s_{co}^O e_o^O + s_{cc}^O e_c^O$. The parameters $s_{ij}^O > 0$, $i, j \in \{o, c\}$, indicate the effect of firm j's effort on firm i's total quality. The magnitudes of coefficients s_{ij}^O are largely determined by the particular software product market and the nature of the originator and contributor firms. These coefficients have a substantial economic and strategic significance. Their relative magnitudes are critical in determining the incentives of the firms to put effort into the software because a firm's competitor benefiting from her effort at a high rate may curb her incentives to bear the costs of exerting that effort. Further, the relative magnitudes of these coefficients between proprietary and open source policies may also affect the originator's licensing policy. As we will see more clearly in the following sections, these coefficients will play an important role in determining the outcome and the results.

Under a proprietary strategy, consumers who purchase the product can choose to use the originator's services or contributor's services. Since the source code is closed under this strategy, the contributor's effort does not create returns toward the originator's quality component, $s_{oc}^P = 0$. Note that s_{oc}^P is related to the benefit that the originator can obtain from developer effort. Under the proprietary strategy, it is harder for the originator to garner developer efforts to her advantage. For example, Microsoft may try to implement certificate programs to nurture new software. Some contributors may invest their efforts to earn the certificates. However, it is not directly related to enhancing the software quality nor to increasing Microsoft's service quality. Consequently, s_{oc}^P is close to 0.[3] Further, the closedness of the source code limits the contributor's ability to improve his own quality factors and related offerings, i.e., $s_{cc}^P < s_{cc}^O$. We consider $s_{oo}^P = s_{oo}^O$ to be constant and independent of the originator firm's strategy. In order to make a fair comparison between the profitability of the two business models, we hold this specific parameter constant and isolate the main effects that help each model to perform relatively better.[4] In summary, for the proprietary case, $Q_o^P = s_{oo}^P e_o^P$ and $Q_c^P = s_{co}^P e_o^P + s_{cc}^P e_c^P$. Note that for simplicity in analysis, we assume that the originator is capable of better harnessing her own efforts in development to improve quality than the contributor can, i.e., $s_{oo}^O > s_{co}^O$ and $s_{oo}^P > s_{co}^P$. In many cases it is reasonable to expect that a software originator can do this because as the creator of the software, she is likely to have fundamental knowledge

[3] Technically, it is not required that $s_{oc}^P = 0$ in order to establish our results.

[4] It is mathematically not a problem to relax this specification, and all of our results would continue to hold if we allowed this parameter to be appropriately constrained to match desired settings.

and expertise as well as control of the project direction, enabling her to better leverage her own efforts.

Consumers' usage decisions are made in the last stage. Under a proprietary strategy, the originator sets a price for the product p^P and a price for her services p_o^P, while a contributor only sets a price for his services offering p_c^P. Under an open source strategy, the pricing of integration and services still occurs (p_o^O and p_c^O), but the product price is zero ($p^O = 0$). For simplicity, throughout the paper, we assume that the cost of providing service for the firms is zero.

In the proprietary case, a consumer with type θ chooses one of the following: not use the software product, purchase both the product and services from the originator, or purchase the product from the originator while obtaining services from the contributor. Her net payoff from each action is given by

$$V^P(\theta) = \begin{cases} Q_o^P\theta - p^P - p_o^P & \text{if purchased and service is contracted with the originator;} \\ Q_c^P\theta - p^P - p_c^P & \text{if purchased and service is contracted with the contributor;} \\ 0 & \text{if not purchased.} \end{cases} \tag{1}$$

Under an open source strategy, the originator forgoes revenues from product sales while focusing on the services market. In this case, the consumer's net payoff is given by

$$V^O(\theta) = \begin{cases} Q_o^O\theta - p_o^O & \text{if service is contracted with the originator;} \\ Q_c^O\theta - p_c^O & \text{if service is contracted with the contributor;} \\ 0 & \text{if not used.} \end{cases} \tag{2}$$

The complete software and services total qualities (Q_o^P and Q_o^O, and Q_c^P and Q_c^O) differ under each source code strategy due to varying investment incentives as well as certain quality cross-effect parameters s_{ij}^P and s_{ij}^O differing for the two source code strategies.

In summary, the timeline for our model is as follows:

1- The originator chooses a licensing strategy, $\rho \in \{P, O\}$, either making the product *proprietary* or *open source*.

2- The originator invests in developing the software, e_o.

3- Knowing whether the software is proprietary or open source and having observed the originator's effort in software development, e_o, the contributor decides whether to offer services for the software and how much to invest in further development, e_c.

4- The originator observes whether the contributor is in the market and, if so, the contributor's level of software development.

5- Both firms simultaneously price their offerings, i.e., the originator and the contributor set their prices for services, p_o^ρ and p_c^ρ, respectively, for $\rho \in \{O, P\}$. In addition, if the product is proprietary, the originator sets the unit price for the product, p^P, simultaneously as well.

6- Each consumer chooses whether to use the product and from which firm to purchase services. The market clears.

4 Source Code Strategies

4.1 Proprietary Strategy

In the commercial software industry, firms have traditionally pursued proprietary strategies which use *product* revenues to recoup software development costs and generate return on investment. Corporations such as SAP, Oracle, and Microsoft have developed leading software products in their respective markets, and all have chosen to keep their products' source code mostly closed and generate revenues from selling licensed copies. However, as we also discussed above, in order to derive full benefits from using the software, consumers also need to spend money and resources to procure integration and support. Software originators who choose to develop their software as a proprietary product typically offer these services as well. Furthermore, service offerings need not necessarily be made separate but may be bundled with the product itself.

By adopting a proprietary strategy, the originator makes a strategic choice to not permit secondary developers to contribute to or build upon the core codebase of the product. Although the firm typically offers Application Programming Interfaces (APIs) that permit selective extensions to software functionality,

custom code written for an API is often user-specific and limited in potential scope. By opting for the firm's services package, a user benefits in several ways. First, by developing and managing the software product, the firm has accumulated expertise which it can leverage to provide a higher quality solution to the customer. Second, the firm can adapt its software product to include specific functionality requested by clients, and, further, this functionality can be built into the product for future releases which then require less maintenance of custom code by users. In this section, we investigate how an originator sets prices for her product and services and chooses investment in overall quality at the product development stage. We characterize the optimal profits obtainable using a proprietary strategy and use it as a benchmark for comparison to the outcome under open source.

4.1.1 Consumer Market Equilibrium

Given the effort levels, e_o^P and e_c^P, and prices, p^P, p_o^P and p_c^P, each consumer of type θ chooses whom to contract in order to maximize her net payoff given in (1) which reflects the choices for obtaining services. The characterization of equilibrium consumer behavior is symmetric, dependent on whether the originator or contributor offers a higher total quality. In either case, the prices are chosen such that one of the following outcomes occurs: (i) no consumer uses the software; (ii) consumers purchase the product from the originator, and all of them elect to be serviced by the firm offering a higher total quality; (iii) consumers purchase the product from the originator, and all of them elect to be serviced by the firm offering a lower total quality; and (iv) consumers purchase the product from the originator, and they further segment such that lower (higher) types elect to be serviced by the lower (higher) quality firm.[5]

Figure 1 illustrates these regions of the consumer market equilibrium. Because of symmetry, the figure focuses on the case where the originator is the quality leader, i.e., $Q_o^P > Q_c^P$. When the originator's service price is sufficiently low compared to the contributor's service price, the originator's service is a more attractive offer as her price is low for her quality level compared to the price/quality offering of the contributor. In such a case, every consumer who purchases the product will prefer to get services from the

[5]The complete mathematical characterization of the consumer market equilibrium and these regions is given in Lemma OS.1 in the online supplement. For ease of exposition, technical statements of all lemmas and propositions, as well as their proofs, are placed in the online supplement.

originator, i.e., the contributor is squeezed out of the market. If the contributor's service price is lower, then consumers with a stronger preference for quality, i.e., those with higher θ values, still prefer to contract with the originator for services, while certain consumers with lower θ values will now choose to acquire the contributor's services; for them, the contributor's price will be more attractive than the originator's, better justifying the quality of his service offering as can be seen in panel (b) of Figure 1. Finally, if the contributor's service price is sufficiently low compared to that of the originator's, all consumers will find purchasing services from the contributor to be more attractive than purchasing services from the originator. In this case, consumers will either not use the software or obtain services from the contributor after purchasing the software. The consumer market structure that results for this case is illustrated in panel (c) of Figure 1.

4.1.2 Consumer Market Pricing

Next, we present and discuss the consumer market pricing equilibrium taking the development effort levels for the originator and the contributor as fixed. Given their respective effort levels, e_o^P and e_c^P, in the pricing stage of the game, the firms compete in the services market by setting their service prices to maximize their profits taking each other's prices as given. The Nash equilibrium outcome of this stage of the game yields the equilibrium service prices denoted by $p_o^{P*}(e_o^P, e_c^P)$ and $p_c^{P*}(e_o^P, e_c^P)$, which are functions of the effort levels the firms chose earlier in the game. The following proposition describes how the originator and contributor set prices in equilibrium.

Proposition 1 *Given the effort levels of the originator and contributor, the equilibrium software and services pricing outcomes are characterized as follows:*

(i) If the originator is the quality leader, she sets the software price as a monopolist and prices her services at cost such that she is the only firm actively servicing consumers in equilibrium. The contributor is forced out of the market.

(ii) If the contributor is the quality leader, then the originator adjusts the software price reflecting the contributor's effort level, and she provides a discount to users who opt to consume her services.

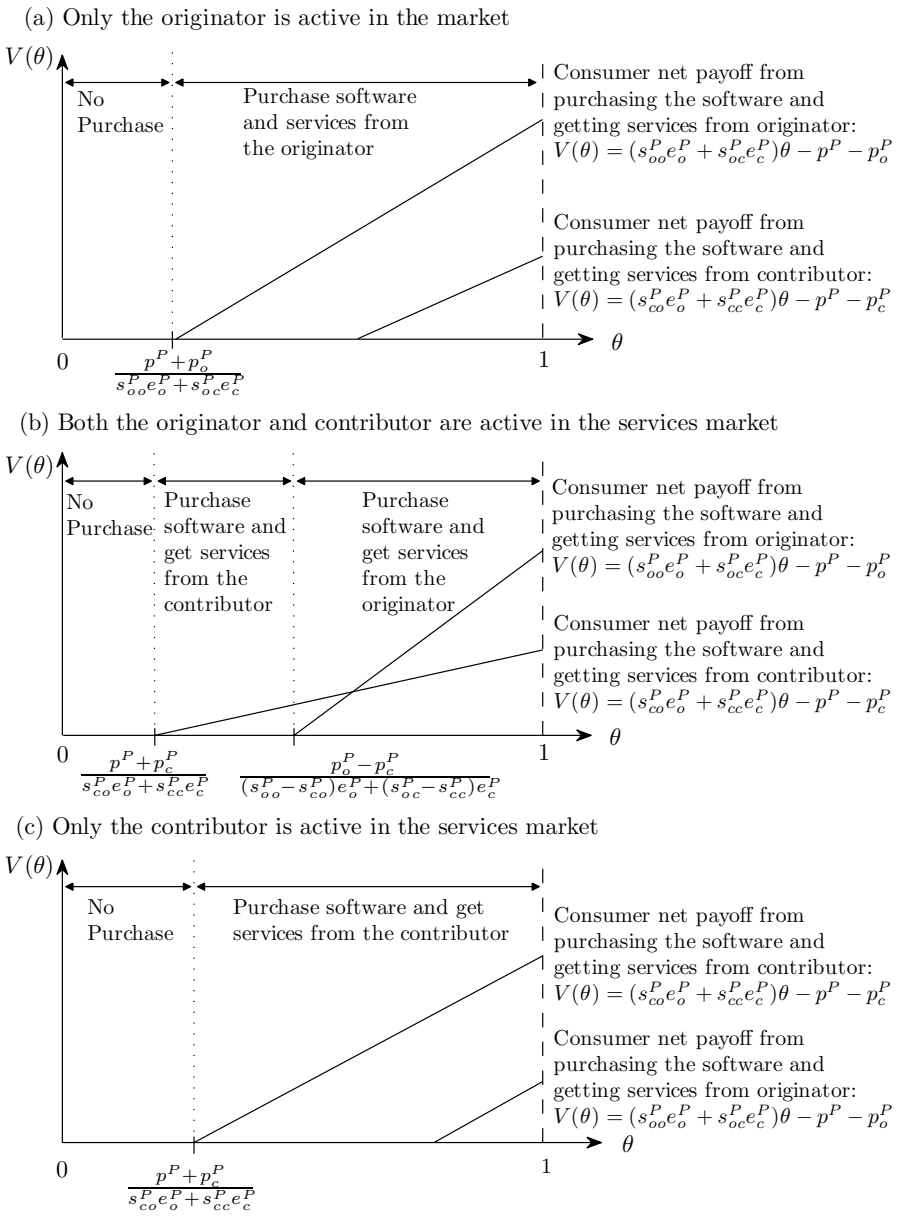

Figure 1: The consumer market equilibrium under a proprietary strategy when the originator has higher total quality.

The contributor sets a service price above cost that mirrors the originator's discount, and both firms actively service consumers in equilibrium.

Part (i) of Proposition 1 states that if the originator's service is of higher quality than the contributor's then the contributor cannot compete with the originator in the services market because the originator will price the product to maximize revenue from software sales, while setting her service price to zero to squeeze the contributor out of the services market. As a consequence, she essentially offers a bundle of product and service as a monopolist and sets the "total price" of the bundle at the monopoly price. At these price

levels, the contributor is forced out of the market completely, since in order to gain any positive market share, she has to price her integration service offering below cost, i.e., below zero, which would result in losses. Therefore, the contributor is effectively forced out of the market.

On the other hand, if the contributor is the quality leader for services, then he has an advantage with the high valuation consumers (i.e., high θ consumers) over the originator. Part (ii) of Proposition 1 states that in this case, in equilibrium, the originator chooses to share the services market with the contributor, utilizing the high quality of the contributor's services to boost sales of her software product at the high end of the market. At the same time, she sets her service price negative to maintain price pressure on the contributor who is offering high total quality, which helps to increase revenues from the product among those who prefer to purchase services from the contributor.[6]

4.1.3 Determination of Development Effort Levels

Using the equilibrium outcome for product and service prices provided in Proposition 1, we can analyze the equilibrium effort investment levels for the two firms. Given the effort of the originator (e_o^P) and using the equilibrium price expressions characterized in the proof of Proposition 1, we can derive the contributor's profit as a function of his effort level, e_c^P.[7] From this function, one can observe that given the originator's effort level, if the contributor's effort level is sufficiently low, the originator is the quality leader; whereas, when the contributor puts sufficient effort in development, he becomes the quality leader. As a best response to the originator's effort level, the contributor chooses an effort level to maximize his profit, and the following proposition summarizes the solution to the decision problem he faces.

Proposition 2 *Given the originator's effort level $e_o^P > 0$, the contributor's best response effort level $e_c^P(e_o^P)$, and the resulting consumer market equilibrium is characterized as follows: There exists a critical*

[6]By part (ii) of Proposition 1, it is in the originator's best interest to enforce that consumers who purchase the service from her and enjoy the discount, indeed get the service from her. This would prevent higher valuation consumers, who prefer the contributor's higher quality service, from purchasing service from the originator just to get the discount and to actually have service performed by the contributor. So, the agreements are set to be binding in that by contracting with the originator for the service, customers agree to being serviced exclusively by the originator. This is naturally enforceable by monitoring the source of service for each customer. This information is verifiable electronically as well as in person because service work is typically ongoing.

[7]The full closed form expression for the contributor's profit as a function of his effort level is given in equation (OS.19) in the online supplement.

threshold value, $\bar{e}_o > 0$, where[8]

(i) If the originator's effort level is less than $\bar{e}_o$, then the contributor's investment in development level is positive and the contributor becomes the quality leader in the market with both firms actively serving segments of the consumer market;

(ii) If the originator's effort level is above $\bar{e}_o$, then the contributor does not invest in development and remains out of the market.

If the originator's effort level is small, then the contributor has an opportunity to leapfrog the originator's quality by investing heavily in development and bringing to market a services package preferred by higher valuation consumers. When the originator's development effort level is low, the contributor seizes this opportunity, and becomes the quality leader. On the other hand, if the originator puts sufficient effort into development, then the contributor faces very different prospects. In this case, his efforts will not yield sufficient returns to cover his costs, so, his best response is not to invest in development at all. Consequently, the originator is the quality leader and becomes the sole firm actively serving the consumer market.

Proposition 2 describes how the contributor responds to feasible effort investments by the originator. Utilizing this characterization, in particular the mathematical characterization detailed in the online supplement, we next examine how the originator then chooses her effort level e_o^P to maximize her own profit Π_o^P. Substituting the contributor's effort best response function and the equilibrium prices, we can write the originator's profit as a function of her effort level e_o^P.[9] Studying this function, one can see that if the originator does not invest sufficiently in the software, i.e., $e_o^P \leq \bar{e}_o$ where $\bar{e}_o$ is as given in Proposition 2, then as we have seen above, the contributor puts substantial effort and becomes the quality leader. In this case, if the contributor is efficient enough in development (low β_c), the originator also benefits from his efforts. In this case, the originator's profit in fact *increases* as β_c decreases. On the other hand, if the originator makes a sufficiently large investment in the software, i.e., $e_o^P \geq \bar{e}_o$, then, by Proposition 2, the contributor

[8]The closed form mathematical expression for $\bar{e}_o$ as well as the investment levels for the contributor are given in the proof of this proposition in the online supplement.

[9]The full closed form expression for the originator's profit as a function of her effort level is given in equation (OS.20) in the online supplement.

is out of the market, and the originator can become the monopolist. Considering and comparing these outcomes, the originator determines the amount of effort to put into software development that maximizes her profits. Her decision, hinges on the development efficiency of the contributor, β_c. We first examine a case where the contributor is highly efficient in development. The following proposition describes how the originator optimally leverages such an efficient contributor.

Proposition 3 *If the contributor's development costs are low, i.e. if β_c is sufficiently small, then, in equilibrium, both the originator and the contributor are active in the services market and the contributor is the quality leader. Only consumers with types in the range $[\frac{1}{2}, 1]$ purchase the software. The originator serves the consumers with types in $[\frac{1}{2}, \frac{2}{3}]$ and the contributor serves those with types in $[\frac{2}{3}, 1]$.*

If the contributor is cost-efficient in development, then the originator invests a relatively small amount, which encourages the contributor to invest effort in the software and capture the high end of the consumer market. This benefits the originator because greater investments by the contributor will also make the originator's software product and service offerings more attractive to customers which, in turn, boosts her revenue. As a result, in this regime, the originator enables the contributor to put in sufficient effort to become the quality leader and actively serve the higher valuation (i.e., high θ) customers. Likewise, the originator not only profits from software sales but also shares the service market by serving a lower valuation segment of the consumer population.

Next, we examine the opposite case in which the contributor is highly inefficient in his cost of effort toward software development. Under a proprietary strategy, a cost-inefficient contributor offers limited value to the originator. As a result, the originator prefers to become the sole provider in the services market; we formally establish this result in the following proposition. Later, we will see that a contributor with similar characteristics is leveraged quite differently under an open source strategy.

Proposition 4 *If the contributor's development costs are high, i.e. if β_c is sufficiently large, then in equilibrium only the originator is active in the services market. Only consumers with types in the range $[\frac{1}{2}, 1]$ purchase the software and services from the originator.*

When the contributor is not very efficient, it is costly for the contributor to put in significant effort, and it becomes much easier for the originator to push him out of the services market. Consequently, the originator invests substantial effort in software development, which in turn discourages the contributor to make any investment in the software; essentially, the contributor does not enter the market. That is, a software originator who faces an inefficient contributor will choose to invest heavily and capture the entire services market.

4.2 Open Source Strategy

As an alternative to the traditional proprietary strategy, a software originator firm can adopt an open source strategy in which the source code is *opened* to other entities such as developers and users. One of the core features of the *Open Source Definition* is that the source code of the software product must be made available (Coar 2006). As a business model, there are many significant advantages to an open approach which leverages the knowledge and resources found in an open community. The originator's cost of development is lower since she can benefit from the contributions of other developers toward both improving the software and writing utilities and other support code helpful in integration and maintenance of the product. In an open approach, the source code is made freely available in which case charging for the base software product itself is not feasible. However, because consumers value services, firms generate the bulk of their revenues by providing integration, support, and consulting.

4.2.1 Characterization of Consumer Market Equilibrium

Similar to Section 4.1, we will start by exploring the consumer market equilibrium, and we will roll back the solution to effort levels and licensing strategy subsequently from there. Given the effort levels, e_o^O and e_c^O, and prices, p_o^O and p_c^O, each consumer of type $\theta \in \Theta$ chooses whom to contract with in order to maximize her net payoff given in (2) which reflects the choices she has for obtaining services. The characterization of consumer behavior in equilibrium is once again symmetric, dependent on whether the originator or contributor offers a higher total quality. Further, the four outcomes described at the beginning of Section 4.1.1 are also all feasible under an open source strategy. Figure 2 illustrates the consumer market

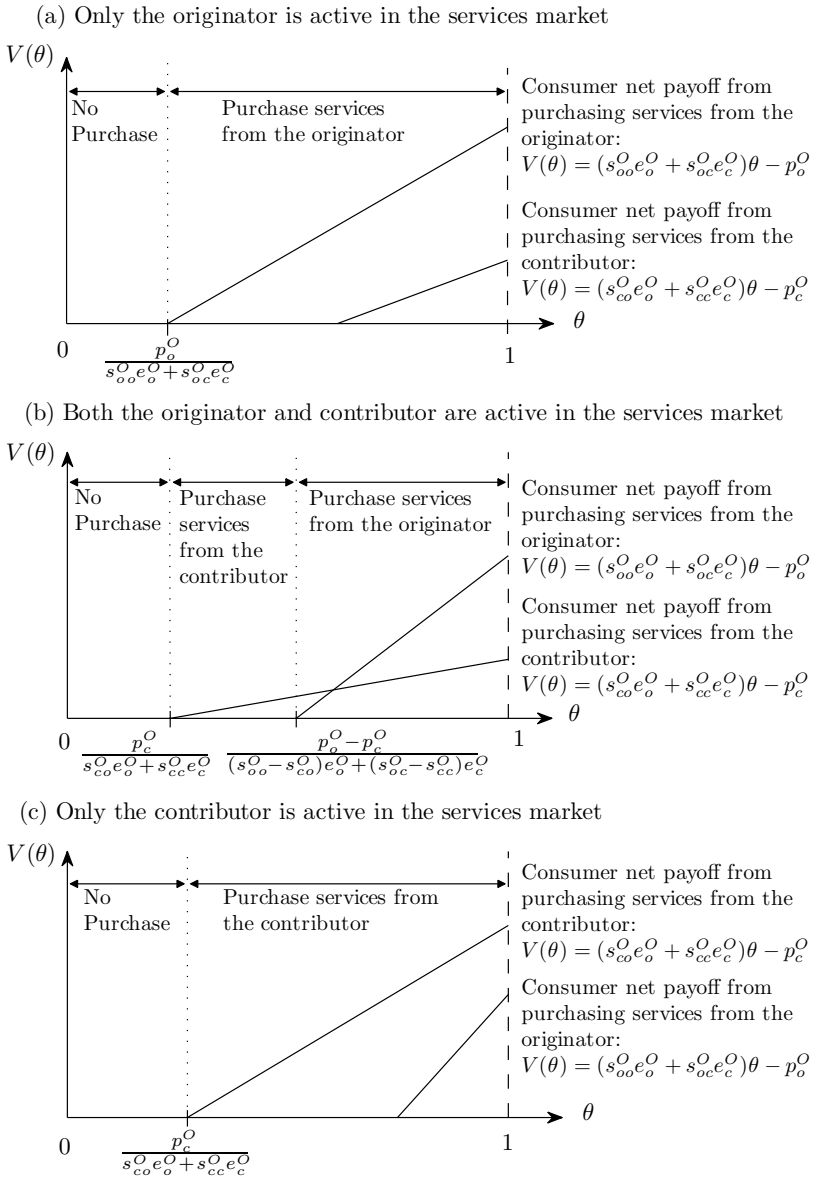

Figure 2: The consumer market equilibrium under an open source strategy when the originator has higher total quality.

equilibrium as is fully mathematically characterized in Lemma OS.2 in the online supplement for the case when the originator is the quality leader, i.e. $Q^O_o > Q^O_c$.

If both firms set their prices too high, no consumer will buy services and hence no consumer will choose to use the software. If either the contributor is pricing too high or if the originator is pricing low enough to make her services more attractive than that of the contributor for all consumer types, then all purchasing customers contract with the originator as is illustrated in panel (a) of Figure 2. On the other hand, if neither firm's price is too high and the originator's price is not too low to drive the contributor

out of the market then the two firms share the services market, with the contributor serving the lower end of the user population. This consumer market structure can be seen in panel (b) of Figure 2. Finally, if the contributor's price is sufficiently low and the originator's price is sufficiently high, then despite the originator having the higher quality, all customers who use the software prefer to contract with the contributor and the originator is out of the market. This can be seen in panel (c) of Figure 2. The outcome for the case when the contributor is the quality leader has exactly the same structure except the roles for the originator and the contributor are reversed.

4.2.2 Pricing in the Consumer Market

Given their respective effort levels, e_o^O and e_c^O, in the last stage of the game, the firms compete in the services market by setting their respective prices to maximize their profits taking each other's prices as given. The Nash equilibrium outcome of this final stage game yields the equilibrium service prices denoted by $p_o^{O*}(e_o^O, e_c^O)$ and $p_c^{O*}(e_o^O, e_c^O)$, which are functions of the effort levels the firms chose in the earlier stages. The following proposition describes how the consumer pricing equilibrium emerges and shapes up the market (full mathematical characterization of the prices are given in the proof of the proposition in the online supplement).[10]

Proposition 5 *Given the effort levels of the originator and contributor, the equilibrium software and services pricing outcomes characterize the market as follows:*

(i) If either the originator or contributor serves as the quality leader, then both firms are active in the services market, each serving a positive mass of the consumer population. The equilibrium prices switch symmetrically when the role as quality leader is reversed.

(ii) If neither firm serves as quality leader (i.e., they offer equivalent total quality), then the firms engage in Bertrand competition which gives rise to both firms pricing at cost in equilibrium.

Proposition 5 presents an interesting statement: When the firms' service qualities are not equal, in equilibrium, the originator and the contributor set prices such that both firms serve a positive market share

[10]Part (i) of Proposition 5 refers to the similar equilibrium outcome as in Gabszewicz and Thisse (1979). Please see the online supplement for more on the technical characterization of this equilibrium.

of the consumer population. That is, in equilibrium, among all possible regions demonstrated in Figure 2, only the outcome illustrated in panel (b) where both firms participate in an active duopoly with positive market shares, can emerge. For each firm there is always some benefit of setting a small, positive price to capture a portion of the low end of the consumer population, and hence, none of the firms can force the other one out of the market. As a consequence, in equilibrium, the firms settle to a shared oligopolistic market. On the other hand, if the firms' qualities are equal, then they engage in Bertrand style competition that drives their equilibrium prices down all the way to their cost of providing service, which is zero.

4.2.3 Equilibrium in Investment Efforts in Development

Using the equilibrium outcome for service prices presented in Proposition 5, we can now analyze the equilibrium in effort investment levels for the two firms. Given e_o^O and substituting the equilibrium service prices, we can write the contributor's profit as a function of his effort investment level. Further, we can fully characterize the best response function of the contributor. We provide the complete, detailed expressions for his profit function and his optimal effort level $e_c^{O*}(e_o^O)$ in Lemma OS.3 in the online supplement and summarize them in this section. If the originator is able to benefit from the contributor's effort better than the contributor can himself (i.e., if $s_{cc}^O \leq s_{oc}^O$), or if the contributor's effort level e_c^O is smaller than a threshold, then the originator is the quality leader. Otherwise, the contributor is the quality leader. Further, in both cases, the contributor's profit function is strictly concave in his own effort e_c^O for any given originator's effort level e_o^O.

The contributor's effort investment level critically depends on two factors, namely (i) how strongly the originator can benefit from development efforts to improve her total quality and (ii) the originator's effort level. First, if $s_{cc}^O \leq s_{oc}^O$, then $Q_o^O = s_{oo}^O e_o + s_{oc}^O e_c \geq Q_c^O = s_{co}^O e_o + s_{cc}^O e_c$, i.e., the originator is the quality leader for all effort levels $e_o, e_c \geq 0$. In this case, the contributor chooses an effort level to boost his own quality and captures a lower segment of the market while the originator serves the top tier of the market. A similar situation arises when the originator puts in a significant amount of effort in development. In such an instance, the originator's quality is too high and even if the contributor can exceed the originator's quality by investing sufficient effort, it does not pay off for him to do so; consequently, e_c^O is small and

can even be zero, again resulting in the originator being the quality leader. However, when the originator cannot benefit from her own effort as efficiently (i.e., when s_{oo}^O is sufficiently small), or when the originator's effort level is low, the contributor faces a dilemma: Putting in substantial effort can make him the quality leader in the market but it could also benefit the originator, who needs the extra development effort from the contributor to boost her own quality. On the other hand, the contributor may put forth a lower effort level and remain the quality follower. In such cases, the contributor has to choose between one of these options and can even choose to invest no effort and become the lower quality service provider in order to not benefit the originator at all. As a result, the contributor compares his options and chooses the one that maximizes his profit.

Using our characterization of the contributor's optimal effort level $e_c^{O*}(e_o^O)$ as a best response to the originator's effort level e_o^O, we can turn attention to the originator's effort investment problem. Based on her own effort level, the originator faces two separate possibilities. First, she may choose not to invest much effort in development and let the contributor become the quality leader. Second, she may choose to invest a substantial effort and compete as the quality leader. Each of the two cases imply a different profit curve for the originator corresponding to these two different effort regions. The originator then chooses her optimal effort level within each region, and subsequently selects the optimal effort that maximizes her profit comparing those two regions. The resulting outcome again hinges on the development efficiency of the contributor. The following two propositions characterize the equilibrium outcome under an open source software license.

Proposition 6 *If the contributor's development costs are low, i.e. if β_c is sufficiently small, then, in equilibrium, the contributor becomes the quality leader. In this case, if the contributor' ability to benefit from his own effort (s_{cc}^O) is sufficiently lower that the originator's ability to benefit from that effort (s_{oc}^O), then the originator invests no effort in development beyond the minimal release of the software, i.e., $e_o^{O*} = 0$. Otherwise, the originator invests a positive effort in development. The contributor serves the top segment of the market and the originator serves a middle segment.*

An illustration of how equilibrium decisions are made in the case of Proposition 6 (for small β_c) is

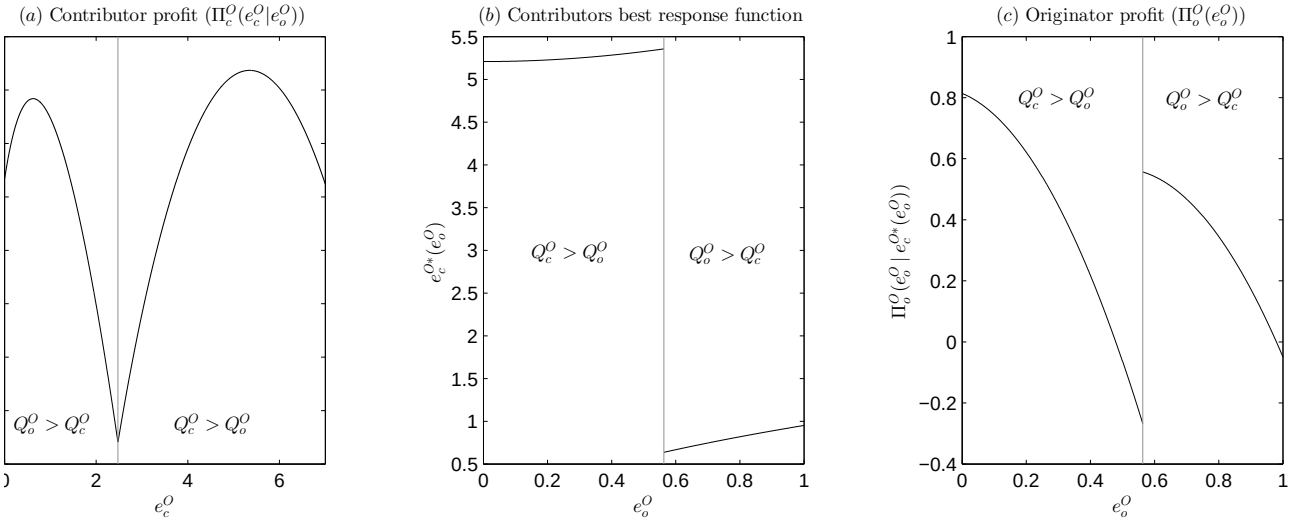

Figure 3: The contributor's profit given the originator's effort, the best response function of the contributor, and the originator's profit.

presented in Figure 3. The parameter values are $s_{oo}^O = 10$, $s_{oc}^O = 8$, $s_{cc}^O = 10$, $s_{co}^O = 1$, $\beta_o = 5$, and $\beta_c = 0.15$. Panel (a) demonstrates the contributor's profit maximization: Given the effort level e_o^O of the originator, the contributor chooses whether to invest sufficient effort to become quality leader ($Q_c^O > Q_o^O$) or to invest less effort and become the low quality service provider ($Q_o^O > Q_c^O$). In this example, the contributor chooses $e_c^{O*}(e_o^O)$ as the interior maximizer of the region in the right-hand portion of panel (a), which leads to the contributor serving as quality leader. Panel (b) shows the best response function of the contributor to the originator's effort investment level. As the originator's effort becomes larger, beyond a certain point, the contributor's optimal effort level jumps down from the maximizer of the case where he is quality leader to the maximizer of the case where he serves as the lower quality provider. Finally, in the first stage of the game, taking the contributor's best response $e_c^{O*}(e_o^O)$ into account, the originator chooses her equilibrium development investment level e_o^{O*} to maximize her profit. In the case illustrated in panel (c) of Figure 3, since β_c is small, the originator chooses not to invest beyond the minimal release of the software (i.e., $e_o^{O*} = 0$), effectively inducing the contributor to invest high and become the quality leader.

In contrast, if the contributor is not very cost efficient in development, the originator becomes the quality leader. The following proposition summarizes the outcome.

Proposition 7 *If the contributor's development costs are high, i.e. if β_c is sufficiently large, then the originator invests a positive effort in development and becomes the quality leader, serving the top segment*

of the consumer population. The contributor as the lower quality service provider is still active and serves a middle consumer segment.

In equilibrium, both firms are active in the consumer services market. This market structure corresponds to the one demonstrated in panel (b) of Figure 2. The lowest segment of the consumer market with types below those that contract with the originator and the contributor does not use the software.

5 Efficiency, Licensing, and Welfare

One argument favoring an open source approach is that there are significant potential gains if the efforts of outside developers are harnessed to develop a superior product. This strategy leverages the input from external contributors to identify and correct bugs and vulnerabilities, as well as govern software design and future direction. Notwithstanding these benefits, the options for a software vendor to generate revenues become limited. Openness of the code essentially means that the software product is free, therefore any consumer surplus that arises as a direct result of value derived from the base product itself is difficult to appropriate. The success of an open source business model must then lie in its ability to associate more of the intrinsic value of the software with the services component. However, the software originator still must compete for the appropriation of this surplus in a contested services market. In this section, we investigate how development cost efficiency impacts the profitability of an open approach and, consequently, the originator's optimal strategy.

Proposition 8 *When contributor's development costs (β_c) are sufficiently low, an open source strategy is attractive for the originator, whereas when contributor's development costs are high, a proprietary strategy is attractive.*

When the contributor's development cost efficiency is high, i.e., when β_c is low, an open source strategy is attractive for the originator because she can utilize his contributions which will significantly increase the quality of her offering through cross-effects. However, since the contributor's development costs are low, under an open source policy, he is also a stronger competitor for the originator in the services market, which is her only source of revenue under an open-source strategy. Proposition 8 states that when the

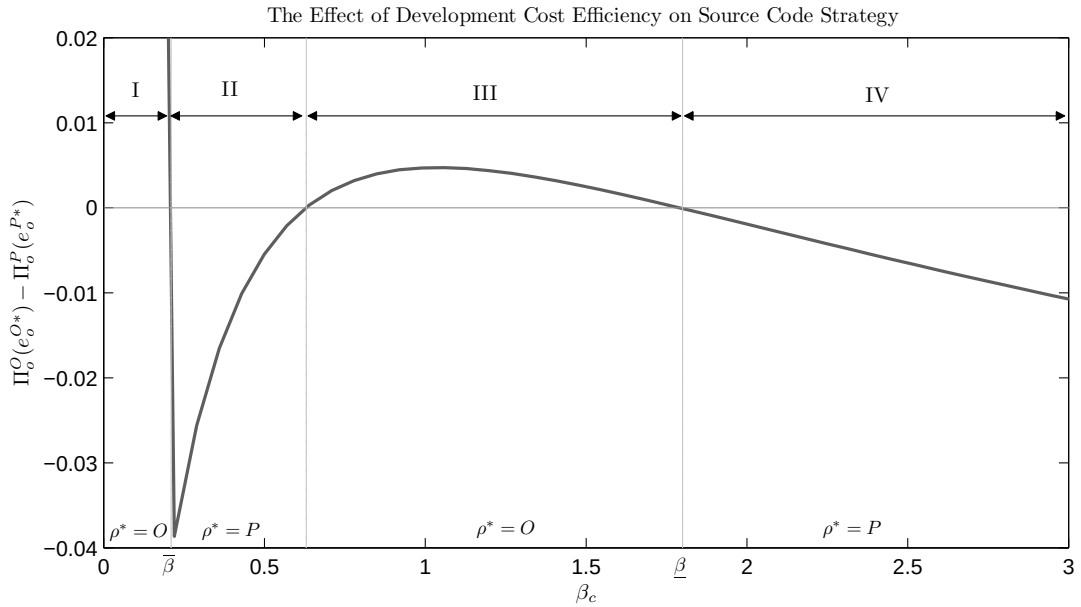

Figure 4: The impact of contributor cost efficiency on the originator's software source code strategy.

contributor is efficient enough in software development, the originator's gains from this outside contribution dominate the losses and reduced profitability associated with facing a strong and efficient competitor. On the other hand, when the contributor's development costs are high, i.e., when β_c is high, his development efforts will be limited, and the benefits the originator can obtain from opening the source code are modest in comparison to the revenue she will lose from not being able to charge for the software itself. Therefore, as is also stated in Proposition 8, for low contributor development efficiency, the optimal policy for the originator is to keep the software proprietary.

However, increased contributor efficiency does not always imply a strategic shift towards an open source policy and vice versa. In fact, increasing contributor efficiency can make a proprietary policy preferable for the originator, as can be seen in Figure 4. This figure plots the difference between the equilibrium profits under an open-source strategy and a proprietary strategy ($\Pi_o^O(e_o^{O*}) - \Pi_o^P(e_o^{P*})$) as a function of the contributor's development cost efficiency parameter (β_c). Thus, in this figure, the positive side of the y-axis indicates that an open-source source strategy is more profitable than a proprietary strategy for the originator, while the negative side of the y-axis reflects that a proprietary strategy is more profitable. Other parameter values in the figure are $s_{oo}^O = 10$, $s_{oc}^O = 8$, $s_{cc}^O = 10$, $s_{co}^O = 1$, $s_{co}^P = s_{cc}^P = 0$, and $\beta_o = 5.0$.

As we discussed above, it is optimal for the software originator to employ an open-source strategy when the contributor's development costs are sufficiently low as stated in Proposition 8 and as shown as Region

I in the figure. What is further interesting is that the originator chooses the open source strategy *when* the contributor's effort benefits himself more than the originator, i.e., $s_{cc}^O > s_{oc}^O$. In fact, in equilibrium, the contributor incurs substantial investment in development and becomes the quality leader in the market for services, while the originator serves the lower tier of the market. Starting from a low β_c, the figure illustrates that a decrease in contributor efficiency (i.e., an increase in β_c) makes the proprietary strategy more preferable. Specifically, as the contributor's development cost (β_c) increases as can be seen in Region II, under an open source policy, neither the contributor has incentives to invest heavily to benefit the originator, nor can the originator make enough revenues in the services market. Instead, the contributor utilizes his cost efficiency to invest in development up to a point where his quality offering is quite attractive relative to the originator's offering, but not high enough so there is still strong price competition to hurt the profits. Therefore, the open source strategy is unattractive to the originator and she chooses the software to stay proprietary.

However, as contributor efficiency even further decreases, i.e., the contributor's development cost (β_c) increases, as can be seen in Region III, an open source strategy can again become optimal. In this region, the originator can establish enough differentiation as the quality leader to make an open source strategy more profitable than solely incurring the entire investment along the proprietary path. As a consequence, decreased contributor efficiency makes the open source policy optimal for the originator. But, as we also discussed above, a further increase in β_c reduces the potential quality improvement coming from the contributor under an open-source strategy and makes the proprietary path preferable for the originator, as also stated in Proposition 8 and illustrated in Region IV in Figure 4.

Proposition 8 and the discussion above provide insight into how the contributor's cost efficiency impacts the originator's source code strategy. Further, under a resultant open source strategy, i.e., when the contributor's development costs are sufficiently low, the originator's development cost efficiency has interesting implications on the equilibrium outcome. The following proposition presents these results.

Proposition 9 *When the contributor's development costs (β_c) are sufficiently low and the originator cannot benefit strongly from the contributor's effort (i.e., when s_{oc}^O is low), increased costs of development for the originator (i.e., increased β_o) can decrease the originator's service price and increase the contributor's*

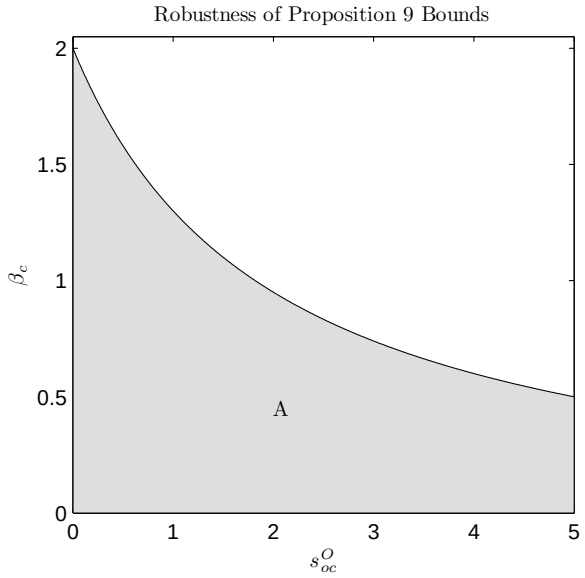

Figure 5: Numerical demonstration of the region of applicability for Proposition 9.

service price and profits.

When determining the effect of the originator's development efficiency on the equilibrium outcome, several factors play important roles. First, as a direct effect, as the originator becomes less cost efficient, i.e., as β_o increases, she invests less effort in development and thus offers a lower quality package in the services market. As stated in Proposition 9, the originator accordingly decreases her price in equilibrium. However, counteracting strategic factors make the net effect of this decreased development efficiency on the contributor bi-directional. First, increased originator development costs reduce the effort the originator puts toward improving software quality. Hence, the total quality of the contributor's offering decreases which places downward pressure on the contributor's price. On the other hand, a decrease in originator effort and her total quality can also imply improved differentiation between the offerings of the two competing firms. Consequently, the firms face relaxed price competition in the services market as they vie for contracts. In particular, if the power of the originator to harness her own investment in the services market (s_{oo}^O) is relatively high compared to that of the contributor to harness the originator's effort (s_{co}^O), the second effect can dominate so that, in equilibrium, the contributor's price increases with decreased originator efficiency. In this case, since the originator cannot benefit much from the contributor's efforts, i.e., since s_{oc}^O is relatively small, the contributor becomes the quality leader in equilibrium (the equilibrium quality levels satisfy $Q_c^O > Q_o^O$). As can be seen in Proposition 9, the same effects are carried to the equilibrium

profits as well; the originator is worse off, despite the fact that, in such a case, she enjoys reduced intensity of price competition due to increased differentiation in the services market. The net effect of decreased originator efficiency can be either positive or negative with respect to contributor profits.

One final thing to note here is that, as demonstrated in Figure 5, Proposition 9 is applicable on a broad parameter region. In Figure 5, the gray shaded area, labeled A, is delineated by the parameter bounds and indicates the region (in terms of β_c and s_{oc}^O) where the proposition statement is applicable. The other parameter values used in the figure are $s_{oo}^O = 10$, $s_{co}^O = 1$, $s_{cc}^O = 10$, and $\beta_o = 5$. As can be seen in the figure, unless either one of β_c and s_{oc}^O becomes quite high, the results of the proposition are applicable.

It is also important to consider the welfare implications of software policy. Social welfare is the sum of consumer surplus and the profits of the firms in the market. We denote the quality of the higher quality firm in equilibrium by Q_H and that of the lower quality firm by Q_L (i.e., if $Q_o \geq Q_c$ then $Q_H = Q_o$ and $Q_L = Q_c$, and vice-versa). Similarly, we define p_H as the service price of the higher quality firm and p_L as the service price of the lower quality firm (and recall that the software purchase price is denoted by p, which is 0 if the software is open source). Finally, we define θ_H as the lowest consumer type $\theta \in [0, 1]$ that contracts service from the higher quality firm and θ_L as the lowest consumer type that contracts service from the lower quality firm ($0 \leq \theta_L \leq \theta_H \leq 1$). That is, the consumers with types above θ_H contract service with the quality leader, and the consumers with types between θ_L and θ_H contract service with the low quality firm. In case there is only one firm providing service in the market in equilibrium, let $\theta_L = \theta_H$. Then the consumer surplus, CS, can be calculated as

$$CS \quad = \quad \int_{\theta_H}^{1} (Q_H\theta - p - p_H)d\theta + \int_{\theta_L}^{\theta_H} (Q_L\theta - p - p_L)d\theta \,. \tag{3}$$

Hence, adding the originator's and contributor's profits, for different licensing strategies $\rho \in \{P, O\}$, total welfare is given by

$$W(e_o, e_c) = CS + \Pi_o^\rho + \Pi_c^\rho \quad = \quad CS + p(1 - \theta_L) + p_H(1 - \theta_H) + p_L(\theta_H - \theta_L) - \frac{1}{2}\beta_c e_c^2 - \frac{1}{2}\beta_o e_o^2 \,. \tag{4}$$

We denote the equilibrium welfare under a proprietary strategy and an open source strategy with $W^P = W(e_o^{P*}, e_c^{P*})$ and $W^O = W(e_o^{O*}, e_c^{O*})$, respectively.

Openness of source code can boost the value software generates for the economy because it can lead to increased software quality; open source software development comes not only from the originator but also from contributors as well. Therefore, governments or regulating bodies may often have incentives to help promote greater contributions to open source development. However, in certain cases, the presence of a potential strong contributor may come with its downsides. The following proposition states this result.

Proposition 10 *An increase in the efficiency of the contributor in both development costs (smaller β_c) and improving the quality of his service (larger s_{cc}^P) may actually result in welfare losses by inducing the originator to strategically choose proprietary licensing instead of open source licensing.*

In order to illustrate the intuition and the meaning behind Proposition 10, consider two cases: (i) the contributor's development costs are low and his ability to improve the quality of his service under the proprietary regime is high; (ii) keeping everything else constant, the contributor's development costs are high and his ability to improve the quality of his service under the proprietary regime is low. Denote the resulting welfare under case (i) by $W^{(i)}$ and the welfare under case (ii) by $W^{(ii)}$. First notice that in terms of direct effects, case (i) is strictly more favorable for welfare generation than case (ii). This is because both a reduction of development costs and an increase in the rate that development efforts generate software quality would be expected to directly contribute to the welfare. Yet, although a stronger contributor is expected to generate substantial value by improving quality, the strategic implications of having such a strong contributor can significantly impact the social benefit generated as stated in the proposition. Specifically, for case (i), with a stronger contributor, the originator chooses to keep the software proprietary while for case (ii), with a weaker contributor, she chooses to open the software. As a result, social welfare improves in case (ii), i.e., $W^{(ii)} = W^O(e_o^{O*}, e_c^{O*}) > W^P(e_o^{P*}, e_c^{P*}) = W^{(i)}$.

To see the intuition here, suppose the contributor is strong and more capable of harnessing his own development efforts to improve his own quality, i.e., $s_{cc}^O > s_{oc}^O$. Then he can capture significant market share of services under the open-source licensing strategy and since there are no revenues from selling the

software itself when the software is open, the originator's profits can suffer by opening up the software. If in addition the contributor can harness his own efforts sufficiently well under a proprietary licensing strategy, (i.e., if s_{cc}^P is sufficiently high) so that he would have strong incentives to contribute, the originator can choose to keep the software proprietary. This is because, in such a case, the originator can collect revenues from selling the software while allowing the contributor to be the quality leader - such an approach enables sufficient differentiation between the service offerings due to high benefits the contributor reaps from his own efforts (i.e., high s_{cc}^P), avoiding cutthroat price competition. On the other hand, in the presence of a relatively weaker contributor, provided that the contributor's development of the open source software does not benefit himself too disproportionately in comparison to the originator, the originator can choose to open the source code. This is because the originator can benefit from the development efforts of the contributor under the open-source strategy while avoiding intense price competition for services due to lack of sufficient differentiation under the proprietary scenario that would arise because s_{cc}^P is low. In summary, since the originator chooses the proprietary licensing strategy under case (i), social welfare can suffer due to loss of value generated for all users by obstructed open availability of the contributor's development efforts and the reduced consumer usage caused by the requirement to pay for the software. Therefore, contrary to what one might expect, a stronger potential contributor may, in fact, be worse for welfare.

6 Discussion

In this paper, we studied the effect of profit-motivated service provision in open source software. We examined a software originator firm's proprietary versus open source licensing problem, facing a contributor firm that can become a competitor. In this context, we explored the effects of several factors including the originator and contributor development efficiencies and the ability of the originator and contributor firms to harness development efforts in improving the quality of their offerings.

Previous literature has studied various aspects of economic incentives of open source software. One dimension related to our study is motivations of individual contributors to open source projects. Studies have identified career and ego-gratification incentives as well as skill and aptitude signaling factors (Lerner

and Tirole 2002), and have demonstrated that paid participation and status motivations improve individuals' development contribution levels to the software (Roberts et al. 2006). In contrast, our model focuses on the incentives of purely profit motivated firms who contribute to open source projects in order to generate revenues from the services market. Therefore, unlike the studies in the previous literature, our model captures the direct monetary profit motivations of contributor firms to a free software project. We explicitly model the collaborative nature of the open-source development efforts and how this gets impacted by the competition in the profit driven services market down the line. Our model enables us to identify some unique strategic motivations that arise from the economic dynamics of this situation. For instance, strong originators of open-source software may, in certain cases, choose not to invest in software development much in order to boost the incentives of potential contributors to invest in development, since low investment and expertise development by the originator creates increased opportunities for contributors to profit from the services market.

Another branch of the existing literature related to our paper is on open source versus proprietary software licensing decisions. Previous studies have found that free-riding can prevent materialization of certain open source software projects (Johnson 2002) and that complementary goods that can generate additional revenues can move incentives towards licensing the software as open-source (Haruvy et al. 2008). In our paper, we explicitly model and study competition from potential contributors in the services market and the effects of endogenous investments in software development, finding that a contributor efficient in development can prevent opening-up of the source code and in fact hurt welfare. There have also been studies that explore competition between firms related to the context of open-source software, but these studies had focused on competition between open-source and proprietary software providers (e.g., Gaudeul 2004b, Casadesus-Masanell and Ghemawat 2006, Lee and Mendelson 2008). Our modeling of the strategic interaction between the software originator and a potential contributor who can simultaneously be a collaborator and competitor also allows us to demonstrate interesting findings such as how increased originator development costs can in certain cases reduce originator's service prices and may decrease the contributors' profits.

In summary, the contribution of our study is two-fold: First, we present the first model to our knowledge

that explicitly studies the role of the market for services in open source software and the associated economic incentives and dynamics between the software originator and development contributor firms that are purely motivated by profits in the open source domain. This business model from services is fast growing in recent years and is becoming the dominant structure in open source software as empirically documented in Suarez et al. (2012). Second, exploring our theoretical model, this study provides many interesting conclusions that also have practical implications for software firms as well as policy makers as we summarized above. These findings also help open avenues for future, related empirical studies that explore the vast subject of open source software.

One finding from our study is that when the contributor is highly efficient in development, the originator is better off making the software open source. Facing a strong contributor, a proprietary licensing approach may not be very profitable for a relatively weak originator as the development of the software and the quality of service will be limited, lacking open contributions of a strong contributor. However, if the originator takes an open-source approach she can benefit from the improved development of the software significantly even if the strong contributor who can highly invest in the software becomes the quality leader and captures the higher value segment of the services market. In this case, the originator can benefit from the contributor's development efforts and can profitably provide services to a lower tier of users. An example of this outcome is that of Apache Geronimo and IBM. Founders of Geronimo have offered services through Covalent Technologies while IBM continues to make large development investments and takes the quality lead. On the other hand, we also found that if the contributor is not efficient in development relative to the originator, the originator will not benefit much from the contributor's development. In this case the originator is often better off keeping the software proprietary and generating revenues from software sales, while taking the quality lead and squeezing out or marginalizing the contributor in the services market. Microsoft is an example of a strong originator who has long utilized a proprietary strategy for its software product line. Our results suggest that one reason Microsoft may not adopt an open source strategy is because of the difficulty to engender a sizable contribution that it can harness from the developer community. Part of the reason is likely Microsoft's bitter history with the open source movement. However, even Microsoft is continuing to adapt to new paradigms such as open source and cloud computing, and so

pursuit of an open source strategy in the near future is certainly possible with increased relative efficiency of the contributors' software development.

Another interesting finding from our study is that an increase in the contributor efficiency is not always good for welfare. If the contributor is efficient in harnessing the originator's development efforts to improve the quality of his own service and becomes highly efficient in development, the originator can be better off by keeping the software proprietary rather than opening the source code up to give an opportunity to a strong contributor who might dominate the market. In such cases, an increase in contributor development efficiency may result in welfare losses by influencing the originator's source code strategy toward a proprietary one. Therefore, regulators should be careful in their approach toward supporting open source contributors' activities. For instance, in certain cases imposing a small tax on open source development or associated revenues can help decrease prices and increase welfare.

Our study has its limitations and restrictions, which give rise to paths for further analysis and future studies on the topic. Our focus in this study has been the market for services and how the corresponding economic incentives of contributors influence the choice of business model. There are several other factors that can also influence these decisions. For example, the success of an open source model also depends on how well coordination, uncertainty, hijacking, forking, and organization sponsorship are managed (see, e.g., Stewart et al. 2006). An important direction for future research is studying how profit-maximizing open source models are influenced by risks of forking and hijacking, and also the role organizational sponsorship and community support play. Bridging together the insights in this work related to services with those found in the other research streams in open source can substantially increase our overall understanding of this domain.

In this paper, for simplicity and clarity we assumed that the cost of providing the service for the firms was zero. Yet another possible avenue for future research is relaxing this assumption and exploring the effects of the changes in the magnitudes of the service costs on the strategic behavior of the firms. Increases in service costs can shrink the potential profits that could be made from the software market. This can alter the strategic positioning of the firms relative to each other with respect to the qualities of their products and in turn can affect the originator's licensing choice. What is more, governments and policy makers can

directly control the magnitude of the service costs by tax and subsidy policies. An extension of our model that studies the effect of the service costs on the equilibrium can shed light into the potential role of policy making on open source licensing decisions and the corresponding implications for welfare.

In our study, we use asymptotic analysis, which is commonly used in microeconomic studies. The use of asymptotic analysis can be expected here, as in many other studies with microeconomic models, due to the complexity of the problem and its solution characterization (some examples of studies that use this technique are, Li et al. 1987, Laffont and Tirole 1988, MacLeod and Malcomson 1993, Pesendorfer and Swinkels 2000, Muller 2000, Tunca and Zenios 2006, August and Tunca 2006, 2008, Pei et al. 2011 among many others). Miller (2006) and Cormen et al. (2009) give comprehensive treatments of the mathematical techniques used in asymptotic analysis. Please also see August and Tunca (2011) for further details and discussion on the rationale of the use of asymptotic analysis. We have characterized the solution and identified the regions under which the results and policy implications related to our research questions arise. Given the complexity of the setting, the boundaries of these regions do not have explicit functional forms. Such boundaries would normally be implicit and would be characterized as such. The goal of the analysis is identifying the characterization of the regions for applicability in terms of parameter characteristics, which is the focus of our results and our propositions.

In recent decades, open source software has become increasingly more prominent, and the economic landscape that governs its development has evolved significantly. The economics underlying open source software are inherently complex and uniquely multifaceted. Further, robust economic structures and instruments by which open source developers can turn their efforts into revenues have just started to clearly emerge as the path for open source software's evolution. Formal analysis and research is needed to build an understanding of the underlying complex structures. Our analysis aims to provide such a formal approach to gain relevant insights into various aspects of this issue. The insights generated can provide guidance for software firms and policy makers alike and help generate future research to further our understanding and benefit from this valuable concept.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. CNS-0954234.

References

Adamson, C. (2005, Mar.). Welcome to a new world: JBoss world 2005. *O'Reilly ONJava.com*.

August, T. and T. I. Tunca (2006). Network software security and user incentives. *Management Science 52*(11), 1703–1720.

August, T. and T. I. Tunca (2008). Let the pirates patch? An economic analysis of software security patch restrictions. *Information Systems Research 19*(1), 48–70.

August, T. and T. I. Tunca (2011). Who should be responsible for software security? A comparative analysis of liability policies in network environments. *Management Science 57*(5), 934–959.

Bessen, J. (2006). Open source software: Free provision of complex public goods. In J. Bitzer and P. J. Shroder (Eds.), *The Economics of Open Source Software Development*, pp. 57–82. Elsevier.

Broersma, M. (2005, Jun.). Open-source databases find their place in the enterprise. *Techworld*.

Brunelli, M. (2005, Jun.). Open source gaining traction in ERP market. *SearchSAP.com*.

Capek, P., S. Frank, S. Gerdt, and D. Shields (2005). A history of IBM's open-source involvement and strategy. *IBM Systems Journal 44*(2), 249–257.

Casadesus-Masanell, R. and P. Ghemawat (2006). Dynamic mixed duopoly: A model motivated by Linux vs. Windows. *Management Science 52*(7), 1072–1084.

Coar, K. (2006). The open source definition (annotated). `http://www.opensource.org/docs/definition.php`.

Cormen, T. H., C. E. Leiserson, R. L. Rivest, and C. Stein (2009, July). *Introduction to Algorithms* (3 ed.). U.S.A.: The MIT Press.

Farber, D. (2004, Mar.). Six barriers to open source adoption. *ZDNet.com*.

Gabszewicz, J. J. and J.-F. Thisse (1979). Price competition, quality and income disparities. *Journal of Economic Theory 20*(3), 340–359.

Gaudeul, A. (2004a, Sep.). Competition between open-source and proprietary software: the LaTeX case study. Working Paper, University of East Anglia, Norwich, UK.

Gaudeul, A. (2004b, Sep.). Open source software development patterns and license terms. Working Paper, University of East Anglia, Norwich, UK.

Hars, A. and S. Ou (2002). Working for free? Motivations for participating in open-source projects. *International Journal of Electronic Commerce 6*(3), 25–39.

Haruvy, E., S. P. Sethi, and J. Zhou (2008). Open source development with a commercial complementary product or service. *Production and Operations Management 17*(1), 29–43.

Herz, J., M. Lucas, and J. Scott (2006, June). Open technology development: Roadmap plan. U.S. Department of Defense.

Hewlett-Packard (2007). Open source and Linux from HP. `http://opensource.hp.com/`.

Iansiti, M. and G. L. Richards (2006). The business of free software: Enterprise, incentives, investment, and motivation in the open source community. Working Paper, Harvard Business School, Boston, MA.

IBM (2007). IBM 2007 annual report.

Johnson, J. P. (2002). Open source software: Private provision of a public good. *Journal of Economics and Management Strategy 11*(4), 637–662.

Kerstetter, J. (2004, Oct.). Redemption for JBoss's boss. *BusinessWeek*.

Koch, S. and G. Schneider (2002). Effort, co-operation and co-ordination in an open source software project: Gnome. *Information Systems Journal 12*(1), 27–42.

Kogut, B. and A. Metiu (2001). Open-source software development and distributed innovation. *Oxford Review of Economic Policy 17*(2), 248–264.

Kontzer, T. (2005, Apr.). Open-source SugarCRM takes on Siebel, Salesforce.com. *Information Week*.

Kuk, G. (2006). Strategic interaction and knowledge sharing in the KDE developer mailing list. *Management Science 52*(7), 1031–1042.

Laffont, J.-J. and J. Tirole (1988). The dynamics of incentive contracts. *Econometrica 56*(5), 1153–1175.

LaMonica, M. (2005, Jul.). Site offers open-source clearinghouse. *CNET News*.

Lee, D. and H. Mendelson (2008). Divide and conquer: Competing with free technology under network effects. *Production and Operations Management 17*(1), 12–28.

Lerner, J. and J. Tirole (2002). Some simple economics of open source. *Journal of Industrial Economics 50*(2), 197–234.

Lerner, J. and J. Tirole (2005a). The economics of technology sharing: Open source and beyond. *Journal of Economic Perspectives 19*(2), 99–120.

Lerner, J. and J. Tirole (2005b). The scope of open source licensing. *Journal of Law, Economics, & Organization 21*(1), 20–56.

Li, L., R. D. McKelvey, and T. Page (1987). Optimal research for Cournot oligopolists. *Journal of Economic Theory 42*(1), 140–166.

MacLeod, W. B. and J. M. Malcomson (1993). Investments, holdup, and the form of market contracts. *American Economic Review 83*(4), 811–837.

Miller, P. D. (2006). *Applied Asymptotic Analysis*, Volume 75 of *Graduate Studies in Mathematics*. Providence, Rhode Island: American Mathematical Society.

Mockus, A., R. Fielding, and J. Herbsleb (2000). A case study of open source software development: The Apache server. *Proc. 22nd International Conference on Software Engineering*, 263–272. ICSE 2000, Limerick Ireland, June 4-11.

Muller, H. M. (2000). Asymptotic efficiency in dynamic principal-agent problems. *Journal of Economic Theory 91*, 292–301.

Oi, W. Y. (1983). Heterogeneous firms and the organization of production. *Economic Inquiry 21*(2), 147–171.

Pei, P. P.-E., D. Simchi-Levi, and T. I. Tunca (2011). Sourcing flexibility, spot trading, and procurement contract structure. *Operations Research 59*(3), 578–601.

Pesendorfer, W. and J. M. Swinkels (2000). Efficiency and information aggregation in auctions. *American Economic Review 90*(3), 499–525.

Raymond, E. S. (1999, Oct.). *The Cathedral and the Bazaar: Musings on Linux and Open Source by an Accidental Revolutionary* (1 ed.). Sebastopol, CA: O'Reilly & Associates, Inc.

Roberts, J. A., I.-H. Hann, and S. A. Slaughter (2006). Understanding the motivations, participation, and performance of open source software developers: A longitudinal study of the Apache projects. *Management Science 52*(7), 984–999.

Sager, I. (2005, Jul.). The myth of open-source. *BusinessWeek*.

Stewart, K. J., A. P. Ammeter, and L. M. Maruping (2006). Impacts of license choice and organizational sponsorship on user interest and development activity in open source software projects. *Information Systems Research 17*(2), 126–144.

Suarez, F. F., M. A. Cusumano, and S. J. Kahl (2012). Services and the business models of product firms: An empirical analysis of the software industry. *Management Science*, Forthcoming.

Tunca, T. I. and S. A. Zenios (2006). Supply auctions and relational contracts for procurement. *M&SOM 8*(1), 43–67.

Vance, A. (2009, Jan. 11). A software populist who doesn't do Windows. *New York Times*.

Virijevich, P. (2005, Jun.). LAMP made easy with XAMPP. *Linux.com*.

von Krogh, G. and E. von Hippel (2006). The promise of research on open source software. *Management Science 52*(7), 975–983.

Watson, R. T., M.-C. Boudreau, P. T. York, M. E. Greiner, and J. Donald Wynn (2008). The business of open source. *Communications of the ACM 51*(4), 41–46.

Zhu, K. and Z. Zhou (2012). Lock-in strategy in software competition: Open-source software vs. proprietary software. *Information Systems Research 23*(2), 536–545.

Online Supplement

Licensing and Competition for Services in Open Source Software

A Notation

For the reader's convenience, we summarize our model notation in the following table:

Table 1: Summary of model notation

Notation	Description
ρ	Originator's software strategy ($\rho = P$ for proprietary, and $\rho = O$ for open source)
θ	Consumer type $\theta \in \Theta = [0, 1]$
e_o^ρ	Originator's development effort level
e_c^ρ	Contributor's development effort level
$\beta_o(e_o^\rho)^2/2$	Originator's cost of effort
$\beta_c(e_c^\rho)^2/2$	Contributor's cost of effort
s_{oo}^ρ	Impact coefficient of originator's effort on her own total quality ($s_{oo}^P = s_{oo}^O$ and is independent of the originator's software strategy)
s_{cc}^ρ	Impact coefficient of contributor's effort on his own total quality
s_{oc}^ρ	Cross-firm impact coefficient of contributor's effort on originator's total quality ($s_{oc}^P = 0$ for proprietary)
s_{co}^ρ	Cross-firm impact coefficient of originator's effort on contributor's total quality
Q_o^ρ	Total quality of the originator's offering (software and service) ($Q_o^O = s_{oo}^O e_o^O + s_{oc}^O e_c^O$ for open source, and $Q_o^P = s_{oo}^P e_o^P$ for proprietary)
Q_c^ρ	Total quality of the contributors offering (software and service) ($Q_c^O = s_{co}^O e_o^O + s_{cc}^O e_c^O$ for open source, and $Q_c^P = s_{co}^P e_o^P + s_{cc}^P e_c^P$ for proprietary)
p_o^ρ	Price charged by the originator for each unit serviced
p_c^ρ	Price charged by the contributor for each unit serviced
p^P	Price charged by the originator for each copy of the software (proprietary case)
$V^P(\theta)$	Net payoff to purchasing consumer with type θ under proprietary strategy ($Q_o^P\theta - p^P - p_o^P$ if serviced by originator, $Q_c^P\theta - p^P - p_c^P$ if serviced by contributor)
$V^O(\theta)$	Net payoff to purchasing consumer with type θ under open source strategy ($Q_o^O\theta - p_o^O$ if serviced by originator, $Q_c^O\theta - p_c^O$ if serviced by contributor)

B Technical Statements and Proofs for Lemmas and Propositions

Lemma OS.1 *For fixed p^P, p_o^P, p_c^P, e_o^P and e_c^P,*

(i) If $Q_o^P = s_{oo}^P e_o^P > Q_c^P = s_{co}^P e_o^P + s_{cc}^P e_c^P$, then the originator is the quality leader and the consumer market has the following characterization of regions:

Region I: If $p_o^P \geq Q_o^P - p^P$ and $p_c^P \geq Q_c^P - p^P$, then no consumer uses the software.

Region II: If $p_o^P < Q_o^P - p^P$ and $p_c^P \geq \frac{p_o^P Q_c^P - p^P(Q_o^P - Q_c^P)}{Q_o^P}$, then only the originator is active in the market, and

(a) consumers with $\theta \in [0, \frac{p^P + p_o^P}{Q_o^P}]$ do not use the software, and

(b) consumers with $\theta \in (\frac{p^P + p_o^P}{Q_o^P}, 1]$ purchase the software from the originator and contract with the originator for the services;

Region III: If $p_o^P < Q_o^P - p^P$ and $p_o^P - (Q_o^P - Q_c^P) \leq p_c^P < \frac{p_o^P Q_c^P - p^P(Q_o^P - Q_c^P)}{Q_o^P}$, then both the originator and the contributor are active in the service market, and

(a) consumers with $\theta \in [0, \frac{p^P + p_c^P}{Q_c^P}]$ do not use the software,

(b) consumers with $\theta \in (\frac{p^P + p_c^P}{Q_c^P}, \frac{p_o^P - p_c^P}{Q_o^P - Q_c^P}]$ purchase the software from the originator and contract with the contributor for the services, and

(c) consumers with $\theta \in (\frac{p_o^P - p_c^P}{Q_o^P - Q_c^P}, 1]$ purchase the software from the originator and contract with the originator for the services;

Region IV: If either $p_o^P < Q_o^P - p^P$ and $p_c^P < p_o^P - (Q_o^P - Q_c^P)$, or $p_o^P \geq Q_o^P - p^P$ and $p_c^P < Q_c^P - p^P$, then only the contributor is active in the service market, and

(a) consumers with $\theta \in [0, \frac{p^P + p_c^P}{Q_c^P}]$ do not use the software, and

(b) consumers with $\theta \in (\frac{p^P + p_c^P}{Q_c^P}, 1]$ purchase the software from the originator and contract with the contributor for the services.

(ii) If $Q_o^P < Q_c^P$, then the contributor is the quality leader and the consumer market has the same characterization as in part (i), except only for switching Q_o^P and Q_c^P, and p_o^P and p_c^P in all the expressions.

(iii) If $Q_o^P = Q_c^P$, then

(a) If $p_o^P < p_c^P$, then consumers with $\theta \in [0, \min\{(p^P + p_o^P)/Q_o^P, 1\}]$ do not use the software, and consumers with $\theta \in (\min\{(p^P + p_o^P)/Q_o^P, 1\}, 1]$ purchase and contract with the originator.

(b) If $p_o^P > p_c^P$, consumers with $\theta \in [0, \min\{(p^P + p_c^P)/Q_c^P, 1\}]$ do not use the software, and consumers with $\theta \in (\min\{(p^P + p_c^P)/Q_c^P, 1\}, 1]$ purchase from the originator and contract with the contributor for the services.

(c) If $p_o^P = p_c^P$, the consumers with $\theta \in [0, \min\{(p^P + p_o^P)/Q_o^P, 1\}]$ do not use the software, and consumers with $\theta \in (\min\{(p^P + p_o^P)/Q_o^P, 1\}, 1]$ purchase from the originator and are indifferent between contracting with the two firms and randomly select a provider for the services.

Proof: For part (i), suppose $Q_o^P > Q_c^P$. By (1), a consumer with type θ,

(a) prefers to contract with the originator for services and to purchase the source code from the originator rather than to not use the software if and only if $Q_o^P\theta - p^P - p_o^P \geq 0$, i.e., $\theta \geq \theta_{oN}^P$, where θ_{oN}^P is defined as $(p^P + p_o^P)/Q_o^P$;

(b) prefers to contract with the contributor for services and to purchase the source code from the originator rather than to not use if and only if $Q_c^P\theta - p^P - p_c^P \geq 0$, i.e., $\theta \geq \theta_{cN}^P = (p^P + p_c^P)/Q_c^P$; and

(c) prefers to contract with the originator rather than the contributor, for services after purchasing the source code from the originator if and only if $Q_o^P\theta - p^P - p_o^P \geq Q_c^P\theta - p^P - p_c^P$, i.e., $\theta \geq \theta_{oc}^P = (p_o^P - p_c^P)/(Q_o^P - Q_c^P)$.

Now suppose $\theta_{oN}^P = (p^P + p_o^P)/Q_o^P \leq \theta_{cN}^P = (p^P + p_c^P)/Q_c^P$, then by carrying out the algebra, it follows that $\theta_{oc}^P = (p_o^P - p_c^P)/(Q_o^P - Q_c^P) \leq \theta_{oN}^P = (p^P + p_o^P)/Q_o^P \leq \theta_{cN}^P = (p^P + p_c^P)/Q_c^P$. In this case, all consumers who prefer to contract with the contributor for services and to purchase the software from the originator rather than to not use, i.e., consumers with $\theta \geq \theta_{cN}^P$, prefer to contract with the originator for services rather than the contributor since for those consumers, $\theta \geq \theta_{cN}^P \geq \theta_{oc}^P$. Thus, in this region, there are only two possible consumer choice outcomes: a consumer either purchases the software and contracts with the originator for the service or does not use the software. Consequently, if $\theta_{oN}^P \geq 1$, i.e., if $p_o^P \geq Q_o^P - p^P$ then no consumer uses the software. Notice that in this case, if $p_o^P \geq Q_o^P - p^P$, it also means that $p_c^P \geq Q_c^P - p^P$, since $\theta_{oN}^P = (p^P + p_o^P)/Q_o^P \leq \theta_{cN}^P = (p^P + p_c^P)/Q_c^P$. That is, in *Region I*, as defined in part (i), no consumer uses the software. On the other hand, if $\theta_{oN}^P < 1$, i.e., if $p^P \leq Q_o^P - p_o^P$, then consumers with types $\theta \in (\theta_{oN}^P, 1]$ contract with the originator for services and purchase the software from the originator, and the rest, i.e., those with $\theta \in [0, \theta_{oN}^P]$ do not use the software. Again notice that $(p^P + p_o^P)/Q_o^P \leq (p^P + p_c^P)/Q_c^P$ implies $p_c^P \geq (p_o^P Q_c^P - p^P(Q_o^P - Q_c^P))/Q_o^P$, i.e., *Region II* as defined above, therefore the statement for this region is also confirmed.

Next, suppose that $\theta_{oN}^P = (p^P + p_o^P)/Q_o^P \geq \theta_{cN}^P = (p^P + p_c^P)/Q_c^P$. Then we have $\theta_{oc}^P = (p_o^P - p_c^P)/(Q_o^P - Q_c^P) \geq \theta_{oN}^P = (p^P + p_o^P)/Q_o^P \geq \theta_{cN}^P = (p^P + p_c^P)/Q_c^P$. Thus, in this case, if $\theta \leq \theta_{oc}^P = (p_o^P - p_c^P)/(Q_o^P - Q_c^P)$, then a consumer with type θ always prefers to contract with the contributor for the services given that she chooses to purchase the software. This means that if $\theta_{oc}^P < 1$, then consumers with types $\theta \in (\theta_{oc}^P, 1]$ contract with the originator for services after purchasing the software from the originator, consumers with types $\theta \in (\theta_{cN}^P, \theta_{oc}^P]$ contract with the contributor for services after purchasing the software from the originator, and the rest with types $\theta \in [0, \theta_{cN}^P]$ do not use the software. Notice that since $Q_o^P > Q_c^P$, $\theta_{oc}^P < 1$ if and only if $p_o^P - (Q_o^P - Q_c^P) \leq p_c^P$. Therefore, if $\theta_{oN}^P = (p^P + p_o^P)/Q_o^P \geq \theta_{cN}^P = (p^P + p_c^P)/Q_c^P$, that is, if $p_c^P < (p_o^P Q_c^P - p^P(Q_o^P - Q_c^P))/Q_o^P$, together with $p_o^P - (Q_o^P - Q_c^P) \leq p_c^P$, i.e., in *Region III*, then the consumer market for services will be split as described in the statement of the lemma. Note that $p_o^P - (Q_o^P - Q_c^P) < (p_o^P Q_c^P - p^P(Q_o^P - Q_c^P))/Q_o^P$ implies that $p_o^P < Q_o^P - p^P$. On the other hand, if either $p_o^P < Q_o^P - p^P$ and $p_c^P < p_o^P - (Q_o^P - Q_c^P)$, or $p_o^P \geq Q_o^P - p^P$ and $p_c^P < Q_c^P - p^P$, i.e., in *Region IV*, then no consumer with type $\theta \in [0, 1]$ will prefer to contract with the originator over the contributor for services, and hence consumers with types $\theta \in (\theta_{cN}^P, 1]$ contract with the contributor for services after purchasing the

software, and the rest with types $\theta \in [0,\, \theta^P_{cN}]$ do not use the software as stated. This proves part (i). Part (ii) follows in exactly similar manner except the originator and the contributor are switched.

For part (iii), when $Q^P_o = Q^P_c$, there is no product differentiation and, as in a standard Bertrand competition case for services, all consumers prefer the lower priced service if there is a difference in service prices. If the prices are the same, all consumers are indifferent and they choose between the two service providers randomly. This completes the proof. □

Proof of Proposition 1: Technically, we will prove that, given e^P_o and e^P_c,

(i) If $Q^P_o(= s^P_{oo}e^P_o) > Q^P_c(= s^P_{co}e^P_o + s^P_{cc}e^P_c)$, then $p^{P*} = \frac{s^P_{oo}e^P_o}{2}$, $p^{P*}_o = 0$, and $p^{P*}_c = 0$;[1]

(ii) If $Q^P_o(= s^P_{oo}e^P_o) \le Q^P_c(= s^P_{co}e^P_o + s^P_{cc}e^P_c)$, then $p^{P*} = \frac{(2s^P_{co}+s^P_{oo})e^P_o + 2s^P_{cc}e^P_c}{6}$, $p^{P*}_o = -\frac{(s^P_{co}-s^P_{oo})e^P_o + s^P_{cc}e^P_c}{3}$ and $p^{P*}_c = \frac{(s^P_{co}-s^P_{oo})e^P_o + s^P_{cc}e^P_c}{3}$.

We focus on the case in which the originator is the quality leader, i.e., $Q^P_o(= s^P_{oo}e^P_o) > Q^P_c = (s^P_{co}e^P_o + s^P_{cc}e^P_c)$. We start by writing the profit functions for the originator. Using Lemma OS.1, in the pricing stage, given the investment levels e^P_o and e^P_c, the originator's profit function (excluding the investment cost) can be written as follows (the indicated regions are as defined in part (i) of Lemma OS.1):

If $p^P_c \ge Q^P_c$, then the contributor is squeezed out of the services market, and

$$\breve{\Pi}^P_o(p^P, p^P_o \,|\, p^P_c, e^P_o, e^P_c) = \begin{cases} (p^P + p^P_o)\left(1 - \frac{p^P + p^P_o}{Q^P_o}\right) & \text{if } p^P + p^P_o \le Q^P_o \text{ (Region II)}; \\ 0 & \text{if } p^P + p^P_o > Q^P_o \text{ (Region I)}. \end{cases} \tag{OS.1}$$

On the other hand, if $p^P_c < Q^P_c$, then

$$\breve{\Pi}^P_o(p^P, p^P_o \,|\, p^P_c, e^P_o, e^P_c) = \begin{cases} (p^P + p^P_o)\left(1 - \frac{p^P + p^P_o}{Q^P_o}\right) & \text{if } p^P_o Q^P_c - p^P(Q^P_o - Q^P_c) \le Q^P_o p^P_c,\ p^P + p^P_o \le Q^P_o, \\ & \quad \text{and } p^P_o \le p^P_c + Q^P_o - Q^P_c \text{ (Region II)}; \\ p^P\left(1 - \frac{p^P + p^P_c}{Q^P_c}\right) + p^P_o\left(1 - \frac{p^P_o - p^P_c}{Q^P_o - Q^P_c}\right) & \text{if } p^P_o Q^P_c - p^P(Q^P_o - Q^P_c) > Q^P_o p^P_c,\ p^P + p^P_o \le Q^P_o, \\ & \quad p^P_o \le p^P_c + Q^P_o - Q^P_c, \text{ and } p^P \le Q^P_c - p^P_c \text{ (Region III)}; \\ p^P\left(1 - \frac{p^P + p^P_c}{Q^P_c}\right) & \text{if } p^P_o > p^P_c + Q^P_o - Q^P_c, \text{ and } p^P \le Q^P_c - p^P_c \text{ (Region IV)}; \\ 0 & \text{if } p^P > Q^P_c - p^P_c, \text{ and } p^P + p^P_o > Q^P_o \text{ (Region I)}. \end{cases} \tag{OS.2}$$

Similarly, again using Lemma OS.1, the contributor's profit (excluding the effort investment costs) is given as follows:

If $p^P \ge Q^P_c$ or $Q^P_c p^P_o \le (Q^P_o - Q^P_c)p^P$, then the contributor is squeezed out of the market and $\breve{\Pi}^P_c(p^P_c \,|\, p^P, p^P_o, e^P_o, e^P_c) = 0$ (Regions I and II).

[1] Note that in this case, there are multiple pricing equilibria; however, in all equilibria, the equilibrium profit outcomes are the same as those presented in this part of the proposition. Consequently, the resulting equilibrium outcomes for the effort investments are also the same. For clarity, here we choose to present the simplest equilibrium (which corresponds to zero service prices). The full set of equilibria are available from the authors upon request.

However, if $p^P < Q_c^P$, $Q_c^P p_o^P > (Q_o^P - Q_c^P)p^P$ and $p^P + p_o^P \geq Q_o^P$, then the originator is squeezed out of the services market and

$$\breve{\Pi}_c^P(p_c^P \,|\, p^P, p_o^P, e_o^P, e_c^P) = \begin{cases} p_c^P \left(1 - \frac{p^P + p_c^P}{s_{co}^P e_o^P + s_{cc}^P e_c^P}\right) & \text{if } p_c^P \leq s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P \text{ (Region IV);} \\ 0 & \text{if } p_c^P > s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P \text{ (Region I).} \end{cases} \tag{OS.3}$$

Finally, if $p^P < Q_c^P$, $Q_c^P p_o^P > (Q_o^P - Q_c^P)p^P$ and $p^P + p_o^P < Q_o^P$, then

$$\breve{\Pi}_c^P(p_c^P \,|\, p^P, p_o^P, e_o^P, e_c^P) = \begin{cases} p_c^P \left(1 - \frac{p^P + p_c^P}{s_{co}^P e_o^P + s_{cc}^P e_c^P}\right) & \text{if } p_c^P \leq p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P) \text{ (Region IV);} \\ p_c^P \left(\frac{p_o^P - p_c^P}{(s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P} - \frac{p^P + p_c^P}{s_{co}^P e_o^P + s_{cc}^P e_c^P}\right) & \text{if } p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P) < p_c^P \\ & \leq \frac{p_o^P(s_{co}^P e_o^P + s_{cc}^P e_c^P) - p^P((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P)}{s_{oo}^P e_o^P} \text{ (Region III);} \\ 0 & \text{if } \frac{p_o^P(s_{co}^P e_o^P + s_{cc}^P e_c^P) - p^P((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P)}{s_{oo}^P e_o^P} < p_c^P \text{ (Region II).} \end{cases} \tag{OS.4}$$

Note that if the contributor is the quality leader, i.e., if $Q_c^P (= s_{co}^P e_o^P + s_{cc}^P e_c^P) > Q_o^P (= s_{oo}^P e_o^P)$, then the profit expressions will be similar with the originator's quality and the contributor's quality roles reversed. Suppose the originator is the quality leader, i.e., $Q_o^P (= s_{oo}^P e_o^P) > Q_c^P (= s_{co}^P e_o^P + s_{cc}^P e_c^P)$. We first derive the originator's best response function $B_o(p_c^P) = (p^P, p_o^P)$ that gives her optimal software price, p^P, and service price, p_o^P, choices for any service price, p_c^P, chosen by the contributor.

1- If $p_c^P \geq Q_c^P$, the originator's profit function is as given in (OS.1). Writing the first order conditions for p_o^P and p^P from this expression leads to the same equation:

$$1 - \frac{2(p^P + p_o^P)}{Q_o^P} = 0. \tag{OS.5}$$

Notice that the second order conditions are always satisfied. Further,

$$B_o(p_c^P) = \left(p^P,\, p_o^P\right) = \left(\frac{Q_o^P}{2}, 0\right) = \left(\frac{s_{oo}^P e_o^P}{2}, 0\right) \tag{OS.6}$$

satisfy the first order conditions. Thus we conclude that $(p^P,\, p_o^P) = (s_{oo}^P e_o^P/2, 0)$ is the best response price pair for this case.

2- If $p_c^P < Q_c^P$, the originator's profit function is given in equation (OS.2). As can be seen in that equation, there are three regions to consider:

(a) If $p_o^P Q_c^P - p^P(Q_o^P - Q_c^P) \leq Q_o^P p_c^P$, and $p^P + p_o^P \leq Q_o^P$, then we are in Region II, so the profit function is identical to given in (OS.1) and the optimality conditions are as stated in the proof of case (1) above. Thus, $(p^P,\, p_o^P) = (Q_o^P/2, 0)$ is the maximizer of the profit function. We only need to verify that this price pair falls into this region and hence it is the true optimizer in this region. For $(p^P,\, p_o^P) = (Q_o^P/2, 0)$, $p_o^P Q_c^P - p^P(Q_o^P - Q_c^P) = -\frac{Q_o^P}{2}(Q_o^P - Q_c^P) \leq 0 \leq Q_o^P p_c^P$, and $p^P + p_o^P = Q_o^P/2 \leq Q_o^P$. It follows that $(p^P,\, p_o^P) = (s_{oo}^P e_o^P/2, 0)$ is indeed a best response price pair

for this case.

(b) If $p_o^P Q_c^P - p^P(Q_o^P - Q_c^P) > Q_o^P p_c^P$, and $p_o^P \leq p_c^P + Q_o^P - Q_c^P$, then we are in Region III. The maximization problem in this region is separable in p^P and p_o^P, and the profit function $\breve{\Pi}_o^P$ is quadratic in both p^P and p_o^P. Writing the first order conditions for p_o^P and p^P, we obtain

$$1 - \frac{2p^P + p_c^P}{Q_c^P} = 0 \quad \text{and} \quad 1 - \frac{2p_o^P - p_c^P}{Q_o^P - Q_c^P} = 0\,. \tag{OS.7}$$

By solving these first order conditions and noticing that the second order conditions are always satisfied, we obtain the interior best response prices, if feasible, as $p^P = (Q_c^P - p_c^P)/2$ and $p_o^P = (Q_o^P - Q_c^P + p_c^P)/2$. However, at these interior best response prices, $p_o^P Q_c^P - p^P(Q_o^P - Q_c^P) = Q_o^P p_c^P/2 < Q_o^P p_c^P$ holds, and hence, the interior best response prices do not satisfy the conditions for this region and fall outside of it. Therefore, the best response price pairs in this region fall on the boundaries of the region.

(c) If $p_o^P > p_c^P + Q_o^P - Q_c^P$, and $p^P \leq Q_c^P - p_c^P$, then we are in Region IV. In this region, as can be seen in ((OS.2)), the originator's profit $\breve{\Pi}_o^P$ is a concave quadratic function of only p^P, and does not depend on p_o^P. The first order condition is

$$1 - \frac{2p^P + p_c^P}{Q_c^P} = 0 \tag{OS.8}$$

and the second order condition is always satisfied. Then, the interior best response price is $p^P = (Q_c^P - p_c^P)/2$, which always falls into the region, since $p^P \leq Q_c^P - p_c^P$. In this case, the corresponding originator's profit is $\breve{\Pi}_o^P = (Q_c^P - p_c^P)^2/4Q_c^P$.

Now, consider the boundary of Regions II and III. This boundary is defined by the line $p_o^P Q_c^P - p^P(Q_o^P - Q_c^P) = Q_o^P p_c^P$, which is equivalent to

$$\frac{p^P + p_o^P}{Q_o^P} = \frac{p^P + p_c^P}{Q_c^P} = \frac{p_o^P - p_c^P}{Q_o^P - Q_c^P}. \tag{OS.9}$$

Plugging this into the profit functions for Regions II and III given in (OS.2), we see that both profit expressions on this boundary are equal to

$$\frac{Q_o^P(p_o^P - p_c^P)}{Q_o^P - Q_c^P}\left(1 - \frac{p_o^P - p_c^P}{Q_o^P - Q_c^P}\right). \tag{OS.10}$$

That is, the originator's profit function is continuous on this boundary. Similarly, consider the boundary of Regions III and IV. This boundary is defined by the line $p_o^P = p_c^P + Q_o^P - Q_c^P$, which is equivalent to $(p_o^P - p_c^P)/(Q_o^P - Q_c^P) = 1$. Plugging this into the profit functions for Regions III and IV given in (OS.2), we obtain both profit expressions on this boundary as

$$p^P\left(1 - \frac{p^P + p_c^P}{Q_c^P}\right). \tag{OS.11}$$

Consequently, the originator's profit function in (OS.2) is continuous in p^P and p_o^P. Since in Regions II and IV the maximum profit is attained at interior points, the maximum values of these peaks are higher than those of the boundary points of these regions with Region III. But as we mentioned above, Region III maximum is attained at its boundaries. Therefore, since the profit function is continuous, the maximum in Region III is dominated by the maxima of Regions II and IV. In summary, the originator's profit function as described in (OS.2) is maximized at the interior peak of either Region II or Region IV, and the overall maximum can be found by comparing these two. By plugging in the interior maximum best response price pairs found for Regions II and IV above, the corresponding maximum profit levels in Regions II and IV are $Q_o^P/4$ and $(Q_c^P - p_c^P)^2/4Q_c^P$, respectively. Since $Q_o^P > Q_c^P$, $Q_o^P/4 \geq (Q_c^P - p_c^P)^2/4Q_c^P$. Therefore, the optimal price pair for the originator falls into Region II and the best response prices are given as in (OS.6).

Next, we derive the contributor's best response function $B_c(p^P, p_o^P) = p_c^P$ that gives his optimal service price p_c^P choice for any given software price p^P and service price p_o^P chosen by the originator.

1- If $p^P \geq Q_c^P$ or $Q_c^P p_o^P \leq (Q_o^P - Q_c^P)p^P$, then the contributor is squeezed out of the market and his profit equals zero for any non-negative service price p_c^P. Consequently, $B_c(p^P, p_o^P)$ is any non-negative service price p_c^P.

2- If $p^P < Q_c^P$ and $p^P + p_o^P \geq Q_o^P$, then the contributor's profit function is as given in (OS.3). Writing the first order condition for p_c^P corresponding to Region IV gives

$$1 - \frac{p^P + 2p_c^P}{s_{co}^P e_o^P + s_{cc}^P e_c^P} = 0 \qquad \text{(OS.12)}$$

and the second order condition is always satisfied. Solving the first order condition, we obtain the optimal service price for the contributor, which is $(s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P)/2$. Note that it falls into the interior of Region IV, since the condition $p_c^P \leq s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P$ is satisfied. Therefore, in this case, $B_c(p^P, p_o^P) = (s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P)/2$.

3- If $p^P < Q_c^P$, $Q_c^P p_o^P > (Q_o^P - Q_c^P)p^P$ and $p^P + p_o^P < Q_o^P$, then the contributor's profit function is given in equation (OS.4). As can be seen in that equation, there are two regions to consider:

(a) If $p_c^P \leq p_o^P - (Q_o^P - Q_c^P)$, then we are in Region IV, so the profit function is identical to given in (OS.3) and the optimality conditions are as stated in the proof of case (2). Thus, $p_c^P = (s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P)/2$, if falls into this region, is the maximizer of the profit function in this region. Note that the condition for this region, i.e., $p_c^P \leq p_o^P - (Q_o^P - Q_c^P)$, is satisfied at this interior maximizer if and only if $2p_o^P + p^P \geq 2Q_o^P - Q_c^P$. In that case, the best response price is $B_c(p^P, p_o^P) = (s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P)/2$ and the corresponding profit is $(Q_c^P - p^P)^2/4Q_c^P$. Otherwise, i.e., if $2p_o^P + p^P < 2Q_o^P - Q_c^P$, then the profit function is increasing in p_c^P and the best response price in this region is at the upper boundary of the region, i.e., $B_c(p^P, p_o^P) = p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P)$.

(b) If $p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P) < p_c^P \leq (p_o^P(s_{co}^P e_o^P + s_{cc}^P e_c^P) - p^P((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P))/s_{oo}^P e_o^P$, then we are in Region III. In this region, the profit function, as can be seen in (OS.4) is strictly concave

and quadratic in p_c^P. Its first order condition is

$$\frac{p_o^P - 2p_c^P}{(s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P} - \frac{p^P + 2p_c^P}{s_{co}^P e_o^P + s_{cc}^P e_c^P} = 0\,, \tag{OS.13}$$

and the second order condition is always satisfied. Solving the first order condition, we obtain that the interior optimizer, it falls into this region, is given by $p_c^P = ((s_{co}^P e_o^P + s_{cc}^P e_c^P)p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P)p^P)/2s_{oo}^P e_o^P$. Note that at this interior optimizer price p_c^P, the condition $p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P) < p_c^P \le (p_o^P(s_{co}^P e_o^P + s_{cc}^P e_c^P) - p^P((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P))/s_{oo}^P e_o^P$ is satisfied if and only if $(2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P < 2Q_o^P(Q_o^P - Q_c^P)$ is satisfied. In that case, the best response price is $B_c(p^P, p_o^P) = ((s_{co}^P e_o^P + s_{cc}^P e_c^P)p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P)p^P)/2s_{oo}^P e_o^P$ and the corresponding profit is $\frac{(Q_c^P p_o^P - (Q_o^P - Q_c^P)p^P)^2}{4Q_o^P Q_c^P (Q_o^P - Q_c^P)}$. Otherwise, i.e., if $(2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P \ge 2Q_o^P(Q_o^P - Q_c^P)$, then the profit function is decreasing in p_c^P and the best response price in this region falls on the lower boundary of the region, i.e., $B_c(p^P, p_o^P) = p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P)$.

In order to find the overall maximizer of the profit function, we need to compare the maximums in these two regions. There are four subcases:

First, if $2p_o^P + p^P \ge 2Q_o^P - Q_c^P$ and $(2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P \ge 2Q_o^P(Q_o^P - Q_c^P)$, then the interior optimizer exists only in Region IV. In Region III, the maximizer is on the boundary. In addition, the boundary condition $p_c^P = p_o^P - ((s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P)$ is equivalent to $\frac{p_o^P - p_c^P}{(s_{oo}^P - s_{co}^P)e_o^P - s_{cc}^P e_c^P} = 1$, and hence, the contributor's profit function is continuous in p_c^P at this boundary. As a result, the best response price in this case is $B_c(p^P, p_o^P) = (Q_c^P - p^P)/2$.

Second, suppose $2p_o^P + p^P \ge 2Q_o^P - Q_c^P$ and $(2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P < 2Q_o^P(Q_o^P - Q_c^P)$ are both satisfied. In this case, the interior optimizers exist for both Regions III and IV. However, in the relevant region of $p^P + p_o^P < Q_o^P$, if $2p_o^P + p^P \ge 2Q_o^P - Q_c^P$ holds then

$$\begin{aligned}(2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P &= Q_o^P(2p_o^P + p^P) - Q_c^P(p^P + p_o^P) > Q_o^P(2Q_o^P - Q_c^P) - Q_c^P(p^P + p_o^P) \\ &= 2Q_o^P(Q_o^P - Q_c^P) + Q_c^P(Q_o^P - p_o^P - p^P) > 2Q_o^P(Q_o^P - Q_c^P)\,. \end{aligned} \tag{OS.14}$$

Hence, it cannot be the case that Regions III and IV both have interior optima.

Third, if $2p_o^P + p^P < 2Q_o^P - Q_c^P$ and $(2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P \ge 2Q_o^P(Q_o^P - Q_c^P)$, then neither Region III nor IV have interior optima. We have shown above that if there is no interior optimum, then profit is increasing in Region IV and decreasing in Region III. It then follows that the profit is maximized at the boundary point of these two regions, i.e., the best response price is $B_c(p^P, p_o^P) = p_o^P - (Q_o^P - Q_c^P)$.

Fourth and finally, if $2p_o^P + p^P < 2Q_o^P - Q_c^P$ and $(2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P < 2Q_o^P(Q_o^P - Q_c^P)$, then Region III has an interior optimum and Region IV does not, and as we showed above, the profit is increasing in Region IV. Further, as we also showed above, the contributor's profit function is continuous in p_c^P, and hence, the best response price is the interior optimizer of Region III, i.e., $B_c(p^P, p_o^P) = (Q_c^P p_o^P - (Q_o^P - Q_c^P)p^P)/(2Q_o^P)$.

In summary we obtain the contributor's best response function as

$$B_c(p^P,\, p_o^P) = \begin{cases} \frac{Q_c^P - p^P}{2}, & \text{if } 2p_o^P + p^P \geq 2Q_o^P - Q_c^P\,; \\ p_o^P - (Q_o^P - Q_c^P), & \text{if } 2p_o^P + p^P < 2Q_o^P - Q_c^P \text{ and } (2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P \geq 2Q_o^P(Q_o^P - Q_c^P)\,; \\ \frac{Q_c^P p_o^P - (Q_o^P - Q_c^P)p^P}{2Q_o^P}, & \text{if } (2Q_o^P - Q_c^P)p_o^P + (Q_o^P - Q_c^P)p^P < 2Q_o^P(Q_o^P - Q_c^P)\,. \end{cases} \tag{OS.15}$$

The Nash Equilibrium prices can then be found by simultaneously solving the best response functions

$$B_c(p^{P*},\, p_o^{P*}) = p_c^{P*} \text{ and } B_o(p_c^{P*}) = (p^{P*},\, p_o^{P*})\,. \tag{OS.16}$$

Since, $p^P = Q_o^P/2$ and $p_o^P = 0$. Therefore, Case 1 for the contributor's best response function applies. In this case, the contributor is out of the services market at any non-negative price he sets. Therefore, for any $\psi \geq 0$, $(p^{P*},\, p_o^{P*}, p_c^P) = (Q_o^P/2, 0, \psi)$ is an equilibrium profile. In the simplest case where $\psi = 0$, which gives us the equilibrium as stated. This completes the proof of part (i).

On the other hand, if the contributor is the quality leader, the originator's profit function (excluding the investment cost) can be written as

$$\breve{\Pi}_o^P(p^P, p_o^P \,|\, p_c^P, e_o^P, e_c^P) = \begin{cases} (p^P + p_o^P)\left(1 - \frac{p^P + p_o^P}{s_{oo}^P e_o^P}\right) & \text{if } p^P + p_o^P \leq s_{oo}^P e_o^P, \text{ and } p_o^P \leq p_c^P\,; \\ p^P\left(1 - \frac{p^P + p_c^P}{s_{co}^P e_o^P + s_{cc}^P e_c^P}\right) & \text{if } p^P \leq s_{co}^P e_o^P + s_{cc}^P e_c^P - p_c^P, \text{and } p_o^P \geq p_c^P\,; \\ 0 & \text{if } p^P + p_o^P > s_{oo}^P e_o^P, \text{and } p^P > s_{co}^P e_o^P + s_{cc}^P e_c^P - p_c^P\,, \end{cases} \tag{OS.17}$$

and the contributor's profit function (again excluding the effort investment cost) can be written as

$$\breve{\Pi}_c^P(p_c^P \,|\, p^P, p_o^P, e_o^P, e_c^P) = \begin{cases} p_c^P\left(1 - \frac{p^P + p_c^P}{s_{co}^P e_o^P + s_{cc}^P e_c^P}\right) & \text{if } p_c^P \leq s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P, \text{ and } p_c^P \leq p_o^P\,; \\ 0 & \text{if } p_c^P > s_{co}^P e_o^P + s_{cc}^P e_c^P - p^P, \text{ or } p_c^P > p_o^P\,. \end{cases} \tag{OS.18}$$

Following similar steps as in part (i) and using (OS.17) and (OS.18), we can also obtain the equilibrium result presented in part (ii) for the case when the contributor is the quality leader, i.e., $Q_o^P (= s_{oo}^P e_o^P) \leq Q_c^P (= s_{co}^P e_o^P + s_{cc}^P e_c^P)$. The details are omitted here because they are repetitive of the proof of part (i) given above. ■

Proof of Proposition 2: Technically, given $e_o^P \geq 0$, the contributor's profit function can be written as

$$\Pi_c^P(e_c^P \,|\, e_o^P) = \begin{cases} -\frac{\beta_c}{2}(e_c^P)^2 & \text{if } e_c^P \leq \frac{s_{oo}^P - s_{co}^P}{s_{cc}^P} e_o^P\,; \\ \frac{s_{cc}^P e_c^P - (s_{oo}^P - s_{co}^P)e_o^P}{9} - \frac{\beta_c}{2}(e_c^P)^2 & \text{if } e_c^P \geq \frac{s_{oo}^P - s_{co}^P}{s_{cc}^P} e_o^P\,. \end{cases} \tag{OS.19}$$

We will prove that, given $e_o^P \geq 0$ and denoting $e_c^{P*}(e_o^P)$ as the solution to maximizing (OS.19),

(i) If $e_o^P < \bar{e}_o$, then $e_c^{P*}(e_o^P) = \frac{s_{cc}^P}{9\beta_c}$ and $Q_c^P > Q_o^P$;

(ii) If $e_o^P \geq \bar{e}_o$, then $e_c^{P*}(e_o^P) = 0$,

where $\bar{e}_o = (s_{cc}^P)^2/(18\beta_c(s_{oo}^P - s_{co}^P))$. From the contributor's profit given in (OS.19), first, in the region of $e_c^P \leq (s_{oo}^P - s_{co}^P)e_o^P/s_{cc}^P$, the contributor's optimal effort e_c^P is zero with zero as the corresponding profit. In the alternative region where $e_c^P \geq (s_{oo}^P - s_{co}^P)e_o^P/s_{cc}^P$, the effort level that satisfies the first order condition is $s_{cc}^P/9\beta_c$, which falls in the region if and only if $e_o^P < \frac{(s_{cc}^P)^2}{9\beta_c(s_{oo}^P - s_{co}^P)}$. Moreover, at this effort level, the contributor's corresponding profit is $\frac{(s_{cc}^P)^2}{162\beta_c} - \frac{(s_{oo}^P - s_{co}^P)e_o^P}{9}$, which is positive if and only if $e_o^P < \bar{e}_o$. If $e_o^P \geq \frac{(s_{cc}^P)^2}{9\beta_c(s_{oo}^P - s_{co}^P)}$, then the contributor's profit is strictly decreasing on $e_c^P \geq \frac{s_{oo}^P - s_{co}^P}{s_{cc}^P}e_o^P$. Finally, note that at $e_c^P = \frac{(s_{oo}^P - s_{co}^P)e_o^P}{s_{cc}^P}$, both profit expressions become identical to $-\frac{\beta_c(s_{oo}^P - s_{co}^P)^2(e_o^P)^2}{2(s_{cc}^P)^2}$, and hence, the contributor's profit is continuous. Now, since, if $e_o^P < \bar{e}_o$, then $e_o^P < \frac{(s_{cc}^P)^2}{9\beta_c(s_{oo}^P - s_{co}^P)}$, we have that, given $e_o^P < \bar{e}_o$, the maximizer for $e_c^P \geq \frac{s_{oo}^P - s_{co}^P}{s_{cc}^P}e_o^P$ is $s_{cc}^P/9\beta_c$ and the corresponding profit is positive. The optimal profit for $e_c^P \leq \frac{s_{oo}^P - s_{co}^P}{s_{cc}^P}e_o^P$ is zero, and hence the overall maximizer of the contributor's profit is $e_c^{P*}(e_o^P) = s_{cc}^P/9\beta_c$. Further, from (OS.19), since $e_c^{P*}(e_o^P) \geq \frac{s_{oo}^P - s_{co}^P}{s_{cc}^P}e_o^P$, the contributor is the quality leader. This proves part (i). On the other hand, if $e_o^P \geq \bar{e}_o$ the maximum value for the curve $\frac{s_{cc}^P e_c^P - (s_{oo}^P - s_{co}^P)e_o^P}{9} - \frac{\beta_c}{2}(e_c^P)^2$ is negative. This means that the profit in the entire region $e_c^P \geq \frac{s_{oo}^P - s_{co}^P}{s_{cc}^P}e_o^P$ is negative, while the maximum profit in the region $e_c^P < \frac{s_{oo}^P - s_{co}^P}{s_{cc}^P}e_o^P$ is zero which is attained at zero. Hence for this case, the overall profit maximizer for the contributor is $e_c^{P*}(e_o^P) = 0$, i.e., the contributor does not invest in the software and is out of the market. This completes the proof. ■

Proofs of Propositions 3 and 4: The originator's profit function can be written as

$$\Pi_o^P(e_o^P) = \begin{cases} \frac{4s_{co}^P + 5s_{oo}^P}{36}e_o^P - \frac{\beta_o}{2}(e_o^P)^2 + \frac{(s_{cc}^P)^2}{81\beta_c} & \text{if} \quad e_o^P \leq \bar{e}_o\,; \\ \frac{s_{oo}^P e_o^P}{4} - \frac{\beta_o(e_o^P)^2}{2} & \text{if} \quad e_o^P > \bar{e}_o\,, \end{cases} \tag{OS.20}$$

where $\bar{e}_o$ is as given in Proposition 2. Also, define

$$\bar{r} = \frac{2(s_{cc}^P)^2\beta_o}{81(s_{oo}^P - s_{co}^P)(s_{oo}^P)^2}\left(13s_{oo}^P - 4s_{co}^P + 2\sqrt{2(s_{oo}^P - s_{co}^P)(11s_{oo}^P - 2s_{co}^P)}\right). \tag{OS.21}$$

Technically, we will first prove that (i) if $\beta_c < \bar{r}$, then $e_o^{P*} = \frac{5s_{oo}^P + 4s_{co}^P}{36\beta_o}$; and (ii) if $\beta_c \geq \bar{r}$, then $e_o^{P*} = \frac{s_{oo}^P}{4\beta_o}$. Part (i) corresponds to the determination of originator effort level in the statement of Proposition 3, and part (ii) corresponds to the same for Proposition 4.

First suppose that $e_o^P \leq \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$. From (OS.20), the originator's profit function in this region is $\frac{4s_{co}^P + 5s_{oo}^P}{36}e_o^P - \frac{\beta_o}{2}(e_o^P)^2 + \frac{(s_{cc}^P)^2}{81\beta_c}$, which is concave and quadratic in e_o^P. So by solving the first order condition, we find that this curve is maximized at $e_o^P = \frac{5s_{oo}^P + 4s_{co}^P}{36\beta_o}$, and the corresponding originator's profit at this maximizer is

$$\frac{(5s_{oo}^P + 4s_{co}^P)^2}{2592\beta_o} + \frac{(s_{cc}^P)^2}{81\beta_c}. \tag{OS.22}$$

Now, by carrying out the algebra, $\frac{5s_{oo}^P + 4s_{co}^P}{36\beta_o} \leq \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$, i.e., the maximizer falls in the region where the first curve in (OS.20) is valid if and only if $\beta_c < \frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P - s_{co}^P)(5s_{oo}^P + 4s_{co}^P)}$. Since the maximizer is positive, if it does not fall in the valid region, then the originator's profit in this region, which is a quadratic, is increasing

in e_o^P, and hence, the maximizer within $[0, \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}]$ is the upper boundary, i.e., $e_o^P = \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$, and the corresponding originator's profit is

$$\frac{(s_{cc}^P)^2}{648\beta_c(s_{oo}^P - s_{co}^P)}\left(4s_{co}^P + 5s_{oo}^P - \frac{\beta_o(s_{cc}^P)^2}{\beta_c(s_{oo}^P - s_{co}^P)}\right) + \frac{(s_{cc}^P)^2}{81\beta_c}. \tag{OS.23}$$

Now consider the case $e_o^P \geq \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$. Again from (OS.20), the originator's profit function in this region is $\frac{s_{oo}^P e_o^P}{4} - \frac{\beta_o(e_o^P)^2}{2}$, which is also concave and quadratic in e_o^P. Therefore, by solving the first order condition, we obtain the maximizer of this curve as $e_o^P = \frac{s_{oo}^P}{4\beta_o}$, with a corresponding profit of $\frac{(s_{oo}^P)^2}{32\beta_o}$. Again carrying out the algebra, this maximizer falls in the region $e_o^P \geq \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$ if and only if $\beta_c > \frac{2(s_{cc}^P)^2\beta_o}{9s_{oo}^P(s_{oo}^P - s_{co}^P)}$. If the maximizer is not in this region, i.e., if $\beta_c \leq \frac{2(s_{cc}^P)^2\beta_o}{9s_{oo}^P(s_{oo}^P - s_{co}^P)}$, then the originator's profit in this region is decreasing in e_o^P; hence, the maximizer on $e_o^P \geq \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$ is the lower boundary, i.e., $e_o^P = \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$, at which point, the corresponding originator's profit is

$$\frac{(s_{cc}^P)^2}{36\beta_c(s_{oo}^P - s_{co}^P)}\left(\frac{s_{oo}^P}{2} - \frac{\beta_o(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}\right). \tag{OS.24}$$

In order to find the global optimizer for the originator's profit function, we need to compare the maximums in these two regions as we derived above. First, notice that since $s_{oo}^P > s_{co}^P$, we have $\frac{2(s_{cc}^P)^2\beta_o}{9s_{oo}^P(s_{oo}^P - s_{co}^P)} < \frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P - s_{co}^P)(5s_{oo}^P + 4s_{co}^P)}$. Then we have three cases for β_c to consider:

(*a*) $\beta_c < \frac{2(s_{cc}^P)^2\beta_o}{9s_{oo}^P(s_{oo}^P - s_{co}^P)}$: In this case, the originator's profit has an interior maximum on $e_o^P \leq \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$ and is maximized at the lower boundary on $e_o^P > \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$. The corresponding profits are given in (OS.22) and (OS.24) as we have shown above. Subtracting the latter from the former we have

$$\begin{aligned}&\frac{(5s_{oo}^P + 4s_{co}^P)^2}{2592\beta_o} + \frac{(s_{cc}^P)^2}{81\beta_c} - \frac{(s_{cc}^P)^2}{36\beta_c(s_{oo}^P - s_{co}^P)}\left(\frac{s_{oo}^P}{2} - \frac{\beta_o(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}\right)\\ &= \frac{(5s_{oo}^P + 4s_{co}^P)^2}{2592\beta_o\beta_c^2}\left(\beta_c - \frac{2(s_{cc}^P)^2(s_{oo}^P + 8s_{co}^P)\beta_o}{(5s_{oo}^P + 4s_{co}^P)^2(s_{oo}^P - s_{co}^P)}\right)^2 + \frac{(s_{cc}^P)^4(s_{oo}^P + 2s_{co}^P)\beta_o}{27\beta_c^2(s_{oo}^P - s_{co}^P)(5s_{oo}^P + 4s_{co}^P)^2} > 0\,,\end{aligned} \tag{OS.25}$$

since $s_{oo}^P > s_{co}^P$. Therefore, (OS.25) is strictly positive, and hence, the originator's profit in (OS.22) is larger than that in (OS.24), i.e., the global maximizer is again the maximizer of the region $e_o^P \leq \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$, which is $e_o^{P*} = \frac{5s_{oo}^P + 4s_{co}^P}{36\beta_o}$. Finally, again as above, in equilibrium, the contributor will be active and will be the quality leader.

(*b*) $\frac{2(s_{cc}^P)^2\beta_o}{9s_{oo}^P(s_{oo}^P - s_{co}^P)} \leq \beta_c < \frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P - s_{co}^P)(5s_{oo}^P + 4s_{co}^P)}$: In this case, the originator's profit has an interior maximum both on $e_o^P \leq \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$ and $e_o^P > \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P - s_{co}^P)}$. As we have derived above, the maximum profit on the first region is given in (OS.22) and on the second region is $\frac{(s_{oo}^P)^2}{32\beta_o}$. Now subtracting the latter from

the former, we obtain

$$\frac{(5s_{oo}^P+4s_{co}^P)^2}{2592\beta_o}+\frac{(s_{cc}^P)^2}{81\beta_c}-\frac{(s_{oo}^P)^2}{32\beta_o}=\frac{1}{\beta_o}\left(\frac{(5s_{oo}^P+4s_{co}^P)^2}{2592}-\frac{(s_{oo}^P)^2}{32}+\frac{(s_{cc}^P)^2}{81}\cdot\frac{\beta_o}{\beta_c}\right)$$
$$>\frac{1}{\beta_o}\left(\frac{(5s_{oo}^P+4s_{co}^P)^2}{2592}-\frac{(s_{oo}^P)^2}{32}+\frac{(s_{cc}^P)^2}{81}\cdot\frac{(s_{oo}^P-s_{co}^P)(5s_{oo}^P+4s_{co}^P)}{2(s_{cc}^P)^2}\right)=\frac{(s_{oo}^P-s_{co}^P)(s_{oo}^P+2s_{co}^P)}{108\beta_o}>0\,, \tag{OS.26}$$

since $s_{oo}^P > s_{co}^P$. Therefore, the originator's maximum profit on the region $e_o^P \le \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P-s_{co}^P)}$ is greater than that on $e_o^P > \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P-s_{co}^P)}$ and the global maximizer in this case is $e_o^{P*}=\frac{5s_{oo}^P+4s_{co}^P}{36\beta_o}$, and as above, in equilibrium, the contributor will be active and will have higher quality.

(c) $\frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P-s_{co}^P)(5s_{oo}^P+4s_{co}^P)} \le \beta_c$: In this case, the originator's profit is monotonically increasing in e_o^P on $e_o^P \le \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P-s_{co}^P)}$ and has an interior maximum on $e_o^P > \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P-s_{co}^P)}$. As we have derived above, the maximum profit on the first region is given in (OS.23) and on the second region is $\frac{(s_{oo}^P)^2}{32\beta_o}$. Now subtracting the former from the latter, we obtain

$$\frac{(s_{oo}^P)^2}{32\beta_o}-\left(\frac{(s_{cc}^P)^2}{648\beta_c(s_{oo}^P-s_{co}^P)}\left(4s_{co}^P+5s_{oo}^P-\frac{\beta_o(s_{cc}^P)^2}{\beta_c(s_{oo}^P-s_{co}^P)}\right)+\frac{(s_{cc}^P)^2}{81\beta_c}\right)$$
$$=\frac{4(s_{cc}^P)^4\beta_o^2-4(s_{cc}^P)^2(s_{oo}^P-s_{co}^P)(13s_{oo}^P-4s_{co}^P)\beta_o\beta_c+81(s_{oo}^P-s_{co}^P)^2(s_{oo}^P)^2\beta_c^2}{2592\beta_o\beta_c^2(s_{oo}^P-s_{co}^P)^2}\,. \tag{OS.27}$$

The denominator in (OS.27) is strictly positive and the numerator in (OS.27) is convex and quadratic in β_c. Furthermore, if we take the derivative of the numerator with respect to β_c, we obtain

$$\begin{aligned}
& 162(s_{oo}^P-s_{co}^P)^2(s_{oo}^P)^2\beta_c-4(s_{cc}^P)^2(s_{oo}^P-s_{co}^P)(13s_{oo}^P-4s_{co}^P)\beta_o \\
> \;& 162(s_{oo}^P-s_{co}^P)^2(s_{oo}^P)^2\cdot\frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P-s_{co}^P)(5s_{oo}^P+4s_{co}^P)}-4(s_{cc}^P)^2(s_{oo}^P-s_{co}^P)(13s_{oo}^P-4s_{co}^P)\beta_o \\
= \;& \frac{64(s_{cc}^P)^2(s_{oo}^P-s_{co}^P)^3\beta_o}{5s_{oo}^P+4s_{co}^P}>0\,.
\end{aligned} \tag{OS.28}$$

That is, the numerator of (OS.27) is strictly increasing in β_c. In addition, the numerator becomes strictly positive as $\beta_c \to \infty$, and equals $-24(s_{oo}^P-s_{co}^P)^2((s_{oo}^P)^2-(s_{co}^P)^2+s_{co}^P(s_{oo}^P-s_{co}^P)) < 0$ at $\beta_c = \frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P-s_{co}^P)(5s_{oo}^P+4s_{co}^P)}$. In summary, it starts negative and crosses to positive exactly once on $\beta_c \ge \frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P-s_{co}^P)(5s_{oo}^P+4s_{co}^P)}$. Therefore, there exists the unique solution $\bar{r} \ge \frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P-s_{co}^P)(5s_{oo}^P+4s_{co}^P)}$, which is the larger root of the quadratic equation of β_c in the numerator of (OS.27), given in (OS.21), such that if $\beta_c \le \bar{r}$, then (OS.27) is negative, and, if $\beta_c > \bar{r}$, then (OS.27) is positive.

Consequently, if $\beta_c \ge \bar{r}$, the maximum originator profit in the region $e_o^P \ge \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P-s_{co}^P)}$ is greater than that in the region $e_o^P < \frac{(s_{cc}^P)^2}{18\beta_c(s_{oo}^P-s_{co}^P)}$ and vice-versa otherwise. That is, the globally optimal effort level for the originator is $e_o^{P*}=\frac{s_{oo}^P}{4\beta_o}$ if $\beta_c \ge \bar{r}$, and $e_o^{P*}=\frac{5s_{oo}^P+4s_{co}^P}{36\beta_o}$ if $\frac{2(s_{cc}^P)^2\beta_o}{(s_{oo}^P-s_{co}^P)(5s_{oo}^P+4s_{co}^P)} \le \beta_c < \bar{r}$. Moreover, the former is in the effort region where the contributor will be out of the market and the latter is in the region where the contributor will be active and the quality leader.

Combining the above three cases, gives us the results stated in parts (i) and (ii). Now, substituting these effort levels into the quality and price expressions given in the Proof of Proposition 1 and contributor effort level expressions given in the Proof of Proposition 2, we obtain

(i) If $\beta_c < \bar{r}$, then $e_o^{P*} = \frac{5s_{oo}^P + 4s_{co}^P}{36\beta_o}$, $e_c^{P*} = \frac{s_{cc}^P}{9\beta_c}$, $Q_o^P = \frac{s_{oo}^P(5s_{oo}^P + 4s_{co}^P)}{36\beta_o}$, $Q_c^P = \frac{s_{co}^P(5s_{oo}^P + 4s_{co}^P)}{36\beta_o} + \frac{(s_{cc}^P)^2}{9\beta_c}$,

$$p^{P*} = \frac{1}{216}\left(\frac{(s_{oo}^P + 2s_{co}^P)(5s_{oo}^P + 4s_{co}^P)}{\beta_o} + \frac{8(s_{cc}^P)^2}{\beta_c}\right), \tag{OS.29}$$

$$p_o^{P*} = \frac{1}{108}\left(\frac{(s_{oo}^P - s_{co}^P)(5s_{oo}^P + 4s_{co}^P)}{\beta_o} - \frac{4(s_{cc}^P)^2}{\beta_c}\right), \tag{OS.30}$$

and

$$p_c^{P*} = \frac{1}{108}\left(\frac{(s_{co}^P - s_{oo}^P)(5s_{oo}^P + 4s_{co}^P)}{\beta_o} + \frac{4(s_{cc}^P)^2}{\beta_c}\right). \tag{OS.31}$$

(ii) If $\beta_c \geq \bar{r}$, then $e_o^{P*} = \frac{s_{oo}^P}{4\beta_o}$, $e_c^{P*} = 0$, $Q_o^P = \frac{(s_{oo}^P)^2}{4\beta_o}$, $p^{P*} = \frac{(s_{oo}^P)^2}{8\beta_o}$, and $p_o^{P*} = 0$.

Finally, substituting the above expressions into the consumer valuation cutoff expressions found in Lemma OS.1, yields the equilibrium consumer segments served by each firm as given in the statements of Propositions 3 and 4. ■

Lemma OS.2 *For fixed p_o^O, p_c^O, e_o^O and e_c^O,*

(i) If $Q_o^O (= s_{oo}^O e_o^O + s_{oc}^O e_c^O) > Q_c^O (= s_{co}^O e_o^O + s_{cc}^O e_c^O)$, then the originator is the quality leader and the consumer market has the following characterization of regions:

Region I: If $p_c^O \geq Q_c^O$ and $p_o^O \geq Q_o^O$, then no consumer uses the software;

Region II: If either $p_c^O \geq Q_c^O$ and $p_o^O < Q_o^O$, or $p_c^O < Q_c^O$ and $p_o^O \leq p_c^O Q_o^O / Q_c^O$, then

(a) consumers with $\theta \in [0, \frac{p_o^O}{Q_o^O}]$ do not use the software, and

(b) consumers with $\theta \in (\frac{p_o^O}{Q_o^O}, 1]$ contract with the originator;

Region III: If $p_c^O < Q_c^O$ and $p_c^O Q_o^O / Q_c^O < p_o^O \leq p_c^O + Q_o^O - Q_c^O$, then

(a) consumers with $\theta \in [0, \frac{p_c^O}{Q_c^O}]$ do not use the software,

(b) consumers with $\theta \in (\frac{p_c^O}{Q_c^O}, \frac{p_o^O - p_c^O}{Q_o^O - Q_c^O}]$ contract with the contributor, and

(c) consumers with $\theta \in (\frac{p_o^O - p_c^O}{Q_o^O - Q_c^O}, 1]$ contract with the originator;

Region IV: If $p_c^O < Q_c^O$ and $p_c^O + Q_o^O - Q_c^O < p_o^O$, then

(a) consumers with $\theta \in [0, \frac{p_c^O}{Q_c^O}]$ do not use any software, and

(b) consumers with $\theta \in (\frac{p_c^O}{Q_c^O}, 1]$ contract with the contributor.

(ii) If $Q_o^O < Q_c^O$, then the contributor is the quality leader and the consumer market has the same characterization as in part (i), except only for switching Q_o^O and Q_c^O, and p_o^O and p_c^O in all the expressions.

(iii) If $Q_o^O = Q_c^O$, then

(a) If $p_o^O < p_c^O$, then consumers with $\theta \leq \min(p_o^O/Q_o^O, 1)$ do not use the software, and consumers with $\theta > \min(p_o^O/Q_o^O, 1)$ contract with the originator.

(b) If $p_o^O > p_c^O$, consumers with $\theta \leq \min(p_c^O/Q_c^O, 1)$ do not use the software, and consumers with $\theta > \min(p_c^O/Q_c^O, 1)$ contract with the contributor.

(c) If $p_o^O = p_c^O$, the consumers with $\theta \leq \min(p_o^O/Q_o^O, 1)$ do not use the software, and consumers with $\theta > \min(p_o^O/Q_o^O, 1)$ are indifferent between contracting with the two firms and randomly select a provider.

Proof: For part (i), let $Q_o^O > Q_c^O$. By (2), a consumer with type θ,

(a) prefers to contract with the originator rather than to not use the software if and only if $Q_o^O\theta - p_o^O \geq 0$, i.e., $\theta \geq \theta_{oN}^O = p_o^O/Q_o^O$;

(b) prefers to contract with the contributor rather than to not use if and only if $Q_c^O\theta - p_c^O \geq 0$, i.e., $\theta \geq \theta_{cN}^O = p_c^O/Q_c^O$; and

(c) prefers to contract with the originator rather than the contributor if and only if $Q_o^O\theta - p_o^O \geq Q_c^O\theta - p_c^O$, i.e., $\theta \geq \theta_{oc}^O = (p_o^O - p_c^O)/(Q_o^O - Q_c^O)$.

Now suppose $\theta_{oN}^O = p_o^O/Q_o^O \leq \theta_{cN}^O = p_c^O/Q_c^O$, then by carrying out the algebra, it follows that $\theta_{oc}^O = (p_o^O - p_c^O)/(Q_o^O - Q_c^O) \leq \theta_{oN}^O = p_o^O/Q_o^O \leq \theta_{cN}^O = p_c^O/Q_c^O$. In this case, all consumers who prefer to contract with the contributor rather than to not use, i.e., consumers with $\theta \geq \theta_{cN}^O$, prefer to contract with the originator rather than the contributor since for those consumers, $\theta \geq \theta_{cN}^O \geq \theta_{oc}^O$. Thus, in this region, there are only two possible consumer choices: a consumer either contracts with the originator or does not use. Consequently, if $\theta_{oN}^O \geq 1$, i.e., if $p_o^O \geq Q_o^O$ then no consumer uses the software. Notice that in this case, if $p_o^O \geq Q_o^O$, it also means that $p_c^O \geq Q_c^O$, since $\theta_{oN}^O = p_o^O/Q_o^O \leq \theta_{cN}^O = p_c^O/Q_c^O$. That is, in *Region I*, as defined in part (i), no consumer uses the software. On the other hand, if $\theta_{oN}^O < 1$, i.e., if $p_o^O \leq Q_o^O$, then consumers with types $\theta \in (\theta_{oN}^O, 1]$ contract with the originator and the rest, i.e., those with $\theta \in [0, \theta_{oN}^O]$ do not use the software. Again notice that $p_o^O/Q_o^O \leq p_c^O/Q_c^O$ and $p_o^O \leq Q_o^O$ implies either $p_c^O \geq Q_c^O$ and $p_o^O < Q_o^O$, or $p_c^O < Q_c^O$ and $p_o^O \leq p_c^O Q_o^O/Q_c^O$, i.e., *Region II* as defined above, therefore the statement for this region is also confirmed.

Next suppose $\theta_{oN}^O = p_o^O/Q_o^O \geq \theta_{cN}^O = p_c^O/Q_c^O$. Then we have $\theta_{oc}^O = (p_o^O - p_c^O)/(Q_o^O - Q_c^O) \geq \theta_{oN}^O = p_o^O/Q_o^O \geq \theta_{cN}^O = p_c^O/Q_c^O$. Thus, in this case, if $\theta \leq \theta_{oc}^O = (p_o^O - p_c^O)/(Q_o^O - Q_c^O)$, then a consumer with type θ always prefers to contract with the contributor given that she chooses to use the software. This means that if $\theta_{oc}^O < 1$, then consumers with types $\theta \in (\theta_{oc}^O, 1]$ contract with the originator; consumers with types $\theta \in (\theta_{cN}^O, \theta_{oc}^O]$ contract with the contributor; and the rest with types $\theta \in [0, \theta_{cN}^O]$ do not use the software. Notice that since $Q_o^O > Q_c^O$, $\theta_{oc}^O < 1$ if and only if $p_o^O \leq p_c^O + Q_o^O - Q_c^O$. Therefore, if $\theta_{cN}^O = p_c^O/Q_c^O < 1$, that is, $p_c^O < Q_c^O$, and $p_c^O Q_o^O/Q_c^O < p_o^O \leq p_c^O + Q_o^O - Q_c^O$, i.e., in *Region III*, the consumer market will be split as described in the statement of the lemma. On the other hand, if $p_o^O > p_c^O + Q_o^O - Q_c^O$, i.e., in *Region IV*, then no consumer with type $\theta \in [0, 1]$ will prefer to contract with the originator over the contributor, and hence consumers with types $\theta \in (\theta_{cN}^O, 1]$ contract with the contributor and the rest with

types $\theta \in [0,\, \theta^O_{cN}]$ do not use the software as stated. This proves part (i). Part (ii) follows in the exact same manner except the originator and the contributor are switched.

For part (iii), when $Q^O_o = Q^O_c$, there is no product differentiation and, as in a standard Bertrand competition case, all consumers prefer the lower priced service if there is a difference in prices. If the prices are the same, all consumers are indifferent and they choose between the two providers randomly. This completes the proof. □

Proof of Proposition 5: Technically, we will prove that, given e^P_o and e^P_c,

(i) If $Q^O_o (= s^O_{oo} e^O_o + s^O_{oc} e^O_c) > Q^O_c (= s^O_{co} e^O_o + s^O_{cc} e^O_c)$, then

$$p^{O*}_o = \frac{2(s^O_{oo} e^O_o + s^O_{oc} e^O_c)((s^O_{oo} - s^O_{co}) e^O_o + (s^O_{oc} - s^O_{cc}) e^O_c)}{(4s^O_{oo} - s^O_{co}) e^O_o + (4s^O_{oc} - s^O_{cc}) e^O_c}\,, \tag{OS.32}$$

$$p^{O*}_c = \frac{(s^O_{co} e^O_o + s^O_{cc} e^O_c)((s^O_{oo} - s^O_{co}) e^O_o + (s^O_{oc} - s^O_{cc}) e^O_c)}{(4s^O_{oo} - s^O_{co}) e^O_o + (4s^O_{oc} - s^O_{cc}) e^O_c}\,. \tag{OS.33}$$

However, if $Q^O_o < Q^O_c$, then the prices are symmetric and reversed.

(ii) If $Q^O_o = Q^O_c$, then $p^{O*}_o = p^{O*}_c = 0$.

We illustrate the profits and pricing game for the case in which the originator is the quality leader, i.e., $Q^O_o (= s^O_{oo} e^O_o + s^O_{oc} e^O_c) > Q^O_c (= s^O_{co} e^O_o + s^O_{cc} e^O_c)$. Using Lemma OS.2, in the pricing stage, given the investment levels e^O_o and e^O_c, the originator's profit function (excluding the effort cost) can be written as follows:

(i) If $p^O_c < Q^O_c$, then

$$\breve{\Pi}^O_o(p^O_o \,|\, p^O_c, e^O_o, e^O_c) = \begin{cases} p^O_o \left(1 - \frac{p^O_o}{s^O_{oo} e^O_o + s^O_{oc} e^O_c}\right) & if \quad p^O_o \le \frac{p^O_c (s^O_{oo} e^O_o + s^O_{oc} e^O_c)}{s^O_{co} e^O_o + s^O_{cc} e^O_c}\,; \\ p^O_o \left(1 - \frac{p^O_o - p^O_c}{(s^O_{oo} - s^O_{co}) e^O_o + (s^O_{oc} - s^O_{cc}) e^O_c}\right) & if \quad \frac{p^O_c (s^O_{oo} e^O_o + s^O_{oc} e^O_c)}{s^O_{co} e^O_o + s^O_{cc} e^O_c} < p^O_o \le p^O_c + (s^O_{oo} - s^O_{co}) e^O_o + (s^O_{oc} - s^O_{cc}) e^O_c\,; \\ 0 & if \quad p^O_c + (s^O_{oo} - s^O_{co}) e^O_o + (s^O_{oc} - s^O_{cc}) e^O_c < p^O_o\,. \end{cases} \tag{OS.34}$$

(ii) If $p^O_c \ge Q^O_c$, then

$$\breve{\Pi}^O_o(p^O_o \,|\, p^O_c, e^O_o, e^O_c) = \begin{cases} p^O_o \left(1 - \frac{p^O_o}{s^O_{oo} e^O_o + s^O_{oc} e^O_c}\right) & if \quad p^O_o < s^O_{oo} e^O_o + s^O_{oc} e^O_c\,; \\ 0 & if \quad p^O_o \ge Q^O_o = s^O_{oo} e^O_o + s^O_{oc} e^O_c\,. \end{cases} \tag{OS.35}$$

Similarly the contributor's profit (excluding effort costs) is:

(i) If $p_o^O < Q_o^O$,

$$\breve{\Pi}_c^O(p_c^O \,|\, p_o^O, e_o^O, e_c^O) = \begin{cases} p_c^O \left(1 - \frac{p_c^O}{s_{co}^O e_o^O + s_{cc}^O e_c^O}\right) & if \quad p_c^O \le p_o^O - (s_{oo}^O - s_{co}^O)e_o^O - (s_{oc}^O - s_{cc}^O)e_c^O \\ p_c^O \left(\frac{p_o^O - p_c^O}{(s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O} - \frac{p_c^O}{s_{co}^O e_o^O + s_{cc}^O e_c^O}\right) & if \quad p_o^O - (s_{oo}^O - s_{co}^O)e_o^O - (s_{oc}^O - s_{cc}^O)e_c^O < p_c^O \le \frac{p_o^O(s_{co}^O e_o^O + s_{cc}^O e_c^O)}{s_{oo}^O e_o^O + s_{oc}^O e_c^O}\,; \\ 0 & if \quad \frac{p_o^O(s_{co}^O e_o^O + s_{cc}^O e_c^O)}{s_{oo}^O e_o^O + s_{oc}^O e_c^O} < p_c^O\,. \end{cases} \tag{OS.36}$$

(ii) If $p_o^O \ge Q_o^O$,

$$\breve{\Pi}_c^O(p_c^O \,|\, p_o^O, e_o^O, e_c^O) = \begin{cases} p_c^O \left(1 - \frac{p_c^O}{s_{co}^O e_o^O + s_{cc}^O e_c^O}\right) & if \quad p_c^O < s_{co}^O e_o^O + s_{cc}^O e_c^O\,; \\ 0 & if \quad p_c^O \ge s_{co}^O e_o^O + s_{cc}^O e_c^O\,. \end{cases} \tag{OS.37}$$

Note that if the contributor is the quality leader, i.e., if $Q_c^O (= s_{co}^O e_o^O + s_{cc}^O e_c^O) > Q_o^O (= s_{oo}^O e_o^O + s_{oc}^O e_c^O)$, then we will have the exact symmetric expressions for the profits with the originator's and the contributor's roles reversed.

To see part (i), we first derive the originator's best response price function, $B_{p_o^O}(p_c^O)$. If $p_c^O < Q_c^O$, then as can be seen from (OS.34), the originator's profit function is piecewise quadratic in p_o^O. The first quadratic part has its unrestricted maximizer at $p_o^{O1} = (s_{oo}^O e_o^O + s_{oc}^O e_c^O)/2$. This local maximizer will be located in the region where the first quadratic curve applies if and only if

$$\frac{s_{oo}^O e_o^O + s_{oc}^O e_c^O}{2} \le \frac{p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)}{s_{co}^O e_o^O + s_{cc}^O e_c^O}\,, \tag{OS.38}$$

which is equivalent to $(s_{co}^O e_o^O + s_{cc}^O e_c^O)/2 < p_c^O$. Similarly, the second quadratic part has its unrestricted maximizer at $p_o^O = ((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O + p_c^O)/2$, and this maximizer falls into the region where this quadratic is applicable if and only if

$$\frac{p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)}{s_{co}^O e_o^O + s_{cc}^O e_c^O} < \frac{(s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O + p_c^O}{2} \le p_c^O + (s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O. \tag{OS.39}$$

Applying simple algebraic manipulations on (OS.39), we can see that the second inequality is always satisfied and the first inequality is equivalent to

$$p_c^O \le \frac{(s_{co}^O e_o^O + s_{cc}^O e_c^O)((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O)}{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O}\,. \tag{OS.40}$$

Next, again by simple algebraic manipulations, one can see that

$$\frac{(s_{co}^O e_o^O + s_{cc}^O e_c^O)((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O)}{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O} < \frac{s_{co}^O e_o^O + s_{cc}^O e_c^O}{2}\,. \tag{OS.41}$$

Therefore, if

$$p_c^O \le \frac{(s_{co}^O e_o^O + s_{cc}^O e_c^O)((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O)}{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O}, \tag{OS.42}$$

then $\breve{\Pi}_o^O(p_o^O \,|\, p_c^O, e_o^O, e_c^O)$ in (OS.34) is increasing in p_o^O in the region of

$$p_o^O \le \frac{(p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O))}{(s_{co}^O e_o^O + s_{cc}^O e_c^O)}, \tag{OS.43}$$

achieves the local maximum and is in the interior of the region

$$\frac{p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)}{(s_{co}^O e_o^O + s_{cc}^O e_c^O)} < p_o^O \le p_c^O + (s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O\,. \tag{OS.44}$$

Hence, in this case, the local maximizer of the second quadratic, which is $((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O + p_c^O)/2$ is the optimum. Furthermore, if

$$\frac{(s_{co}^O e_o^O + s_{cc}^O e_c^O)((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O)}{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O} < p_c^O \le \frac{(s_{co}^O e_o^O + s_{cc}^O e_c^O)}{2}, \tag{OS.45}$$

then $\breve{\Pi}_o^O(p_o^O \,|\, p_c^O, e_o^O, e_c^O)$ in (OS.34) is increasing in p_o^O in the region of

$$p_o^O \le \frac{p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)}{(s_{co}^O e_o^O + s_{cc}^O e_c^O)}, \tag{OS.46}$$

and is decreasing in p_o^O in the region of

$$\frac{p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)}{(s_{co}^O e_o^O + s_{cc}^O e_c^O)} < p_o^O \le p_c^O + (s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O\,. \tag{OS.47}$$

Consequently, the global optimum is the boundary p_c^O value between the two quadratic parts, which, by (OS.34), is $p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)/(s_{co}^O e_o^O + s_{cc}^O e_c^O)$. Lastly, if $(s_{co}^O e_o^O + s_{cc}^O e_c^O)/2 < p_c^O$, then $\breve{\Pi}_o^O(p_o^O \,|\, p_c^O, e_o^O, e_c^O)$ in (OS.34) achieves the local maximum in the region of

$$p_o^O \le \frac{p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)}{(s_{co}^O e_o^O + s_{cc}^O e_c^O)}, \tag{OS.48}$$

while it is decreasing in the region of

$$\frac{p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)}{(s_{co}^O e_o^O + s_{cc}^O e_c^O)} < p_o^O \le p_c^O + (s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O\,. \tag{OS.49}$$

Therefore the optimum p_o^O becomes the local maximizer in the first region, which is $(s_{oo}^O e_o^O + s_{oc}^O e_c^O)/2$.

In short, the best response price function for the originator for $p_c^O < Q_c^O$ can be summarized as

$$B_{p_o^O}(p_c^O) = \begin{cases} \frac{(s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O + p_c^O}{2} & \text{if } p_c^O \le \frac{(s_{co}^O e_o^O + s_{cc}^O e_c^O)((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O)}{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O}\,; \\ \frac{p_c^O(s_{oo}^O e_o^O + s_{oc}^O e_c^O)}{s_{co}^O e_o^O + s_{cc}^O e_c^O} & \text{if } \frac{(s_{co}^O e_o^O + s_{cc}^O e_c^O)((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O)}{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O} < p_c^O \le \frac{s_{co}^O e_o^O + s_{cc}^O e_c^O}{2}\,; \\ \frac{s_{oo}^O e_o^O + s_{oc}^O e_c^O}{2} & \text{if } \frac{s_{co}^O e_o^O + s_{cc}^O e_c^O}{2} < p_c^O\,. \end{cases} \tag{OS.50}$$

On the other hand, if $p_c^O \geq Q_c^O$, then maximizing (OS.35) with respect to p_o^O, we obtain

$$B_{p_o^O}(p_c^O) = \frac{s_{oo}^O e_o^O + s_{oc}^O e_c^O}{2}\,, \tag{OS.51}$$

which is the same for the final case given in (OS.50). Combining both cases, we can then conclude that the originator's best response price given the contributor's price, is the same as stated in (OS.50).

Similarly, the contributor's best response price function $B_{p_c^O}(p_o^O)$, can be written as:

$$B_{p_c^O}(p_o^O) = \begin{cases} \frac{(s_{co}^O e_o^O + s_{cc}^O e_c^O) p_o^O}{2(s_{oo}^O e_o^O + s_{oc}^O e_c^O)} & \text{if } p_o^O \leq \frac{2(s_{oo}^O e_o^O + s_{oc}^O e_c^O)((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O)}{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O}\,; \\ p_o^O - (s_{oo}^O - s_{co}^O)e_o^O - (s_{oc}^O - s_{cc}^O)e_c^O & \text{if } \frac{2(s_{oo}^O e_o^O + s_{oc}^O e_c^O)((s_{oo}^O - s_{co}^O)e_o^O + (s_{oc}^O - s_{cc}^O)e_c^O)}{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O} < p_o^O \leq \frac{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O}{2}\,; \\ \frac{s_{co}^O e_o^O + s_{cc}^O e_c^O}{2} & \text{if } \frac{(2s_{oo}^O - s_{co}^O)e_o^O + (2s_{oc}^O - s_{cc}^O)e_c^O}{2} < p_o^O\,. \end{cases} \tag{OS.52}$$

The Nash Equilibrium prices, can then be found by simultaneously solving the equations

$$B_{p_c^O}(p_o^{O*}) = p_c^{O*} \text{ and } B_{p_o^O}(p_c^{O*}) = p_o^{O*} \tag{OS.53}$$

which, carrying the algebra, leads to (OS.32) and (OS.33). The analysis for the case where the contributor is the quality leader uses the exact symmetric argument. This completes the proof of part (i). For part (ii), notice that when the firms have no difference in quality of service, the competition boils down to standard Bertrand setting, which means both firms in equilibrium set their prices equal to their cost, which is zero. □

Proof of Propositions 6 and 7: Before analyzing the originator's optimal effort investment level, we first study the contributor's optimal effort level e_c^O given the originator's effort level e_o^O. Given e_o^O, combining equations (OS.36) and (OS.37) with Lemma OS.2 and Proposition 5, the contributor's profit function and effort investment maximization problem can be written as

$$\max_{e_c^O \geq 0} \;\; \Pi_c^O(e_c^O \,|\, e_o^O) = \breve{\Pi}_c^O(p_c^{O*}(e_o^O, e_c^O) \,|\, p_o^{O*}(e_o^O, e_c^O), e_o^O, e_c^O) - \tfrac{1}{2}\beta_c (e_c^O)^2\,, \tag{OS.54}$$

where $\breve{\Pi}_c^O(e_c^O \,|\, e_o^O) = 0$ if $Q_o = Q_c$, and otherwise

$$\breve{\Pi}_c^O(e_c^O \,|\, e_o^O) = \begin{cases} \frac{(e_o^O s_{co}^O + e_c^O s_{cc}^O)(e_o^O s_{oo}^O + e_c^O s_{oc}^O)(e_o^O(s_{oo}^O - s_{co}^O) + e_c^O(s_{oc}^O - s_{cc}^O))}{(e_o^O(4s_{oo}^O - s_{co}^O) + e_c^O(4s_{oc}^O - s_{cc}^O))^2} \text{ (Curve Ic)} & \text{if } s_{cc}^O \leq s_{oc}^O, \text{ or } s_{cc}^O > s_{oc}^O \text{ and } e_c^O < \frac{s_{oo}^O - s_{co}^O}{s_{cc}^O - s_{oc}^O} e_o^O\,; \\ \frac{4(e_o^O s_{co}^O + e_c^O s_{cc}^O)^2 (e_o^O(s_{co}^O - s_{oo}^O) + e_c^O(s_{cc}^O - s_{oc}^O))}{(e_o^O(4s_{co}^O - s_{oo}^O) + e_c^O(4s_{cc}^O - s_{oc}^O))^2} \text{ (Curve IIc)} & \text{if } s_{cc}^O > s_{oc}^O \text{ and } e_c^O > \frac{s_{oo}^O - s_{co}^O}{s_{cc}^O - s_{oc}^O} e_o^O\,. \end{cases} \tag{OS.55}$$

Note that in the region valid for Curve Ic as defined in (OS.55), the originator is the quality leader, while in the region valid for Curve IIc, the contributor is the quality leader. Further, both Curves Ic and IIc are strictly concave. We first present the contributor's optimal effort level characterization in the following Lemma:

Lemma OS.3 *Define $\bar{s}_{oo}$ as the unique solution on $s^O_{oo} \geq s^O_{co}$ of the third degree polynomial equation*

$$4s^O_{cc}(s^O_{oo})^3 - 7s^O_{co}s^O_{cc}(s^O_{oo})^2 + 2s^O_{oc}(s^O_{co})^2 s^O_{oo} + s^O_{oc}(s^O_{co})^3 = 0, \tag{OS.56}$$

and let Curve Ic and Curve IIc be as defined in (OS.55). Given $e^O_o \geq 0$, the solution $e^{O*}_c(e^O_o) \geq 0$ to the contributor's problem (OS.54) is characterized as follows:

(i) If $s^O_{cc} \leq s^O_{oc}$, or $s^O_{cc} > s^O_{oc}$ and $s^O_{oo} \geq \bar{s}_{oo}$ and $e^O_o > \frac{4(s^O_{cc}-s^O_{oc})^2}{9\beta_c(s^O_{oo}-s^O_{co})}$, then the contributor's profit as a function of e^O_c is unimodal and $e^{O*}_c(e^O_o) \geq 0$ is the interior maximizer of Curve Ic, and the originator is the quality leader.[2]

(ii) If $s^O_{cc} > s^O_{oc}$ and $s^O_{oo} \geq \bar{s}_{oo}$ and $0 < e^O_o \leq \frac{4(s^O_{cc}-s^O_{oc})^2}{9\beta_c(s^O_{oo}-s^O_{co})}$, then the contributor's profit is bimodal and piecewise concave with the two local maximizers. The contributor chooses his optimal effort level between the interior optimizers of Curve Ic and Curve IIc, whichever yields a higher profit.

(iii) If $s^O_{cc} > s^O_{oc}$ and $s^O_{oo} < \bar{s}_{oo}$ and $e^O_o > \frac{4(s^O_{cc}-s^O_{oc})^2}{9\beta_c(s^O_{oo}-s^O_{co})}$, then the contributor's profit as a function of e^O_c is decreasing. Therefore $e^{O*}_c = 0$, and the originator is the quality leader.

(iv) If $s^O_{cc} > s^O_{oc}$ and $s^O_{oo} < \bar{s}_{oo}$ and $e^O_o \leq \frac{4(s^O_{cc}-s^O_{oc})^2}{9\beta_c(s^O_{oo}-s^O_{co})}$, then the contributor's profit is decreasing on $e^O_c \leq \frac{s^O_{oo}-s^O_{co}}{s^O_{cc}-s^O_{oc}} e^O_o$ and unimodal afterwards with an interior optimizer. The contributor chooses his optimal effort level between $e^O_c = 0$ and the interior optimizer of Curve IIc, whichever yields a higher profit.

Proof: First, suppose that the originator is the quality leader, i.e., $Q^O_o = s^O_{oo}e_o + s^O_{oc}e_c > Q^O_c = s^O_{co}e_o + s^O_{cc}e_c$. In this case, the contributor's profit (except the investment cost) corresponds to Curve Ic presented in (OS.55), and including the investment cost, we obtain the contributor's profit function as follows:

$$\Pi^O_c(e_c|e_o) = \frac{\left(e_o s^O_{co} + e_c s^O_{cc}\right)\left(e_o s^O_{oo} + e_c s^O_{oc}\right)\left(e_o\left(s^O_{oo} - s^O_{co}\right) + e_c\left(s^O_{oc} - s^O_{cc}\right)\right)}{\left(e_o\left(4s^O_{oo} - s^O_{co}\right) + e_c\left(4s^O_{oc} - s^O_{cc}\right)\right)^2} - \frac{1}{2}\beta_c e_c^2\,. \tag{OS.57}$$

By taking the derivative of (OS.57) with respect to e_c, we obtain the following first order condition:

$$A_0 + A_1 e_c + A_2 e_c^2 + A_3 e_c^3 + A_4 e_c^4 = 0\,, \tag{OS.58}$$

[2]The equations that the interior optimizers of Curves Ic and IIc satisfy are fourth order polynomials. We give the full precise expressions of these polynomials in the proof of this proposition, which is provided in the proof.

where

$$
\begin{aligned}
A_0 &= -e_o^3\left(s_{cc}^O(s_{oo}^O)^2\left(7s_{co}^O-4s_{oo}^O\right)-(s_{co}^O)^2s_{oc}^O\left(s_{co}^O+2s_{oo}^O\right)\right),\\
A_1 &= e_o^2\Big(2(s_{co}^O)^2(s_{oc}^O)^2-7(s_{cc}^O)^2(s_{oo}^O)^2+s_{cc}^Os_{oc}^O\left(3(s_{co}^O)^2-10s_{co}^Os_{oo}^O+12(s_{oo}^O)^2\right)+e_o\left(s_{co}^O-4s_{oo}^O\right)^3\beta_c\Big),\\
A_2 &= 3e_o\left(s_{co}^O-4s_{oo}^O\right)\left((s_{cc}^O)^2s_{oc}^O-4e_os_{oc}^O\left(s_{co}^O-4s_{oo}^O\right)\beta_c+s_{cc}^O\left(-(s_{oc}^O)^2+e_o\left(s_{co}^O-4s_{oo}^O\right)\beta_c\right)\right),\\
A_3 &= \left(s_{cc}^O-4s_{oc}^O\right)\left((s_{cc}^O)^2s_{oc}^O-12e_os_{oc}^O\left(s_{co}^O-4s_{oo}^O\right)\beta_c-s_{cc}^O\left((s_{oc}^O)^2-3e_o\left(s_{co}^O-4s_{oo}^O\right)\beta_c\right)\right),\\
A_4 &= \left(s_{cc}^O-4s_{oc}^O\right)^3\beta_c\,. \qquad \text{(OS.59)}
\end{aligned}
$$

From (OS.57), the second order condition is e_c:

$$
\frac{d^2\,\Pi_c^O(e_c|e_o)}{d\,e_c^2}=-\frac{2e_o^2\left(s_{oo}^Os_{cc}^O-s_{co}^Os_{oc}^O\right)^2\left(e_o\left(8s_{oo}^O+7s_{co}^O\right)+e_c\left(8s_{oc}^O+7s_{cc}^O\right)\right)}{\left(e_o\left(4s_{oo}^O-s_{co}^O\right)+e_c\left(4s_{oc}^O-s_{cc}^O\right)\right)^4}-\beta_c<0\,, \qquad \text{(OS.60)}
$$

i.e., Curve Ic is strictly concave in e_c.

Further, note that again from (OS.57), the first derivative of Curve Ic at $e_c=0$ is

$$
\left.\frac{d\,\Pi_c^O(e_c|e_o)}{d\,e_c}\right|_{e_c=0}=\frac{h(s_{oo}^O)}{(4s_{oo}^O-s_{co}^O)^3}\,, \qquad \text{(OS.61)}
$$

where

$$
h(s_{oo}^O)=(s_{oo}^O)^2s_{cc}^O(4s_{oo}^O-7s_{co}^O)+(s_{co}^O)^2s_{oc}^O(2s_{oo}^O+s_{co}^O)\,. \qquad \text{(OS.62)}
$$

Second, consider the case in which the contributor is the quality leader, i.e., $Q_o^O=s_{oo}^Oe_o+s_{oc}^Oe_c<Q_c^O=s_{co}^Oe_o+s_{cc}^Oe_c$. In this case, the contributor's profit (except the investment cost) corresponds to Curve IIc presented in (OS.55), and including the investment cost, we obtain the contributor's profit function as follows:

$$
\Pi_c^O(e_c|e_o)=\frac{4\left(e_os_{co}^O+e_cs_{cc}^O\right)^2\left(e_o\left(s_{co}^O-s_{oo}^O\right)+e_c\left(s_{cc}^O-s_{oc}^O\right)\right)}{\left(e_o\left(4s_{co}^O-s_{oo}^O\right)+e_c\left(4s_{cc}^O-s_{oc}^O\right)\right)^2}-\frac{1}{2}\beta_ce_c^2\,. \qquad \text{(OS.63)}
$$

Taking the derivative of (OS.63) with respect to e_c and simplifying, we obtain the following first order condition:

$$
B_0+B_1e_c+B_2e_c^2+B_3e_c^3+B_4e_c^4=0\,, \qquad \text{(OS.64)}
$$

where

$$
\begin{aligned}
B_0 &= 4e_o^3s_{co}^O(s_{cc}^O(2(s_{oo}^O)^2-3s_{co}^Os_{oo}^O+4(s_{co}^O)^2)-s_{co}^Os_{oc}^O(s_{oo}^O+2s_{co}^O)),\\
B_1 &= e_o^2(8(s_{cc}^O)^2((s_{oo}^O)^2-3s_{co}^Os_{oo}^O+6(s_{co}^O)^2)-4s_{co}^Os_{oc}^O(s_{co}^Os_{oc}^O+4s_{cc}^O(9s_{co}^O-2s_{oo}^O))+(s_{oo}^O-4s_{co}^O)^3\beta_ce_o),\\
B_2 &= 3e_o(s_{oo}^O-4s_{co}^O)((4s_{cc}^O-s_{oc}^O)(4s_{co}^O-s_{oo}^O)\beta_ce_o-4(s_{cc}^O)^2(s_{cc}^O-s_{oc}^O)),\\
B_3 &= \left(4s_{cc}^O-s_{oc}^O\right)\left(4(s_{cc}^O)^2(s_{cc}^O-s_{oc}^O)+3(4s_{cc}^O-s_{oc}^O)(s_{oo}^O-4s_{co}^O)\beta_ce_o\right)\,,\\
B_4 &= -(4s_{cc}^O-s_{oc}^O)^3\beta_c\,. \qquad \text{(OS.65)}
\end{aligned}
$$

Taking the second order condition using (OS.64)

$$\frac{d^2\,\Pi_c^O(e_c|e_o)}{d\,e_c^2} = -\frac{8e_o^2\left(s_{cc}^O s_{oo}^O - s_{co}^O s_{oc}^O\right)^2\left(e_o\left(5s_{co}^O + s_{oo}^O\right) + e_c\left(5s_{cc}^O + s_{oc}^O\right)\right)}{\left(e_o\left(4s_{co}^O - s_{oo}^O\right) + e_c\left(4s_{cc}^O - s_{oc}^O\right)\right)^4} - \beta_c < 0\,, \tag{OS.66}$$

which shows that Curve IIc is also strictly concave in e_c.

Now, first consider the case in which $s_{cc}^O \le s_{oc}^O$. Since $s_{oo}^O > s_{co}^O$, in this case, we have $Q_o^O = s_{oo}^O e_o + s_{oc}^O e_c \ge Q_c^O = s_{co}^O e_o + s_{cc}^O e_c$, i.e., the originator is the quality leader for all $e_c \ge 0$ for any given $e_o \ge 0$ and the contributor's profit curve is Curve Ic for all $e_c \ge 0$. We first show that the derivative of the contributor's profit with respect to e_c evaluated at $e_c = 0$, which is given in (OS.61), is positive for all $s_{oo}^O > s_{co}^O$. Note that

$$h'(s_{oo}^O) = 2(6s_{cc}^O(s_{oo}^O)^2 - 7s_{co}^O s_{cc}^O s_{oo}^O + (s_{co}^O)^2 s_{oc}^O)\,, \tag{OS.67}$$

and

$$h''(s_{oo}^O) = 2s_{cc}^O(12s_{oo}^O - 7s_{co}^O)\,. \tag{OS.68}$$

Therefore, for all $s_{oo}^O > s_{co}^O$, we have $h''(s_{oo}^O) > 0$, i.e., $h'(s_{oo}^O)$ is strictly increasing in s_{oo}^O. Moreover, since $s_{oc}^O \ge s_{cc}^O$,

$$h'(s_{co}^O) = 2(s_{co}^O)^2(s_{oc}^O - s_{cc}^O) \ge 0. \tag{OS.69}$$

Consequently, $h(s_{oo}^O)$ is strictly increasing in s_{oo}^O for $s_{oo}^O > s_{co}^O$. Finally, we also have

$$h(s_{co}^O) = 3(s_{co}^O)^3(s_{oc}^O - s_{cc}^O) \ge 0. \tag{OS.70}$$

To sum it up, h is positive and increasing at s_{co}^O, and the first derivative of h is strictly increasing for all $s_{oo}^O \ge s_{co}^O$, which means h will be increasing for all $s_{oo}^O \ge s_{co}^O$. Therefore, $h(s_{oo}^O) > 0$ for all $s_{oo}^O > s_{co}^O$. Hence, we have shown that by (OS.61), $\left.\frac{d\,\Pi_c^O(e_c|e_o)}{d\,e_c}\right|_{e_c=0} \ge 0$ for $s_{cc}^O \le s_{oc}^O$. Moreover, also note that, by (OS.57), for Curve Ic

$$\lim_{e_c\to\infty} \Pi_c^O(e_c|e_o) = \lim_{e_c\to\infty}\left\{\frac{s_{cc}^O s_{oc}^O(s_{oc}^O - s_{cc}^O)}{(4s_{oc}^O - s_{cc}^O)^2}e_c - \frac{1}{2}\beta_c e_c^2\right\} = -\infty\,. \tag{OS.71}$$

Since the second order condition is strictly negative in this region as shown in (OS.60), it follows that the contributor's profit function in this regime is unimodal and that there exists the unique non-negative solution that satisfies the first order condition (OS.58). Furthermore, this solution is the optimal contributor's effort level e_c^{O*} in this region of $s_{cc}^O \le s_{oc}^O$. This proves the first case given in part (i).

Next, consider the case in which $s_{cc}^O > s_{oc}^O$. In order to show the remaining statements in parts (i)-(iv), we first have to study some characteristics of Curves Ic and IIc, which together form the entire profit curve for the contributor.

We start by showing that the derivative of Curve Ic with respect to e_c evaluated at $e_c = \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$ is negative. Taking the first derivative of (OS.57) with respect to e_c, and since $s_{oo}^O > s_{co}^O$, we have

$$\left.\frac{d\,\Pi_c^O(e_c|e_o)}{d\,e_c}\right|_{e_c=\frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}} = -\frac{(s_{cc}^O - s_{oc}^O)^2 + 9(s_{oo}^O - s_{co}^O)\beta_c e_o}{9(s_{cc}^O - s_{oc}^O)} < 0\ . \tag{OS.72}$$

Further, evaluating the contributor's profit for Curve IIc in (OS.55) as $e_c \to \infty$, we have

$$\lim_{e_c\to\infty} \Pi_c^O(e_c|e_o) = \lim_{e_c\to\infty} \left\{ \frac{4(s_{cc}^O)^2(s_{cc}^O - s_{oc}^O)}{(4s_{cc}^O - s_{oc}^O)^2} e_c - \frac{1}{2}\beta_c e_c^2 \right\} = -\infty. \tag{OS.73}$$

Next, we show that the derivative of the contributor's profit corresponding to Curve Ic in (OS.55) with respect to e_c evaluated at $e_c = 0$ presented in (OS.61) is positive if and only if $s_{oo}^O > \bar{s}_{oo}$. Note that as we have shown in (OS.68), $h(s_{oo}^O)$ is strictly convex in s_{oo}^O. Moreover, in this case,

$$h(s_{co}^O) = 3(s_{co}^O)^3(s_{oc}^O - s_{cc}^O) \leq 0, \tag{OS.74}$$

and $\lim_{s_{oo}^O\to\infty} h(s_{oo}^O) = \infty$. Consequently, there exists the unique threshold $\bar{s}_{oo} > s_{co}^O$, which solves $h(s_{oo}^O) = 0$ in (OS.56), and for this threshold, $h(s_{oo}^O) < 0$ if $s_{co}^O < s_{oo}^O < \bar{s}_{oo}$, and $h(s_{oo}^O) > 0$ if $s_{oo}^O > \bar{s}_{oo}$. Therefore, for Curve Ic,

$$\left.\frac{d\,\Pi_c^O(e_c|e_o)}{d\,e_c}\right|_{e_c=0} > 0 \text{ if and only if } s_{oo}^O > \bar{s}_{oo}\,. \tag{OS.75}$$

Finally, let us also examine the derivative of Curve IIc at the junction of the two curves Ic and IIc, i.e., at $e_c = \frac{s_{oo}^O - s_{co}^O}{s_{cc}^O - s_{oc}^O} e_o$. From (OS.63), the derivative of Curve IIc with respect to e_c evaluated at this junction point is

$$\left.\frac{d\,\Pi_c^O(e_c|e_o)}{d\,e_c}\right|_{e_c=\frac{s_{oo}^O - s_{co}^O}{s_{cc}^O - s_{oc}^O} e_o} = \frac{4(s_{cc}^O - s_{oc}^O)^2 - 9(s_{oo}^O - s_{co}^O)\beta_c e_o}{9(s_{cc}^O - s_{oc}^O)}\,. \tag{OS.76}$$

Since $s_{oo}^O > s_{co}^O$, it follows that this derivative is positive if and only if $e_o < \frac{4(s_{cc}^O - s_{oc}^O)^2}{9(s_{oo}^O - s_{co}^O)\beta_c}$.

Now, by using these observations, we can complete the proofs of remaining statement of parts (i)-(iv). First, if $s_{oo}^O \geq \bar{s}_{oo}$, as we have shown above, Curve Ic is increasing in e_c at $e_c = 0$. Since it is also decreasing at $e_c = \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$, and Curve Ic is strictly concave, there is a unique interior maximizer of the profit curve on $0 \leq e_c < \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$. Moreover, if $e_o > \frac{4(s_{cc}^O - s_{oc}^O)^2}{9(s_{oo}^O - s_{co}^O)\beta_c}$, then again as we have shown above, Curve IIc is decreasing at $e_c = \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$, and since Curve IIc is also strictly concave, it is monotonically decreasing on the entire range $e_c > \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$. Therefore, we can conclude that if $s_{cc}^O > s_{co}^O$, $s_{oo}^O \geq \bar{s}_{oo}$ and $e_o > \frac{4(s_{cc}^O - s_{oc}^O)^2}{9(s_{oo}^O - s_{co}^O)\beta_c}$, then the contributor's profit function is unimodal and is maximized at the interior maximizer of Curve Ic and the originator is the quality leader. This completes the proof of part (i).

To see part (ii), first, for $s_{oo}^O \geq \bar{s}_{oo}$, as we have shown above, Curve Ic is increasing in e_c at $e_c = 0$ and identical to our argument above, it is unimodal and has a unique interior maximizer on $0 \leq e_c < \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$. In this case however, for $e_o \leq \frac{4(s_{cc}^O - s_{oc}^O)^2}{9(s_{oo}^O - s_{co}^O)\beta_c}$, as we have shown above, Curve IIc is increasing at $e_c = \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$, and since Curve IIc is strictly concave, it is also unimodal and has a unique interior maximizer on the range $e_c > \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$. Therefore, we can conclude that if $s_{cc}^O > s_{co}^O$, $s_{oo}^O \geq \bar{s}_{oo}$ and $e_o \leq \frac{4(s_{cc}^O - s_{oc}^O)^2}{9(s_{oo}^O - s_{co}^O)\beta_c}$, then the contributor's overall profit function is bimodal and piecewise concave with two local maximizers. The global maximizer is the one that yields the higher profit.

For part (iii), given $s_{oo}^O < \bar{s}_{oo}$, Curve Ic is decreasing in e_c at $e_c = 0$. Since it is strictly concave, it is monotonically decreasing on $0 \leq e_c < \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$. Further, since $e_o > \frac{4(s_{cc}^O - s_{oc}^O)^2}{9(s_{oo}^O - s_{co}^O)\beta_c}$, then similar to part (i), Curve IIc is monotonically decreasing on the entire range $e_c > \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$. Therefore, the contributor's

profit function is monotonically decreasing on the entire range $e_c \geq 0$. Hence it is maximized at $e_c = 0$ and the originator is the quality leader.

Finally, for part (iv), given $s_{oo}^O < \bar{s}_{oo}$, as we have shown above, Curve Ic, and hence the contributor's profit function, is monotonically decreasing on $0 \leq e_c < \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$. So, the maximizer of the profit function on $0 \leq e_c < \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$ is $e_c = 0$. On the other hand, for $e_o \leq \frac{4(s_{cc}^O - s_{oc}^O)^2}{9(s_{oo}^O - s_{co}^O)\beta_c}$ as we have shown above for part (ii), Curve IIc is unimodal and has a unique interior maximum. Therefore, the global maximizer of the contributor's profit function can be found by comparing the profit levels at $e_c = 0$ and the interior maximizer of Curve IIc on $e_c > \frac{(s_{oo}^O - s_{co}^O)e_o}{s_{cc}^O - s_{oc}^O}$ as stated in part (iv). This completes the proof. ■

Given the contributor's effort level $e_c^{O*}(e_o^O) \geq 0$ as the best response to e_o^O as presented in Lemma OS.3, the originator's profit function can be written as

$$\begin{aligned} \max_{e_o^O \geq 0} \quad & \Pi_o^O(e_o^O) = \breve{\Pi}_o^O\left(p_o^{O*}(e_o^O, e_c^{O*}(e_o^O)) \,|\, p_c^{O*}(e_o^O, e_c^{O*}(e_o^O)), e_o^O, e_c^{O*}(e_o^O)\right) - \tfrac{1}{2}\beta_o(e_o^O)^2 \\ s.t. \quad & e_c^{O*}(e_o^O) = \arg\max_{e_c^O \geq 0} \Pi_c^O(e_c^O \,|\, e_o^O)\,, \end{aligned} \tag{OS.77}$$

where $\breve{\Pi}_o^O(e_o^O\,,\, e_c^{O*}(e_o^O)) = 0$ if $Q_o^O = Q_c^O$, and otherwise

$$\breve{\Pi}_o^O(e_o^O\,,\, e_c^{O*}(e_o^O)) = \begin{cases} \frac{\left(e_o^O s_{co}^O + e_c^{O*}(e_o^O)s_{cc}^O\right)\left(e_o^O s_{oo}^O + e_c^{O*}(e_o^O)s_{oc}^O\right)\left(e_o^O\left(s_{oo}^O - s_{co}^O\right) + e_c^{O*}(e_o^O)\left(s_{oc}^O - s_{cc}^O\right)\right)}{\left(e_o^O(4s_{oo}^O - s_{co}^O) + e_c^{O*}(e_o^O)(4s_{oc}^O - s_{cc}^O)\right)^2} & \textit{(Curve Io)} \text{ if } e_o^O < \frac{s_{cc}^O - s_{oc}^O}{s_{oo}^O - s_{co}^O} e_c^{O*}(e_o^O)\,; \\ \frac{4\left(e_o^O s_{oo}^O + e_c^{O*}(e_o^O)s_{oc}^O\right)^2\left(e_o^O\left(s_{oo}^O - s_{co}^O\right) + e_c^{O*}(e_o^O)\left(s_{oc}^O - s_{cc}^O\right)\right)}{\left(e_o^O(4s_{oo}^O - s_{co}^O) + e_c^{O*}(e_o^O)(4s_{oc}^O - s_{cc}^O)\right)^2} & \textit{(Curve IIo)} \quad \text{if } e_o^O > \frac{s_{cc}^O - s_{oc}^O}{s_{oo}^O - s_{co}^O} e_c^{O*}(e_o^O)\,. \end{cases} \tag{OS.78}$$

In equation (OS.78) Curve Io is valid for the region where the originator does not invest much effort and the contributor is the quality leader; Curve IIo is valid for the region where the originator puts substantial effort and becomes the quality leader. The originator chooses his optimal effort level solving (OS.77) by comparing the maximizers of these two curves.

Now, for the proof of Proposition 6, we first explore the case where β_c is small. We start by determining the behavior of e_c^{O*} and e_o^{O*} as $\beta_c \to 0$. Define $z = 1/\beta_c$, i.e., as $\beta_c \to 0$, $z \to \infty$. Further define $k, p \in \mathbb{R}$ as the values that satisfy

$$\lim_{z\to\infty} \frac{e_c^{O*}}{z^k} = K_1 \text{ and } \lim_{z\to\infty} \frac{e_o^{O*}}{z^p} = K_2, \tag{OS.79}$$

where $K_1, K_2 \in \mathbb{R}$ are constants. In other words, as $z \to \infty$, e_c^{O*} is in the order of z^k, and e_o^{O*} is in the order of z^p ($O(z^k)$ and $O(z^p)$ in standard notation, respectively). Our goal is determining the values of k and p in equilibrium, which will tell us the behavior of e_c^{O*} and e_o^{O*} as $z \to \infty$, i.e., as $\beta_c \to 0$.

First suppose $p < k$. Looking at the first order condition of the contributor's profit with respect to e_c, in the two possible regions in the contributor's profit curve as given by (OS.55), first, in the first region, by (OS.57)-(OS.59), plugging in $e_c^{O*} = O(z^k)$ and $e_o^{O*} = O(z^p)$, and collecting the terms in powers of z, we find that the first order condition can be written as

$$E_1 z^{4k-1} + E_2 z^{3k} + Y_1(z) = 0 \tag{OS.80}$$

where $E_1, E_2 \in \mathbb{R}$ and $Y_1(z)$ is a polynomial in z, with terms in orders of z^{3p}, z^{2p+k}, z^{3p+k-1}, z^{p+2k}, $z^{2p+2k-1}$ and z^{3k+p-1}. Notice that as $z \to \infty$, the term in the order of z^{3p} in $Y_1(z)$ will be less than $E_2 z^{3k}$ in absolute value, and all the other terms in $Y_1(z)$ will be less than $E_1 z^{4k-1}$ in absolute value.

Now, as $z \to \infty$, (OS.80) has to hold for all z values. But this is not possible if $4k-1 \neq 3k$, because, in that case, the larger of $E_1 z^{4k-1}$ and $E_2 z^{3k}$ in absolute value will dominate all other terms for sufficiently large z and (OS.80) will never be satisfied. Therefore, in equilibrium $E_1 z^{4k-1}$ and $E_2 z^{3k}$ have to cancel each other, i.e., $4k-1 = 3k$ has to hold, which in turn implies that $k = 1$. Repeating the same analysis for the second region, this time using (OS.63)-(OS.65), we again find that the first order condition is equivalent to (OS.80) as described above, and hence it again follows that $k = 1$.

Second suppose $p > k$. In this case, again utilizing the first order condition for the first region as given in (OS.57)-(OS.59), we find that the first order condition can be written as

$$E_3 z^{3p+k-1} + E_4 z^{3p} + Y_2(z) = 0 \tag{OS.81}$$

for $E_3, E_4 \in \mathbb{R}$ and $Y_2(z)$ is a polynomial in z, with again terms in orders of z^{3p}, z^{2p+k}, z^{3p+k-1}, z^{p+2k}, $z^{2p+2k-1}$ and z^{3k+p-1}. Notice in this case that as $z \to \infty$, the term in the order of z^{3k} in $Y_2(z)$ will be less than $E_4 z^{3p}$ and all the other terms in $Y_2(z)$ will be less than $E_3 z^{3p+k-1}$ in absolute value. Therefore, with the same argument as above, we conclude that $3p+k-1 = 3p$ must hold, which implies that $k = 1$. Once again repeating the same analysis for the second region using (OS.63)-(OS.65) and following identical steps, we find that if the global optimum e_c^{O*} is in this region, $k = 1$ follows.

In summary, we have so far shown that if $p \neq k$, then $k = 1$ holds, i.e., e_c^{O*} is in the order of $z(= 1/\beta_c)$. This means that we can write

$$e_c^{O*} = \kappa_0 + \kappa_1 z + O(1/z)\,, \tag{OS.82}$$

for constants $\kappa_1, \kappa_2 \in \mathbb{R}$.

Now let us look at the optimization problem of the originator, as given in (OS.77)-(OS.78). Taking the total derivative of the originator's profit with respect to e_o, we obtain the first order condition as

$$\begin{aligned}\left.\frac{d\Pi_o^O(e_o, e_c^{O*}(e_o))}{de_o}\right|_{e_o = e_o^{O*}} &= \left.\frac{d}{de_o}\left(\breve{\Pi}_o^O(e_o, e_c^{O*}(e_o)) - \frac{1}{2}\beta_o e_o^2\right)\right|_{e_o = e_o^{O*}} \\ &= \left.\frac{\partial \breve{\Pi}_o^O(e_o, e_c^{O*}(e_o))}{\partial e_o}\right|_{e_o = e_o^{O*}} + \left.\frac{\partial \breve{\Pi}_o^O(e_o, e_c^{O*}(e_o))}{\partial e_c^{O*}} \cdot \frac{de_c^{O*}(e_o)}{de_o}\right|_{e_o = e_o^{O*}} - \beta_o e_o^{O*} = 0\,. \end{aligned} \tag{OS.83}$$

Now we know by Lemma OS.3 that the contributor's optimal effort will be the local maximizer of one of the two curves given in (OS.55). Therefore, we know that

$$\left.\frac{\partial}{\partial e_c}\breve{\Pi}_c^O(e_o, e_c)\right|_{e_c = e_c^{O*}(e_o)} - \beta_c e_c^{O*}(e_o) = 0\,. \tag{OS.84}$$

Then by the implicit function function theorem

$$\frac{de_c^{O*}(e_o)}{de_o} = -\frac{\frac{\partial^2}{\partial e_c \partial e_o}\breve{\Pi}_c^O(e_o, e_c)}{\frac{\partial^2}{\partial e_c^2}\breve{\Pi}_c^O(e_o, e_c) - \beta_c}\,. \tag{OS.85}$$

Plugging (OS.85) in (OS.83), we then obtain

$$\frac{\partial \breve{\Pi}_o^O(e_o, e_c^{O*}(e_o))}{\partial e_o}\bigg|_{e_o=e_o^{O*}} - \frac{\partial \breve{\Pi}_o^O(e_o, e_c^{O*}(e_o))}{\partial e_c^{O*}} \cdot \frac{\frac{\partial^2}{\partial e_c \partial e_o}\breve{\Pi}_c^O(e_o, e_c)}{\frac{\partial^2}{\partial e_c^2}\breve{\Pi}_c^O(e_o, e_c) - \beta_c}\bigg|_{e_o=e_o^{O*}} - \beta_o e_o^{O*} = 0\,. \tag{OS.86}$$

Now, by (OS.55) and (OS.78), $\breve{\Pi}_c^O$ and $\breve{\Pi}_o^O$ can take two possible functional forms each, depending on whether the originator or the contributor is the quality leader. For each of these two cases, plugging in the corresponding functional forms for $\breve{\Pi}_c^O$ and $\breve{\Pi}_o^O$, and calculating the partial derivatives needed in (OS.86), substituting in and calculating (OS.86) and finally substituting the functional form $e_c^{O*} = \kappa_0 + \kappa_1 z + O(1/z)$, and that e_o^{O*} is in the order of z^p, we find that[3]

$$K_1 - \beta_o e_o^{O*} = 0, \tag{OS.87}$$

for some $K_1 \in \mathbb{R}$ for both cases of $p > k$ and $p < k$. That is, for all cases, if $p \neq k$, then it follows that e_o^{O*} is a constant, which means that $p = 0$. Combining this with the finding that $k = 1$ holds for $p \neq k$, it follows that the only possible outcome for the case $p \neq k$ is $p < k$ and that $p = 0$ and $k = 1$.

Finally suppose $p = k$. Then plugging in e_c^{O*} and e_o^{O*} are both in the order of z^p and once again carrying out the calculations as above, we find that (OS.87) is satisfied, i.e., in this case $p = k = 0$ has to hold. Note however that e_c^{O*} has to satisfy the first order condition given in (OS.84) for one of the two curves given in (OS.55). Now suppose in equilibrium, the contributor's quality is lower than the originator's quality. Then, given e_o^{O*}, by (OS.55), $(s_{cc}^O - s_{oc}^O)e_c^{O*} < (s_{oo}^O - s_{co}^O)e_o^{O*}$ must hold. But in that case, again by (OS.55), for small enough β_c, there exists $(s_{cc}^O - s_{oc}^O)\hat{e}_c > (s_{oo}^O - s_{co}^O)e_o^{O*}$ such that

$$\Pi_c^O(\hat{e}_c | e_o^{O*}) > \Pi_c^O(e_c^{O*} | e_o^{O*})\,, \tag{OS.88}$$

that is, the contributor cannot have lower quality than the originator in equilibrium. So the contributor is the quality leader and his profit can be written as in (OS.63) and by (OS.84) the first order condition for the contributor's maximization problem can be written as

$$\begin{aligned}\frac{d\Pi_c^O(e_o, e_c)}{de_c} &= \frac{Q_c^O(24(Q_c^O)^2(4s_{cc}^O - s_{oc}^O) + 8s_{cc}^O(4Q_c^O - Q_o^O)^2 - 4(13s_{cc}^O - s_{oc}^O)Q_c^O(4Q_c^O - Q_o^O))}{(4Q_c^O - Q_o^O)^3} - \beta_c e_c \\ &> \frac{8Q_c^O(Q_c^O - Q_o^O)^2}{(4Q_c^O - Q_o^O)^3} - \beta_c e_c\,. \end{aligned} \tag{OS.89}$$

From (OS.89), we can see that the first order condition becomes positive as $\beta_c \to 0$, and hence, it cannot

[3]The exact expressions that are obtained after these calculations are very large, so we skip them here. These expressions are available from the authors upon request.

be satisfied as $\beta_c \to 0$; this is because the first term in (OS.89) converges to a strictly positive constant since e_o^{O*} and e_c^{O*} converge to constants (since $k = p = 0$) while the second term converges to zero as $\beta_c \to 0$. In conclusion, in equilibrium, $k = p = 0$ cannot hold either and the only possible case is $k = 1$ and $p = 0$, that is,

$$e_c^{O*} = \kappa_0 + \kappa_1 z + O\left(1/z\right) \text{ and } e_o^{O*} = \kappa_2 + O\left(1/z\right) . \tag{OS.90}$$

But then, calculating the service qualities of the originator and the contributor respectively, we obtain

$$\begin{aligned} Q_o^O = s_{oo}^O e_o^{O*} + s_{oc}^O e_c^{O*} = s_{oc}^O \kappa_1 z + s_{oo}^O \kappa_2 + s_{oc}^O \kappa_0 + O\left(1/z\right) \\ \leq Q_c^O = s_{co}^O e_o^{O*} + s_{cc}^O e_c^{O*} = s_{cc}^O \kappa_1 z + s_{co}^O \kappa_2 + s_{cc}^O \kappa_0 + O\left(1/z\right) , \end{aligned} \tag{OS.91}$$

since $s_{cc}^O > s_{oc}^O$. Hence as $z \to \infty$, i.e., as $\beta_c \to 0$, the contributor is the quality leader. Therefore, the contributor's and the originator's profit functions equal to Curve IIc in (OS.55) and Curve Io defined in (OS.78). Therefore, substituting (OS.90) for $\kappa_0, \kappa_1, \kappa_2 \in \mathbb{R}$, into the first order conditions for Curve IIc for the contributor as given in (OS.64)-(OS.65) and those for Curve Io for the originator,[4] and collecting the terms in powers of z, we obtain the two first order conditions that need to be satisfied in equilibrium as polynomials of z. Since these two conditions have to be satisfied for all β_c values, their lead coefficients have to be identically zero. Thus, equating the lead coefficients to zero, we find that the following two equations in two unknowns have to be satisfied by κ_1 and κ_2:

$$\frac{4(s_{cc}^O - s_{oc}^O){s_{cc}^O}^2}{(4s_{cc}^O - s_{oc}^O)^2} - \kappa_1 = 0,$$

$$\begin{aligned} - \frac{3s_{cc}^O s_{oc}^O (s_{cc}^O - s_{oc}^O)(4s_{co}^O - s_{oo}^O)(4(s_{cc}^O)^2(s_{cc}^O - s_{oc}^O) - \kappa_1(4s_{cc}^O - s_{oc}^O)^2)}{4(4s_{cc}^O - s_{oc}^O)^3(3(s_{cc}^O)^2(s_{cc}^O - s_{oc}^O) - \kappa_1(4s_{cc}^O - s_{oc}^O)^2)} \\ + \frac{s_{oo}^O (s_{cc}^O)^2(4s_{cc}^O - 7s_{oc}^O) + (s_{oc}^O)^2 s_{co}^O (s_{oc}^O + 2s_{cc}^O)}{(4s_{cc}^O - s_{oc}^O)^3} - \kappa_2 \beta_o = 0 . \end{aligned} \tag{OS.92}$$

Solving the system (OS.92), we obtain

$$\kappa_1 = \frac{4(s_{cc}^O - s_{oc}^O){s_{cc}^O}^2}{(4s_{cc}^O - s_{oc}^O)^2} , \tag{OS.93}$$

$$\kappa_2 = \frac{s_{oo}^O (s_{cc}^O)^2(4s_{cc}^O - 7s_{oc}^O) + (s_{oc}^O)^2 s_{co}^O (s_{oc}^O + 2s_{cc}^O)}{(4s_{cc}^O - s_{oc}^O)^3 \beta_o} . \tag{OS.94}$$

Note that $\kappa_1 \geq 0$. If $\kappa_2 < 0$, it corresponds to the boundary solution of $e_o^{O*} = 0$. Hence,

$$\lim_{\beta_c \to 0} e_o^{O*} = \max\left(\frac{s_{oo}^O (s_{cc}^O)^2(4s_{cc}^O - 7s_{oc}^O) + (s_{oc}^O)^2 s_{co}^O (s_{oc}^O + 2s_{cc}^O)}{(4s_{cc}^O - s_{oc}^O)^3 \beta_o}, 0 \right) . \tag{OS.95}$$

Finally, substituting the equilibrium effort levels e_o^{O*} and e_c^{O*} into the equilibrium prices and consumer

[4] The first order condition expressions for the originator are very large and hence omitted here. These expressions are available from the authors upon request.

equilibrium outcomes presented in Proposition 5 and Lemma OS.2, respectively, and taking the limit, we then obtain the consumer market demand outcome in equilibrium. This completes the proof of Proposition 6.

Similarly, for Proposition 7, we start by determining the behavior of e_c^{O*} and e_o^{O*} as $\beta_c \to \infty$. Define q, $u \in \mathbb{R}$ as the values that satisfy

$$\lim_{\beta_c \to \infty} \frac{e_c^{O*}}{\beta_c^q} = K_3 \text{ and } \lim_{\beta_c \to \infty} \frac{e_o^{O*}}{\beta_c^u} = K_4, \tag{OS.96}$$

where $K_3, K_4 \in \mathbb{R}$ are constants. In other words, as $\beta_c \to \infty$, e_c^{O*} is in the order of β_c^q, and e_o^{O*} is in the order of β_c^u, or equivalently, $O(\beta_c^q)$ and $O(\beta_c^u)$ in standard notation respectively. We aim to determine the values of q and u in equilibrium, in order to tell the behavior of e_c^{O*} and e_o^{O*} as $\beta_c \to \infty$.

First, suppose that $q < u$. Looking at the first order condition of the contributor's profit with respect to e_c, in the two possible regions of the contributor's profit curve as given by (OS.55), first, in the first region, by (OS.57)-(OS.59), plugging in $e_c^{O*} = O(\beta_c^q)$ and $e_o^{O*} = O(\beta_c^u)$, and collecting the terms in powers of β_c, we find that the first order condition can be written as

$$E_5 \beta_c^{3u} + E_6 \beta_c^{3u+q+1} + Y_3(\beta_c) = 0 \tag{OS.97}$$

where $E_5, E_6 \in \mathbb{R}$ and $Y_3(\beta_c)$ is a polynomial in β_c, with terms in orders of β_c^{3q}, β_c^{2q+u}, β_c^{q+2u}, β_c^{4q+1}, β_c^{3q+u+1} and $\beta_c^{2(q+u)+1}$. Notice that as $\beta_c \to \infty$, the term in the order of β_c^{3q} in $Y_3(\beta_c)$ will be less than $E_5 \beta_c^{3u}$ in absolute value, and all the other terms in $Y_3(\beta_c)$ will be less than $E_6 \beta_c^{3u+q+1}$ in absolute value.

Now, as $\beta_c \to \infty$, (OS.97) has to hold for all β_c values. But this is not possible if $3u \neq 3u + q + 1$, because, in that case, the larger of $E_5 \beta_c^{3u}$ and $E_6 \beta_c^{3u+q+1}$ in absolute value will dominate all other terms for sufficiently large β_c and (OS.97) will never be satisfied. Therefore, in equilibrium $E_5 \beta_c^{3u}$ and $E_6 \beta_c^{3u+q+1}$ have to cancel each other, i.e., $3u = 3u + q + 1$ has to hold, which in turn implies that $q = -1$. Repeating the same analysis for the second region, this time using (OS.63)-(OS.65), we again find that the first order condition is equivalent to (OS.97) as described above, and hence it again follows that $q = -1$.

Second suppose $q > u$. In this case, again utilizing the first order condition for the first region as given in (OS.57)-(OS.59), we find that the first order condition can be written as

$$E_7 \beta_c^{3q} + E_8 \beta_c^{4q+1} + Y_4(\beta_c) = 0 \tag{OS.98}$$

for $E_7, E_8 \in \mathbb{R}$ and $Y_4(\beta_c)$ is a polynomial in β_c, with again terms in orders of β_c^{3u}, β_c^{2u+q}, β_c^{u+2q}, β_c^{u+3q+1}, $\beta_c^{2(u+q)-1}$ and β_c^{3u+q+1}. Notice in this case that as $\beta_c \to \infty$, the term in the order of β_c^{3u} in $Y_4(\beta_c)$ will be less than $E_7 \beta_c^{3q}$ and all the other terms in $Y_4(\beta_c)$ will be less than $E_8 \beta_c^{4q+1}$ in absolute value. Therefore, with the same argument as above, we conclude that $3q = 4q + 1$ must hold, which implies that $q = -1$. Once again repeating the same analysis for the second region using (OS.63)-(OS.65) and following identical steps, we find that if the global optimum e_c^{O*} is in this region, $q = -1$ follows.

In summary, we have so far shown that if $r \neq q$, then $q = -1$ holds, i.e., e_c^{O*} is in the order of $1/\beta_c$.

This means that we can write

$$e_c^{O*} = \kappa_3/\beta_c + O(1/\beta_c^2)\,, \tag{OS.99}$$

for constants $\kappa_3 \in \mathbb{R}$.

To determine the behavior of the originator's effort investment e_o^{O*}, note that by (OS.55) and (OS.78), $\breve{\Pi}_c^O$ and $\breve{\Pi}_o^O$ can take two possible functional forms each, depending on who is the quality leader, either the originator or the contributor. Following the same steps in part (i), calculating (OS.86) and substituting the functional form $e_c^{O*} = \kappa_3/\beta_c + O(1/\beta_c^2)$ into the first order conditions of e_o^O, which is obtained by taking the derivative of Π_o^O in (OS.77) and (OS.78) with respect to e_o^O, and that e_o^{O*} is in the order of β_c^u, we find that

$$K_2 - \beta_o e_o^{O*} = 0, \tag{OS.100}$$

for some $K_2 \in \mathbb{R}$ for both cases of $u > q$ and $u < q$. That is, for all cases, if $u \neq q$, then it follows that e_o^{O*} is a constant, which means that $u = 0$. Combining this with the finding that $q = -1$ holds for $u \neq q$, it follows that the only possible outcome for the case $u \neq q$ is $u > q$ and that $u = 0$ and $q = -1$, i.e., the originator becomes the quality leader in equilibrium.

Finally suppose $u = q$. Then plugging in e_c^{O*} and e_o^{O*} are both in the order of β_c^u and once again carrying out the calculations as above, we find that (OS.100) is satisfied, i.e., in this case $u = q = 0$ has to hold. Note however that e_c^{O*} has to satisfy the first order condition given in (OS.84) for one of the two curves given in (OS.55). In both curves, as $\beta_c \to \infty$, the contributor's first order condition (OS.84) becomes negative for $u = q = 0$, and hence, it cannot be satisfied as $\beta_c \to \infty$. In conclusion, in equilibrium, $u = q = 0$ cannot hold either and the only possible case is $u = 0$ and $q = -1$, that is,

$$e_c^{O*} = \kappa_3/\beta_c + O\left(1/\beta_c^2\right) \text{ and } e_o^{O*} = \kappa_4 + \kappa_5/\beta_c + O\left(1/\beta_c^2\right)\,, \tag{OS.101}$$

for $\kappa_3, \kappa_4, \kappa_5 \in \mathbb{R}$. Then, calculating the service qualities of the originator and the contributor respectively, we obtain

$$\begin{aligned} Q_o^O = s_{oo}^O e_o^{O*} + s_{oc}^O e_c^{O*} = s_{oo}^O \kappa_4 + (s_{oc}^O \kappa_3 + s_{oo}^O \kappa_5)/\beta_c + O\left(1/\beta_c^2\right) \\ \geq Q_c^O = s_{co}^O e_o^{O*} + s_{cc}^O e_c^{O*} = s_{co}^O \kappa_4 + (s_{cc}^O \kappa_3 + s_{co}^O \kappa_5)/\beta_c + O\left(1/\beta_c^2\right)\,, \end{aligned} \tag{OS.102}$$

since $s_{oo}^O > s_{co}^O$. Hence as $\beta_c \to \infty$, the originator is the quality leader. Therefore, the contributor's and the originator's profit functions equal to Curve Ic in (OS.55) and Curve IIo defined in (OS.78), respectively. Substituting (OS.101) for $\kappa_3, \kappa_4, \kappa_5 \in \mathbb{R}$, into the first order conditions for Curve IIc for the contributor as given in (OS.58)-(OS.59) and those for Curve IIo for the originator,[5] and collecting the terms in powers of β_c, we obtain the two first order conditions that need to be satisfied in equilibrium as polynomials of β_c. Because these two conditions have to be satisfied for all β_c values, their lead coefficients have to be identically zero. Thus, following the same steps in the proof of part (i) by equating the lead coefficients to

[5]The first order condition expressions for the originator are again very large and hence omitted. These expressions are also available from the authors upon request.

zero, we find that the following two equations in two unknowns have to be satisfied by κ_3 and κ_4:

$$\begin{aligned} \frac{(s_{oo}^O)^2 s_{cc}^O(4s_{oo}^O - 7s_{co}^O) + (s_{co}^O)^2 s_{oc}^O(2s_{oo}^O + s_{co}^O)}{(4s_{oo}^O - s_{co}^O)^3} - \kappa_3 &= 0, \\ \frac{4(s_{oo}^O)^2(s_{oo}^O - s_{co}^O)}{(4s_{oo}^O - s_{co}^O)^2} - \kappa_4\beta_o &= 0\,. \end{aligned} \tag{OS.103}$$

Solving the system (OS.103), we obtain

$$\kappa_3 = \frac{(s_{oo}^O)^2 s_{cc}^O(4s_{oo}^O - 7s_{co}^O) + (s_{co}^O)^2 s_{oc}^O(2s_{oo}^O + s_{co}^O)}{(4s_{oo}^O - s_{co}^O)^3}\,, \tag{OS.104}$$

$$\kappa_4 = \frac{4(s_{oo}^O)^2(s_{oo}^O - s_{co}^O)}{(4s_{oo}^O - s_{co}^O)^2\beta_o}\,. \tag{OS.105}$$

If $\kappa_3 < 0$, then it corresponds to the boundary solution of $e_c^{O*} = 0$. Therefore, we conclude that

$$\lim_{\beta_c \to \infty} e_o^{O*} = \frac{4(s_{oo}^O)^2(s_{oo}^O - s_{co}^O)}{(4s_{oo}^O - s_{co}^O)^2\beta_o}\,. \tag{OS.106}$$

Lastly, we replace these equilibrium effort levels e_o^{O*} and e_c^{O*} into the equilibrium prices and consumer equilibrium outcomes presented in Proposition 5 and Lemma OS.2, respectively, and take the limit of $\beta_c \to \infty$ to obtain the consumer market demand outcome in equilibrium, which completes the proof. ■

Proof of Proposition 8: Define

$$\bar{s} = \frac{18 s_{cc}^O(s_{cc}^O - s_{oc}^O)}{(4s_{cc}^O - s_{oc}^O)^2}\sqrt{s_{oc}^O s_{cc}^O}\,. \tag{OS.107}$$

Technically, we show that there exists $\underline{\beta} > \overline{\beta} > 0$ such that if $s_{cc}^P < \bar{s}$, and $s_{cc}^O > s_{oc}^O$, then

(i) If $\beta_c < \overline{\beta}$, an open source strategy is optimal for the originator. In equilibrium, the contributor is the quality leader.

(ii) If $\beta_c > \underline{\beta}$ then a proprietary strategy is optimal for the originator. In this case, the originator chooses to push the contributor out of the market and serves as a monopolist for services as well as the product.

We consider the cases as $\beta_c \to 0$ (part (i)) and $\beta_c \to \infty$ (part (ii)). We have already calculated the equilibrium effort levels for the originator and the contributor for these cases for open source strategy, in the proof of Propositions 6 and 7. In this proof, we will utilize these findings in order to calculate equilibrium originator profit levels for the open source strategy and compare them to the corresponding profits under the proprietary strategy to determine the originator's best software strategy choice.

For part (i), to calculate the originator profits under the open source strategy, plugging (OS.93) and (OS.94) into (OS.90), substituting these in turn into $\Pi_o^O(\cdot)$ as given in (OS.77) with $\breve{\Pi}_o^O(\cdot)$ corresponding to Curve Io as defined in (OS.78) and collecting the terms of z, or equivalently $1/\beta_c$, we obtain the equilibrium

profit for the originator for the open source strategy as

$$\Pi_o^O(e_o^{O*}) = \frac{4s_{oc}^O(s_{cc}^O)^3(s_{cc}^O - s_{oc}^O)^2}{\beta_c(4s_{cc}^O - s_{oc}^O)^4} + \frac{(s_{oo}^O(s_{cc}^O)^2(4s_{cc}^O - 7s_{oc}^O) + (s_{oc}^O)^2 s_{co}^O(s_{oc}^O + 2s_{cc}^O))^2}{2\beta_o(4s_{cc}^O - s_{oc}^O)^6} + O(\beta_c)\,. \quad \text{(OS.108)}$$

For the proprietary strategy on the other hand, plugging the optimal effort expressions given in Propositions 2 and 3 into the originator's profit expression in (OS.20), we obtain the originator's optimal profit in equilibrium as

$$\Pi_o^{P*} = \begin{cases} \frac{(5s_{oo}^P + 4s_{co}^P)^2}{2592\beta_o} + \frac{(s_{cc}^P)^2}{81\beta_c}, & \text{if } \beta_c < \bar{r}\,, \\ \frac{(s_{oo}^P)^2}{32\beta_o}, & \text{if } \beta_c \geq \bar{r}\,; \end{cases} \quad \text{(OS.109)}$$

where $\bar{r}$ is as defined in (OS.21). Consequently, for sufficiently small β_c, by Proposition 3 and (OS.109), we have $e_o^{P*} = \frac{4s_{co}^P + 5s_{oo}^P}{36\beta_o}$ and $\Pi_o^P(e_o^{P*}) = \frac{(s_{cc}^P)^2}{81\beta_c} + \frac{(4s_{co}^P + 5s_{oo}^P)^2}{2592\beta_o}$. As a result, for sufficiently small β_c, $\Pi_o^{O*} > \Pi_o^{P*}$ if and only if the coefficient of $1/\beta_c$ in the originator's profit expression for the open-source case is larger than the corresponding coefficient for the proprietary case, i.e.,

$$\Pi_o^O(e_o^{O*}) = \frac{4s_{oc}^O(s_{cc}^O)^3(s_{cc}^O - s_{oc}^O)^2}{(4s_{cc}^O - s_{oc}^O)^4} > \Pi_o^P(e_o^{P*}) = \frac{(s_{cc}^P)^2}{81}. \quad \text{(OS.110)}$$

Simplifying (OS.110), we obtain the desired condition as

$$s_{cc}^P < \frac{18s_{cc}^O(s_{cc}^O - s_{oc}^O)}{(4s_{cc}^O - s_{oc}^O)^2}\sqrt{s_{oc}^O s_{cc}^O}\,. \quad \text{(OS.111)}$$

This completes the proof of part (i).

Similarly, for part (ii), plugging (OS.104) and (OS.105) into (OS.101), substituting these in turn into $\Pi_o^O(\cdot)$ as given in (OS.77) with $\breve{\Pi}_o^O(\cdot)$ corresponding to Curve IIo as defined in (OS.78) and collecting the terms of β_c, we obtain the equilibrium profit for the originator for the open source strategy as

$$\Pi_o^O(e_o^{O*}) = \frac{8(s_{oo}^O)^4(s_{oo}^O - s_{co}^O)^2}{(4s_{oo}^O - s_{co}^O)^4\beta_o} + O(1/\beta_c)\,. \quad \text{(OS.112)}$$

For the proprietary strategy on the other hand, from Propositions 2 and 3, and equation (OS.109), we have $e_o^{P*} = s_{oo}^P/(4\beta_o)$ and $\Pi_o^{P*} = (s_{oo}^P)^2/(32\beta_o)$. Comparing these two profit expressions, since $s_{oo}^P = s_{oo}^O > s_{co}^O$, for sufficiently large β_c, it follows that

$$\begin{aligned} \Pi_o^{P*} - \Pi_o^{O*} &= \frac{(s_{oo}^P)^2}{32\beta_o} - \left(\frac{8(s_{oo}^O)^4(s_{oo}^O - s_{co}^O)^2}{(4s_{oo}^O - s_{co}^O)^4\beta_o} + O(1/\beta_c)\right) \\ &= \frac{(s_{oo}^O)^2}{32(4s_{oo}^O - s_{co}^O)^4\beta_o}\left(16s_{oo}^O s_{co}^O\left(10s_{oo}^O(s_{oo}^O - s_{co}^O) + 6(s_{oo}^O)^2 - (s_{co}^O)^2\right) + (s_{co}^O)^4\right) - O(1/\beta_c) > 0\,. \quad \text{(OS.113)} \end{aligned}$$

This completes the proof. ■

Proof of Proposition 9: Technically, we show that there exist bounds $\overline{\beta_c}$, $\overline{s}_{oc} > 0$ such that if $s_{oc}^O < \overline{s}_{oc}$ and $\beta_c < \overline{\beta_c}$, then

(i) p_o^{O*} strictly decreases in β_o, while p_c^{O*} strictly increases in β_o if $s_{oo}^O \geq (4/3)s_{co}^O$ and strictly decreases

otherwise.

(ii) Π_o^O strictly decreases in β_o, while Π_c^O strictly increases in β_o if $s_{oo}^O \geq 2s_{co}^O$ and strictly decreases otherwise.

First, as we have shown in the proof of part (i) of Proposition 8, as $\beta_c \to 0$, in equilibrium the contributor is the quality leader. From Proposition 5, by substituting the symmetric coefficients in the price expression given in equation (OS.32) for the case in which originator is the quality leader, we find that the originator's equilibrium service price is

$$p_o^{O*} = \frac{2(s_{cc}^O e_c + s_{co}^O e_o)((s_{cc}^O - s_{oc}^O)e_c + (s_{co}^O - s_{oo}^O)e_o)}{(4s_{cc}^O - s_{oc}^O)e_c + (4s_{co}^O - s_{oo}^O)e_o}\,. \tag{OS.114}$$

Now plugging (OS.90), (OS.93) and (OS.94) in (OS.114), we obtain

$$p_o^{O*} = \frac{4s_{oc}^O(s_{cc}^O)^2(s_{oc}^O - s_{cc}^O)^2}{(4s_{cc}^O - s_{oc}^O)^3\beta_c} + \frac{\left((s_{oc}^O)^2(s_{oo}^O + 3s_{co}^O) + 4s_{oo}^O s_{cc}^O(s_{cc}^O - 2s_{oc}^O)\right)\lambda_1}{\beta_o(4s_{cc}^O - s_{oc}^O)^5} + O\left(\beta_c\right)\,, \tag{OS.115}$$

where $\lambda_1 = (s_{oc}^O)^2 s_{co}^O(2s_{cc}^O + s_{oc}^O) + 4s_{oo}^O(s_{cc}^O)^2(4s_{cc}^O - 7s_{oc}^O)$. Differentiating (OS.115) with respect to β_o and taking the limit as $s_{oc}^O \to 0$, we obtain

$$\lim_{s_{oc}^O \to 0} \frac{dp_o^{O*}}{d\beta_o} = \lim_{s_{oc}^O \to 0} -\frac{\left((s_{oc}^O)^2(s_{oo}^O + 3s_{co}^O) + 4s_{oo}^O s_{cc}^O(s_{cc}^O - 2s_{oc}^O)\right)\lambda_1}{\beta_o^2(4s_{cc}^O - s_{oc}^O)^5} + O(\beta_c) = -\frac{(s_{oo}^O)^2}{64\beta_o^2} + O(\beta_c) < 0\,, \tag{OS.116}$$

as $\beta_c \to 0$. That is, when β_c and s_{oc}^O are sufficiently small, p_o^{O*} decreases in β_o, i.e., there exist $\overline{\beta_c}, \overline{s}_{oc} > 0$ such that if $\beta_c < \overline{\beta_c}$ and $s_{oc}^O < \overline{s}_{oc}$, then p_o^{O*} decreases in β_o.

Similarly, for p_c^{O*}, again by substituting the symmetric coefficients in the price expression given in equation (OS.33) in Proposition 5, we find that the contributor's equilibrium service price is

$$p_c^{O*} = \frac{(s_{oc}^O e_c + s_{oo}^O e_o)((s_{cc}^O - s_{oc}^O)e_c + (s_{co}^O - s_{oo}^O)e_o)}{(4s_{cc}^O - s_{oc}^O)e_c + (4s_{co}^O - s_{oo}^O)e_o}\,. \tag{OS.117}$$

Again plugging in (OS.90), (OS.93) and (OS.94),

$$p_c^{O*} = \frac{8(s_{cc}^O)^3(s_{cc}^O - s_{oc}^O)^2}{(4s_{cc}^O - s_{oc}^O)^3\beta_c} - \frac{2\left(s_{oc}^O s_{co}^O(2s_{cc}^O - s_{oc}^O) + (s_{cc}^O)^2(3s_{oo}^O - 4s_{co}^O)\right)\lambda_2}{\beta_o(4s_{cc}^O - s_{oc}^O)^5} + O\left(\beta_c\right)\,, \tag{OS.118}$$

where $\lambda_2 = s_{oo}^O(s_{cc}^O)^2(4s_{cc}^O - 7s_{oc}^O) + (s_{oc}^O)^2 s_{co}^O(s_{oc}^O + 2s_{cc}^O)$. Differentiating (OS.118) with respect to β_o and again taking the limit as $s_{oc}^O \to 0$, we obtain

$$\lim_{s_{oc}^O \to 0} \frac{dp_c^{O*}}{d\beta_o} = \lim_{s_{oc}^O \to 0} \frac{2\left(s_{oc}^O s_{co}^O(2s_{cc}^O - s_{oc}^O) + (s_{cc}^O)^2(3s_{oo}^O - 4s_{co}^O)\right)\lambda_2}{\beta_o^2(4s_{cc}^O - s_{oc}^O)^5} + O(\beta_c) = \frac{s_{oo}^O(3s_{oo}^O - 4s_{co}^O)}{128\beta_o^2} + O(\beta_c)\,. \tag{OS.119}$$

It follows that, for sufficiently small β_c and s_{oc}^O, p_c^{O*} is strictly increasing in β_o if and only if $s_{oo}^O \geq (4/3)s_{co}^O$. This completes the proof of part (i).

For part (ii), taking the derivative of $\Pi_o^O(e_o^{O*}, e_c^{O*})$ given in (OS.108) with respect to β_o and taking the limit as $s_{oc}^O \to 0$, we obtain

$$\lim_{s_{oc}^O \to 0} \frac{d\Pi_o^O(e_o^{O*}, e_c^{O*})}{d\beta_o} = \lim_{s_{oc}^O \to 0} -\frac{(s_{oo}^O(s_{cc}^O)^2(s_{cc}^O - 7s_{oc}^O) + (s_{oc}^O)^2 s_{co}^O(s_{oc}^O + s_{cc}^O))^2}{2\beta_o^2(4s_{cc}^O - s_{oc}^O)^6} + O(\beta_c) = -\frac{(s_{oo}^O)^2}{512\beta_o^2} + O(\beta_c) < 0\,. \tag{OS.120}$$

Consequently, $\Pi_o^O(e_o^{O*}, e_c^{O*})$ is strictly decreasing in β_o for sufficiently small β_c and s_{oc}^O.

Finally, substituting (OS.90), (OS.93), (OS.94), (OS.115), and (OS.118) into (OS.54), and carrying out the algebra, we obtain

$$\Pi_c^O(e_o^{O*}, e_c^{O*}) = \frac{8(s_{cc}^O)^4(s_{cc}^O - s_{oc}^O)^2}{(4s_{cc}^O - s_{oc}^O)^4\beta_c} - \frac{4s_{cc}^O\left((s_{oo}^O + 3s_{co}^O)s_{oc}^O s_{cc}^O + 2(s_{oo}^O - 2s_{co}^O)(s_{cc}^O)^2 - 2(s_{oc}^O)^2 s_{co}^O\right)\lambda_2}{\beta_o(4s_{cc}^O - s_{oc}^O)^6} + O\left(\beta_c\right)\,. \tag{OS.121}$$

Once again, differentiating (OS.121) with respect to β_o, and taking the limit as $s_{oc}^O \to 0$, we find that

$$\begin{aligned}\lim_{s_{oc}^O \to 0} \frac{d\Pi_c^O(e_o^{O*}, e_c^{O*})}{d\beta_o} &= \lim_{s_{oc}^O \to 0} \frac{4s_{cc}^O\left((s_{oo}^O + 3s_{co}^O)s_{oc}^O s_{cc}^O + 2(s_{oo}^O - 2s_{co}^O)(s_{cc}^O)^2 - 2(s_{oc}^O)^2 s_{co}^O\right)\lambda_2}{\beta_o^2(4s_{cc}^O - s_{oc}^O)^6} + O(\beta_c) \\ &= \frac{s_{oo}^O(s_{oo}^O - 2s_{co}^O)}{128\beta_o^2} + O(\beta_c)\,. \end{aligned} \tag{OS.122}$$

Hence, for small enough s_{oc}^O and β_c, $\Pi_c^O(e_o^{O*},\ e_c^{O*})$ is strictly increasing in β_o if and only if $s_{oo}^O \geq 2s_{co}^O$. This completes the proof. ■

Proof of Proposition 10: For any $a > 1$, define

$$\bar{h} = \frac{9(s_{oc}^O)^{1/2}(s_{cc}^O)^{3/2}(s_{cc}^O - s_{oc}^O)}{(4s_{cc}^O - s_{oc}^O)^2}\left(\sqrt{\frac{3s_{cc}^O - s_{oc}^O}{a(s_{cc}^O - s_{oc}^O)}} - 2\right), \tag{OS.123}$$

and let $\bar{s}$ be given as in (OS.107). Technically, we show that there exists $\bar{b} > 0$ such that for all $a > 1$, if $b < \bar{b}$, $s_{oc}^O < s_{cc}^O < \frac{4a-1}{4a-3}s_{oc}^O$, and $0 < s_l < \bar{s} < s_h < \bar{s} + \bar{h}$, then

$$W^{\rho^*}\Big|_{\beta_c = b,\ s_{cc}^P = s_h} < W^{\rho^*}\Big|_{\beta_c = a\cdot b,\ s_{cc}^P = s_l}\,. \tag{OS.124}$$

We start by establishing the originator's optimal licensing strategy for the two parameter combinations, $(\beta_c,\ s_{cc}^P)$ given in the statement of the proposition. First, as given in equation (OS.111) in the proof of Proposition 8, for sufficiently small β_c, the originator's optimal licensing strategy is open source ($\rho^* = O$) if $s_{cc}^P < \bar{s}$, where $\bar{s}$ is as defined in (OS.107), and proprietary ($\rho^* = P$) otherwise. Therefore for all $a > 1$, for sufficiently small $b > 0$, any $s_h > \bar{s}$ and $0 < s_l < \bar{s}$, the policy choice is $\rho^* = P$ if $\beta_c = b$ and $s_{cc}^P = s_h$; while the policy choice is $\rho^* = O$ if $\beta_c = a \cdot b$ and $s_{cc}^P = s_l$.

We next have to calculate and compare the social welfare under these two parameter combinations. First, for sufficiently small $b > 0$, $\beta_c = b$ and $s_{cc}^P = s_h$, $\rho^* = P$. Then, by Proposition 3, $\theta_H = 2/3$ and $\theta_L = 1/2$. By substituting $\beta_c = b$ and $s_{cc}^P = s_h$ into the equilibrium expressions given in the proofs of

Propositions 3 and 4,

$$e_o^{P*} = \frac{5s_{oo}^P + 4s_{co}^P}{36\beta_o}\,, \quad e_c^{P*} = \frac{s_h}{9\beta_c}, \tag{OS.125}$$

$$Q_o^P = \frac{s_{oo}^P(5s_{oo}^P + 4s_{co}^P)}{36\beta_o}\,, \quad Q_c^P = \frac{s_{co}^P(5s_{oo}^P + 4s_{co}^P)}{36\beta_o} + \frac{s_h^2}{9b}\,, \tag{OS.126}$$

$$p^{P*} = \frac{1}{216}\left(\frac{(s_{oo}^P + 2s_{co}^P)(5s_{oo}^P + 4s_{co}^P)}{\beta_o} + \frac{8s_h^2}{b}\right), \tag{OS.127}$$

$$p_o^{P*} = \frac{1}{108}\left(\frac{(s_{oo}^P - s_{co}^P)(5s_{oo}^P + 4s_{co}^P)}{\beta_o} - \frac{4s_h^2}{b}\right), \text{ and } p_c^{P*} = \frac{1}{108}\left(\frac{(s_{co}^P - s_{oo}^P)(5s_{oo}^P + 4s_{co}^P)}{\beta_o} + \frac{4s_h^2}{b}\right). \tag{OS.128}$$

Further, in this case, again by Proposition 3, the contributor is the quality leader, and hence $Q_H = Q_c^P$, $Q_L = Q_o^P$, $p_H = p_c^{P*}$ and $p_L = p_o^{P*}$. Therefore substituting these into the social welfare expression given by (3) and (4), defining $h = s_h - \bar{s}$, and simplifying, we obtain

$$\lim_{b\to 0} b \cdot W^{P*} = \frac{8s_{oc}^O(s_{cc}^O)^3(s_{cc}^O - s_{oc}^O)^2}{(4s_{cc}^O - s_{oc}^O)^4} + \frac{8hs_{cc}^O(s_{cc}^O - s_{oc}^O)}{9(4s_{cc}^O - s_{oc}^O)^2}\sqrt{s_{oc}^O s_{cc}^O} + \frac{2h^2}{81}\,. \tag{OS.129}$$

Second, for the case with $a > 1$, sufficiently small $b > 0$, $\beta_c = a \cdot b$ and $s_{cc}^P = s_l$, for $s_l < \bar{s}$, as we have stated above $\rho^* = O$ and hence, since $s_{oc}^O < s_{cc}^O$, by Proposition 6, again the contributor is the quality leader, i.e., $Q_H = Q_c^O$, $Q_L = Q_o^O$, $p_H = p_c^{O*}$ and $p_L = p_o^{O*}$. Now, from the proof of part (i) of Proposition 8, when $\beta_c = a \cdot b$ is small, plugging (OS.93) and (OS.94) into (OS.90), we obtain

$$e_c^{O*} = \kappa_0 + \frac{4(s_{cc}^O - s_{oc}^O){s_{cc}^O}^2}{a \cdot b(4s_{cc}^O - s_{oc}^O)^2} + O(b) \text{ and } e_o^{O*} = \frac{s_{oo}^O(s_{cc}^O)^2(4s_{cc}^O - 7s_{oc}^O) + (s_{oc}^O)^2 s_{co}^O(s_{oc}^O + 2s_{cc}^O)}{(4s_{cc}^O - s_{oc}^O)^3\beta_o} + O(b)\,. \tag{OS.130}$$

Substituting these equilibrium effort levels in turn into $Q_H = Q_c^O = s_{co}^O e_o^{O*} + s_{cc}^O e_c^{O*}$ and $Q_L = Q_o^O = s_{oo}^O e_o^{O*} + s_{oc}^O e_c^{O*}$, and into $p_H = p_c^{O*}$ and $p_L = p_o^{O*}$ as given in Proposition 5, collecting the terms of b and simplifying, we obtain:

$$\lim_{b\to 0} b \cdot Q_c^O = \frac{4(s_{cc}^O - s_{oc}^O)(s_{cc}^O)^3}{a(4s_{cc}^O - s_{oc}^O)^2}\,, \tag{OS.131}$$

$$\lim_{b\to 0} b \cdot Q_o^O = \frac{4s_{oc}^O(s_{cc}^O - s_{oc}^O)(s_{cc}^O)^2}{a(4s_{cc}^O - s_{oc}^O)^2}\,, \tag{OS.132}$$

$$\lim_{b\to 0} b \cdot p_c^O = \frac{8(s_{cc}^O - s_{oc}^O)^2(s_{cc}^O)^3}{a(4s_{cc}^O - s_{oc}^O)^3}\,, \tag{OS.133}$$

and

$$\lim_{b\to 0} b \cdot p_o^O = \frac{4s_{oc}^O(s_{cc}^O - s_{oc}^O)^2(s_{cc}^O)^2}{a(4s_{cc}^O - s_{oc}^O)^3}\,. \tag{OS.134}$$

Finally, from Proposition 6, we also obtain

$$\lim_{b\to 0} \theta_H = \frac{2s_{cc}^O - s_{oc}^O}{4s_{cc}^O - s_{oc}^O}\,, \tag{OS.135}$$

and

$$\lim_{b\to 0} \theta_L = \frac{s_{cc}^O - s_{oc}^O}{4s_{cc}^O - s_{oc}^O}\,. \tag{OS.136}$$

Again substituting (OS.130)-(OS.136) into the social welfare expression given by (3) and (4), and simplifying, we obtain

$$\lim_{b\to 0} b \cdot W^{O*} = \frac{2s_{oc}^O (s_{cc}^O)^3 (3s_{cc}^O - s_{oc}^O)(s_{cc}^O - s_{oc}^O)}{a(4s_{cc}^O - s_{oc}^O)^4}\,. \tag{OS.137}$$

Now we can compare the two welfare levels. By subtracting (OS.129) from (OS.137), and simplifying, it follows that for any $a > 1$, $0 < s_l < \bar{s}$, and $h > 0$,

$$\begin{aligned}\lim_{b\to 0} b \cdot \left(W^{\rho^*}\Big|_{\beta_c = a\cdot b,\ s_{cc}^P = s} - W^{\rho^*}\Big|_{\beta_c = b,\ s_{cc}^P = \bar{s}+h} \right) \\ = \frac{2s_{oc}^O (s_{cc}^O)^3 (s_{cc}^O - s_{oc}^O)((4a-1)s_{oc}^O - (4a-3)s_{cc}^O)}{a(4s_{cc}^O - s_{oc}^O)^4} - \frac{8hs_{cc}^O (s_{cc}^O - s_{oc}^O)}{9(4s_{cc}^O - s_{oc}^O)^2}\sqrt{s_{oc}^O s_{cc}^O} - \frac{2h^2}{81}\,.\end{aligned} \tag{OS.138}$$

(OS.138) is a quadratic function in h with a negative lead coefficient. Therefore, it will be positive for some $h > 0$ values if and only if its constant is positive, which is satisfied if and only if $s_{cc}^O < \frac{4a-1}{4a-3} s_{oc}^O$. If this condition is satisfied, then (OS.138) has a unique strictly positive root $\bar{h}$ as given in (OS.123) and is positive for all $0 < h < \bar{h}$, i.e., (since $h = s_h - \bar{s}$) for $\bar{s} < s_h < \bar{s} + h$. Hence, for all $a > 1$, there exists a $\bar{b} > 0$ such that when $0 < b < \bar{b}$, and $0 < s_l < \bar{s} < s_h < \bar{s} + \bar{h}$, $W^{\rho^*}\big|_{\beta_c = a\cdot b,\ s_{cc}^P = s_l} > W^{\rho^*}\big|_{\beta_c = b,\ s_{cc}^P = s_h}$, i.e., welfare is higher with a weak contributor than a strong contributor. This completes the proof. ■